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The economic benefits of market timing the style allocation of characteristic-based portfolios

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Many exchange traded funds track simple characteristic-based equity portfolios such as the market capitalization, the fundamental value or the inverse volatility portfolio. This paper provides theoretical and empirical evidence for the economic benefits in exploiting the timing-gains that result from the time-varying relative performance of these characteristic-based portfolios. Under a factor model for expected returns, we show that this dynamic portfolio allocation can be efficient across the low-dimensional set of characteristic-based portfolios. We assess the out-of-sample performance on the S&P 100 universe over the period 1990–2013 and show gains in stability and significant positive risk-adjusted returns for the dynamic style portfolio. We conduct several robustness tests and extensions confirming the benefits of dynamic style allocation across characteristic-based portfolios.

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1. Introduction

Skepticism about the net value of active portfolio management has spurred the increasing popularity of exchange traded funds (ETFs), which track the performance of simple characteristic-based portfolios. Often, portfolio weights are set to normalized versions of the individual stock characteristics. The best known is the market capitalization portfolio but over the past few years, alternative characteristic-based portfolios have become increasingly popular in the investment market (Bloomberg, 2014; Flood, 2013). Examples include fundamental value portfolios where the stocks' weights are a function of accounting measures such as revenues and dividends, and low risk portfolios where the weights are inversely related to the stock's volatility. In this paper, we propose and evaluate dynamic style investment strategies invested in these characteristic-based portfolios to exploit the time-varying differences in relative performance.

The proposed dynamic style portfolios aim to achieve mean-variance efficiency by not investing directly in the underlying stocks, but in the characteristic-based portfolios. This approach of dynamic style investing instead of optimizing the portfolio over all assets assumes that, at least one of the characteristic-based portfolio is (close to being) mean-variance efficient. This assumption matches with the common view that, while they do not explicitly require a return forecast, the portfolio weights based on these stock characteristics are good proxies for a mean-variance efficient portfolio in the long run.[1] However, it is unlikely that one characteristic will always lead to a mean-variance efficient portfolio. We argue that it is more likely that there is time-variation in the mean-variance efficient choice of the characteristic used for weighting the equity portfolio. This intuition is supported by ample empirical and theoretical evidence of the time-variation in the relative performance of characteristic-based portfolios (see, e.g., Amenc, Goltz, & Le Sourd, 2009; Barberis & Shleifer, 2003; Chen & De Bondt, 2004; Lucas, van Dijk, & Kloek, 2002). Thus, this time-variation creates the opportunity to exploit the differences in performance by market timing the style allocation to the underlying characteristic-based portfolios. In fact, low risk stocks usually have a beta lower than unity, and thus they tend to outperform the market capitalization portfolio in down markets and underperform during a market rally (Ang, Hodrick, Xing, & Zhang, 2006; Baker, Bradley, & Wurgler, 2011). Fundamental value portfolios tend to underperform the market capitalization portfolio in the run-up to a speculative bubble, such as the high-tech bubble (Hsu & Campollo, 2006). DeMiguel et al. (2009) show that optimization-based portfolios outperform an equally weighted portfolio when assets exhibit high idiosyncratic volatility. Thus, intuitively, it seems unlikely that one characteristic is sufficient to construct an optimal portfolio and if this were the case, that this characteristic is always the same over time.

In the proposed dynamic style portfolios, we model the mean-variance efficient weights as a linear combination of a set of characteristic-based portfolio weights. We then find the weights that are efficient under this restriction. The dynamic feature of the portfolio is crucial for allowing investors to benefit from the market timing opportunity associated with the life-cycle specificity of each characteristic-based portfolio. The time-varying weights placed on each of the proxies provide information about the expected individual performance of the characteristic-based portfolios. We show that the optimality of this portfolio is consistent with both the factor model of Haugen and Baker (1996), which predicts a time-varying payoff related to the different stock characteristics, and the widespread evidence of time-variation in the conditional distribution of asset returns. Our optimality result is also related to the multi-factor extensions of the CAPM model where firm characteristics are seen as drivers of stock returns (see, e.g., Hjalmarsson & Manchev, 2012 and the references therein).

The *mean-variance dynamic style* (hereafter MVDS) portfolio allocation methodology is thus a rule-based investment strategy aiming at exploiting the market timing opportunity associated with the time-variation in relative performance. An important feature of its design is that it can be applied

[1] For the market capitalization portfolio, this intuition is related to the well-known result that under the Capital Asset Pricing Model (CAPM), the market capitalization portfolio invested in all assets (not only equities) is the maximum Sharpe ratio portfolio. See Arnott, Hsu, and Moore (2005) and Treynor (2005) for studies of the optimality of the fundamental value portfolio, Haugen and Baker (1991) and Baker and Haugen (2012) for low risk stocks, and DeMiguel, Garlappi, and Uppal (2009) and Windcliff and Boyle (2004) for the equally weighted portfolio.

directly in practice by ETF investors.[2] In fact, the basic feature of our analysis is the value of the stock's characteristic normalized using the actual standardization procedure employed by ETFs, i.e., the (positively truncated) value of the characteristic divided by the total value of that characteristic over the investment universe. This contrasts with the use of z-score standardization in Brandt, Santa-Clara, and Valkanov (2009) and rank standardization in Hjalmarsson and Manchev (2012). The latter may have some theoretical appeal, but no investable product with this design is available at present. We also note that an important advantage of our approach is that the portfolio weights given by the linear combination of the standardized characteristics are guaranteed to be positive and to sum up to unity.[3]

The actual success of the MVDS portfolio in exploiting the life-cycle dependency is an empirical question, which we test on the S&P 100 investment universe over the period 1990–2013. We assess the performance and risk of the portfolios on multiple levels including risk-adjusted performance measures, sector biases, factor model alphas and downside risk measures. We show that the absolute outperformance of the MVDS portfolios is similar to the best performing characteristic-based portfolio. Moreover, the dynamic allocation approach has the advantage of addressing the selection problem in a purely data-driven framework. We also examine the economic benefits of style allocation across the characteristic-based portfolios rather than optimizing the mean-variance allocation at the stock level. The former is clearly more restrictive. Thus, there should be an in-sample loss, but, out-of-sample, the mean-variance allocation approach must address the challenging issue of estimating the expected return and covariance matrix. Under our setup, we safeguard our analysis against large estimation errors by requiring that the portfolio weights are a linear combination of the K characteristic-based portfolios ($K \ll N$).[4] For the S&P 100 universe, we show that there is a substantial improvement in the out-of-sample portfolio performance when an investor allocates across the characteristic-based portfolios rather than across the stocks.[5] This result is shown to be robust to the use of alternative diversification-based allocation approaches, the inclusion of a turnover constraint, adding a risk-free investment to the investment universe, and to the choice of covariance estimator and estimation window.

The remainder of the paper is organized as follows. In Section 2, we present the modeling framework. We then describe the method used to construct the dynamic mean-variance optimized style portfolio and we show that the proposed dynamic style portfolio can be mean-variance efficient under the factor model of Haugen and Baker (1996). In Section 3, we describe the data, review previous studies of characteristic-based portfolios, and discuss the estimation approaches. The out-of-sample performance results are presented in Sections 4 and 5. In Section 6, we give our conclusions.

2. Characteristic-based portfolios and dynamic style allocation

We consider an investor who can decide to invest either directly in the N stocks belonging to his investment universe, or indirectly by investing in K ETF-mimicking portfolios.[6] Each of the

[2] We focus on the economic value of style allocation for the individual investor, but the method should also be of interest to multi-style index developers, ETFs tracking multi-style indices, and multi-style equity funds.

[3] The base portfolio in Brandt et al. (2009) allows for short positions, but for long-only portfolios they recommend the ex-post truncation of the portfolio weights. The implementation of this approach requires direct investment in stocks whereas the style allocation portfolios that we consider are invested in a lower dimensional set of ETF-mimicking portfolios.

[4] An alternative for reducing the estimation error in the portfolio weights is to impose bound constraints on the portfolio weights (see, e.g., Jagannathan & Ma, 2003), to construct portfolios based on principal components analysis (see, e.g., Clarke, De Silva, & Thorley, 2006), and/or to impose a factor model structure when estimating the mean and covariance matrix of the stock returns. We tested the robustness of our findings using this alternative approach for estimating the (co)moments of stock returns.

[5] In an independent research, Fletcher (2016) obtains a similar result when testing the out-of-sample performance of the characteristic-based portfolio allocation of Brandt et al. (2009). Unlike Brandt et al. (2009) and Fletcher (2016), who directly optimized the portfolio weights invested in each stock, we employ a two-step approach where the single characteristic-based portfolios are computed first and then the optimal style allocation is determined across these portfolios. This two-step approach mimics the behavior of an investor in ETFs.

[6] Note that the ETF-mimicking portfolios represent physical ETFs, which take real positions in the equity market.

Table 1
Examples of commercially available style indices and ETFs that track the style index when the characteristic-based index is market capitalization, fundamental value, risk-based, or equally weighted.

Style	Index	ETF
Market capitalization	S&P 100	iShares S&P 100 ETF
	FTSE Developed Europe Index	Vanguard FTSE Europe ETF
Fundamental value	FTSE RAFI US 1000	PowerShares FTSE RAFI US 1000 ETF
	FTSE RAFI Europe	Lyxor FTSE RAFI Europe ETF
Risk-based	S&P 500 Low Volatility	PowerShares S&P 500 Low Volatility ETF
	MSCI Europe Risk Weighted	First Asset MSCI Europe Low Risk Weighted ETF
Equally weighted	S&P 100 Equal Weight Index	Guggenheim S&P 100 Equal Weight ETF
	Stoxx Europe 600	Ossiam Stoxx Europe 600 Equal Weight NR

ETF-mimicking portfolios corresponds to a stock characteristic, and thus we refer to them as K *characteristic-based* portfolios. In the first case, the decision variable is a $(N \times 1)$ vector of weights invested in the stocks in the universe, whereas in the second case, the decision variable is a $(K \times 1)$ vector of weights invested in the characteristic-based portfolios. Portfolios are rebalanced on discrete dates. On each rebalancing date t, the next rebalancing date $t+1$ is known. The $(N \times 1)$ vector of stocks' arithmetic (nominal) return between the rebalancing dates t and $t+1$ is a random variable denoted by $\mathbf{r}_{t+1} \equiv (r_{1,t+1}, \ldots, r_{N,t+1})'$. Similarly, $\widetilde{\mathbf{r}}_{t+1} \equiv (\widetilde{r}_{1,t+1}, \ldots, \widetilde{r}_{K,t+1})'$ is the $(K \times 1)$ vector of random variables corresponding to the characteristic-based portfolios' arithmetic (nominal) return.

In the modern portfolio theory described by Markowitz (1952), the investor finds, for each rebalancing date t, the portfolio that assigns the weight vector to the N stocks in the universe in order to optimize his mean-variance utility function. The input parameters are thus the risk aversion parameter γ, $0<\gamma<\infty$, the $(N \times 1)$ vector of expected returns $\boldsymbol{\mu}_{t+1|t}$ and the $(N \times N)$ covariance matrix $\boldsymbol{\Sigma}_{t+1|t}$ of the vector of next-period arithmetic (nominal) returns $\mathbf{r}_{t+1}$, both computed conditionally on the information set up to time t.

In contrast, under the characteristic-based portfolio allocation approach, the weights are not directly optimized but set as an explicit function of the stock characteristic. In this section, we first review the definition of characteristic-based portfolios. We then explore the conditions they need to fulfill in order to be mean-variance efficient. Next we introduce the dynamic style allocation portfolio in which the optimized portfolio is a linear combination of characteristic-based portfolios. We derive conditions under which this dynamic style allocation portfolio is efficient when the commonality in the stock returns is driven by an underlying factor model.

2.1. Definition of characteristic-based portfolios

The weights of the characteristic-based portfolios mentioned in Table 1 are all set equal to a normalized version of the stock's characteristic. We employ this approach throughout this study. In particular, we denote the generic characteristic k of stock i based on the information set up to time t as $z_{k,i,t}$. In a portfolio based on characteristic k for a universe of N stocks, the portfolio weight assigned to stock i for the one-period investment horizon starting at time t is:

$$x_{k,i,t} \equiv \frac{\max\{0, z_{k,i,t}\}}{\sum_{i=1}^{N} \max\{0, z_{k,i,t}\}} \,. \tag{1}$$

The lower truncation at zero is needed to avoid short positions and the normalization ensures that the portfolio is fully invested (see, e.g., Arnott et al., 2005; Walkshäusl & Lobe, 2010).

Thus, the $(N \times 1)$ vector $\mathbf{x}_{k,t} \equiv (x_{k,1,t}, \ldots, x_{k,N,t})'$ denotes the characteristic k-based portfolio weights for a universe of N stocks and $\mathbf{X}_t \equiv (\mathbf{x}_{1,t} \cdots \mathbf{x}_{K,t})$ is the $(N \times K)$ matrix where column k contains the stock weights based on characteristic k.

Other characteristic-based portfolio weight definitions have been considered in previous studies. In particular, Brandt et al. (2009) propose a characteristic-based portfolio definition where portfolio weights are modeled as deviations from a benchmark portfolio and the spread is proportional to the z-score of the stock characteristics. To the best of our knowledge, no investable index follows their modeling approach at present.

In the application, we consider a market capitalization, fundamental value, inverse volatility, and equally weighted portfolio. In Table 1, we provide examples of such indices and the ETFs which track their performance for both the American and European equity markets.

2.2. Mean-variance efficiency of characteristic-based portfolios

In this section, we investigate the conditions that the characteristic-based portfolio must fulfill in order to be mean-variance efficient for a level of risk aversion equal to γ, $0<\gamma<\infty$. Recall that $\boldsymbol{\mu}_{t+1|t}$ denotes the $(N \times 1)$ vector of expected returns and $\boldsymbol{\Sigma}_{t+1|t}$ the corresponding $(N \times N)$ covariance matrix. Then, the characteristic-based portfolio $\mathbf{x}_{k,t}$ is mean-variance efficient, when it maximizes the mean-variance utility under a full investment constraint:

$$\mathbf{x}^*_{k,t} \equiv \underset{\mathbf{w}\in\Theta}{argmax} \left\{ \boldsymbol{\mu}'_{t+1|t}\mathbf{w} - \frac{\gamma}{2}\,\mathbf{w}'\boldsymbol{\Sigma}_{t+1|t}\mathbf{w} \right\}, \tag{2}$$

where $\Theta \equiv \{\mathbf{w} \in \mathbb{R}^N \mid \boldsymbol{\iota}'_N\mathbf{w} = 1\}$ is the set of portfolio weights satisfying the full-investment constraint and $\boldsymbol{\iota}_N$ is a $(N \times 1)$ vector of ones.[7] The corresponding Lagrangian is given by:

$$\mathcal{L}(\mathbf{w}, l) \equiv \mathbf{w}'\boldsymbol{\mu}_{t+1|t} - \frac{\gamma}{2}\,\mathbf{w}'\boldsymbol{\Sigma}_{t+1|t}\mathbf{w} - l(\boldsymbol{\iota}'_N\mathbf{w} - 1), \tag{3}$$

where $l \in \mathbb{R}$ is the Lagrangian multiplier. The first order conditions are:

$$\boldsymbol{\mu}_{t+1|t} - \gamma\boldsymbol{\Sigma}_{t+1|t}\mathbf{w} - l\boldsymbol{\iota}_N = \mathbf{0}_N \tag{4}$$

$$\boldsymbol{\iota}'_N\mathbf{w} = 1, \tag{5}$$

where $\mathbf{0}_N$ is a $(N \times 1)$ vector of zeros. For $\mathbf{x}^*_{k,t}$ to be optimal, it must thus satisfy that:

$$\mathbf{x}^*_{k,t} = \frac{1}{\gamma}\boldsymbol{\Sigma}^{-1}_{t+1|t}(\boldsymbol{\mu}_{t+1|t} - l\boldsymbol{\iota}_N), \tag{6}$$

where l is such that $\boldsymbol{\iota}'_N\mathbf{x}^*_{k,t} = 1$. It also follows that, as a condition for optimality, there must exist a linear relationship between $\boldsymbol{\mu}_{t+1|t}$ and $\boldsymbol{\Sigma}_{t+1|t}\mathbf{x}^*_{k,t}$:

$$\boldsymbol{\mu}_{t+1|t} = l\boldsymbol{\iota}_N + \gamma\,\boldsymbol{\Sigma}_{t+1|t}\mathbf{x}^*_{k,t}\,. \tag{7}$$

Investors who claim mean-variance efficiency of a particular characteristic-based portfolio thus have the implied view that the expected stock returns are a linear combination of that stock characteristic. In practice, it is more realistic to expect that the characteristic leading to a mean-variance efficient characteristic-based portfolio changes over time. It may even be that, in some selection months, none of the characteristics lead to a mean-variance efficient solution but a more efficient solution can be obtained by combining them. Therefore, we hypothesize the existence of performance gains in dynamic characteristic-based portfolio investing and propose in the next section dynamic style portfolios to exploit the time-variation in the relative performance of the characteristic-based portfolios.

2.3. Mean-variance dynamic style portfolio construction

The proposed MVDS allocation approach aims at exploiting the information in the characteristic-based portfolios by dynamically investing in a portfolio that combines multiple characteristic-based portfolios. In particular, let $\widetilde{\boldsymbol{\mu}}_{t+1|t}$ and $\widetilde{\boldsymbol{\Sigma}}_{t+1|t}$ be the $(K \times 1)$ vector of expected returns and the $(K \times K)$

[7] The characteristic-based portfolios that we consider in (1) are constrained to be long-only investments in equities. In the general formulation of mean-variance efficiency in this section, we do not impose the no short sells restriction. This thus implies that for the characteristic-based portfolio in (1) to be mean-variance efficient, the solution in (2) must have no negative position. In the application, we will also consider the possibility of allowing for short selling positions in the dynamic style portfolio.

covariance matrix, respectively, of the $(K \times 1)$ vector $\widetilde{\mathbf{r}}_{t+1}$ of next-period characteristic-based portfolios return, given the information set up to time t. Then, for a risk aversion parameter γ, the MVDS allocation is given by:

$$\widetilde{\mathbf{w}}_t^* \equiv \underset{\widetilde{\mathbf{w}} \in \widetilde{\Theta}}{argmax} \left\{ \widetilde{\mathbf{w}}' \, \widetilde{\boldsymbol{\mu}}_{t+1|t} - \frac{\gamma}{2} \widetilde{\mathbf{w}}' \, \widetilde{\boldsymbol{\Sigma}}_{t+1|t} \, \widetilde{\mathbf{w}} \right\} , \quad (8)$$

where $\widetilde{\Theta} \equiv \{\widetilde{\mathbf{w}} \in \mathbb{R}^K \mid \boldsymbol{\iota}_K' \widetilde{\mathbf{w}} = 1\}$ is the set of portfolio weights satisfying the full-investment constraint. In the empirical application, we include additional weight constraints. Our main results are for long-only investors, as this is in line with the typical behavior of ETF investors. In addition, we present results for the 130/30 MVDS portfolio, in which the investor can short up to 30% of his portfolio value.

2.4. Mean-variance efficiency of MVDS portfolios

The portfolios that we recommend in Section 2.3 are obtained by a time-varying linear combination of the characteristic-based portfolios. In this section, we consider a framework under which this approach is optimal.

Suppose that, for the investment horizon between rebalancing dates t and $t+1$, the true mean-variance optimal portfolio weight vector is $\mathbf{w}_t^*$ and that the $(N \times K)$ matrix of characteristic-based portfolio weights over this investment horizon is $\mathbf{X}_t$. When there is no estimation uncertainty, the investor is indifferent between optimizing over the $(N \times 1)$ portfolio weight vector $\mathbf{w}_t$ and the $(K \times 1)$ weights vector $\widetilde{\mathbf{w}}_t$ whenever a $\widetilde{\mathbf{w}}_t^*$ exists such that $\mathbf{w}_t^* = \mathbf{X}_t \widetilde{\mathbf{w}}_t^*$, i.e., the optimal portfolio weights are a linear combination of the characteristic-based portfolio weights.

As we demonstrate below, a particular case where this situation arises is when the investor's preferences can be characterized by the usual mean-variance utility function with risk aversion parameter γ, the stock returns follow the factor model of Haugen and Baker (1996) with homoskedastic and uncorrelated error terms, and $\gamma = \boldsymbol{\iota}_N' \boldsymbol{\Sigma}_{t+1|t}^{-1} \boldsymbol{\mu}_{t+1|t}$.

Proof. As before, we denote $\boldsymbol{\mu}_{t+1|t}$ and $\boldsymbol{\Sigma}_{t+1|t}$ as the $(N \times 1)$ vector of expected returns and the $(N \times N)$ covariance matrix of the $(N \times 1)$ stock return vector $\mathbf{r}_{t+1}$, given the information set up to time t. Under the factor model of Haugen and Baker (1996), the return of stock i at time $t+1$, $r_{i,t+1}$, is a weighted average of the payoffs related to the stock characteristics at time t and an error term $\epsilon_{i,t+1}$:

$$r_{i,t+1} = \sum_{k=1}^{K} \delta_{k,t} \, z_{k,i,t} + \epsilon_{i,t+1} , \quad (9)$$

where $\delta_{k,t}$ corresponds to the risk premium of stock characteristic k over the one-period investment period starting at time t. From the first order conditions in (4) and (5) and provided that $\gamma = \boldsymbol{\iota}_N' \boldsymbol{\Sigma}_{t+1|t}^{-1} \boldsymbol{\mu}_{t+1|t}$, it follows that $\mathbf{w}_t^* = \frac{1}{\gamma} \boldsymbol{\Sigma}_{t+1|t}^{-1} \boldsymbol{\mu}_{t+1|t}$.[8] Without loss of generality, we henceforth use the rescaled premiums $\widetilde{\delta}_{j,t} \equiv \delta_{j,t} \big/ \sum_{i=1}^{N} \max\{0, z_{k,i,t}\}$. Under (9), and provided $z_{k,i,t}$ is positive valued, we obtain $\boldsymbol{\mu}_{t+1|t} \equiv \mathbb{E}[\mathbf{r}_{t+1} | \mathbf{X}_t] = \mathbf{X}_t \widetilde{\boldsymbol{\delta}}_t$ and thus:

$$\mathbf{w}_t^* = \frac{1}{\gamma} \boldsymbol{\Sigma}_{t+1|t}^{-1} \mathbf{X}_t \widetilde{\boldsymbol{\delta}}_t , \quad (10)$$

where $\widetilde{\boldsymbol{\delta}}_t \equiv (\widetilde{\delta}_{1,t}, \ldots, \widetilde{\delta}_{K,t})'$. Therefore, in this setting, the optimal equity allocation is completely determined by the conditional covariance matrix $\boldsymbol{\Sigma}_{t+1|t}$, the characteristic-based portfolio weights $\mathbf{X}_t$, and the risk premiums $\boldsymbol{\delta}_t$ associated with the K stock characteristics. The optimal portfolio changes over time because of the time-variation in each of these factors.

[8] Note that from (4), we have that $\mathbf{w}_t^* = \frac{1}{\gamma} \boldsymbol{\Sigma}_{t+1|t}^{-1} (\boldsymbol{\mu}_{t+1|t} - l \boldsymbol{\iota}_N)$. Substituting this in (5) yields that $\gamma = \boldsymbol{\iota}_N' \boldsymbol{\Sigma}_{t+1|t}^{-1} \boldsymbol{\mu}_{t+1|t} - l \boldsymbol{\iota}_N' \boldsymbol{\Sigma}_{t+1|t}^{-1} \boldsymbol{\iota}_N$, which can be rewritten as $l = (-\gamma + \boldsymbol{\iota}_N' \boldsymbol{\Sigma}_{t+1|t}^{-1} \boldsymbol{\mu}_{t+1|t}) / \boldsymbol{\iota}_N' \boldsymbol{\Sigma}_{t+1|t}^{-1} \boldsymbol{\iota}_N$, which is zero when $\gamma = \boldsymbol{\iota}_N' \boldsymbol{\Sigma}_{t+1|t}^{-1} \boldsymbol{\mu}_{t+1|t}$.

In order to show optimality of the MVDS portfolio, we need additional conditions under which the solution in (10) becomes linear in $\mathbf{X}_t$. A sufficient condition is that $\mathbf{\Sigma}_{t+1|t}$ is a diagonal matrix and that all stocks have the same variance $\sigma^2_{t+1|t}$, i.e., $\mathbf{\Sigma}_{t+1|t} \equiv \sigma^2_{t+1|t}\mathbf{I}_N$ with $\mathbf{I}_N$ the $(N \times N)$ identity matrix. Then it follows that:

$$\mathbf{w}^*_t = \mathbf{X}_t \widetilde{\mathbf{w}}^*_t , \tag{11}$$

with $\widetilde{\mathbf{w}}^*_t \equiv \frac{1}{\gamma}\frac{1}{\sigma^2_{t+1|t}}\widetilde{\boldsymbol{\delta}}_t$. Thus, under the above assumptions (particularly in the special case of homoscedasticity and no correlation across the idiosyncratic terms $\epsilon_{i,t+1}$), we find in (11) that the optimal portfolio is a linear combination of the characteristic-based portfolios. □

In practice, the assumptions needed to prove optimality of the proposed dynamic style portfolios are almost surely violated on real data. Evaluating whether an optimized portfolio obtained from characteristic-based portfolios outperforms an optimized portfolio out-of-sample with full flexibility over the $(N \times 1)$ weights is an empirical question. When N is large compared with the number of time series observations T, the estimates may contain significant errors (see, e.g., Ledoit & Wolf, 2003). Estimation errors cause unstable, extreme portfolio weights and poor out-of-sample performance (see, e.g., Best & Grauer, 1991; DeMiguel et al., 2009; Jobson & Korkie, 1980; Litterman, 2003). Compared with the traditional portfolio optimization over N equities, style allocation reduces the dimension of the original mean-variance problem from optimizing over N equity weights to optimizing over the allocation to K characteristic-based portfolios, with $K \ll N$.

3. Data, estimation and performance evaluation

In this section, we describe the data used in the empirical application. We discuss the covariance matrix estimation approaches and introduce the performance evaluation criteria employed in the analysis.

3.1. Investment universe and characteristics

The investment universe is restricted to the month-end S&P 100 constituents. Daily total return data are obtained from the COMPUSTAT database and expressed in U.S. dollars. As is common in practice, the characteristic-based portfolios are rebalanced at the end of every month.[9] The evaluation period ranges from January 1990 to December 2013.

We focus on the characteristics that are among the most popular at present when designing characteristic-based portfolios, i.e., the stock's market capitalization, its fundamental value, its volatility, and its inclusion in the investment universe. The latter characteristic corresponds to the equally weighted portfolio. The market capitalization portfolio is the standard choice for investments in equity. According to the EDHEC-Risk North American index survey, almost 60% of professional investors use alternatively weighted indices to complement the market capitalization portfolio rather than to replace it (Amenc, Goltz, Tang, & Vaidyanathan, 2012). These alternative weighting schemes are categorized into two groups. First, the alternative indices that apply a weighting scheme based on a more representative measure of company size, e.g., fundamental value portfolios. Second, alternative weighting schemes that avoid focusing on large sized companies. The inverse volatility portfolio seeks diversification in terms of risk, while the equally weighted portfolio is the most diversified portfolio in terms of weights.

Market capitalization weighting. The market capitalization portfolio is the standard choice for investment in equity for many investors (Amenc, Goltz, Martellini, & Retkowsky, 2011). It represents

[9] Characteristic-based portfolios are typically rebalanced at a low frequency such as the monthly, quarterly, semi-annual or annual frequency. In order to exploit the differences in relative performance, we obtain our main results in Section 4 when optimizing the MVDS portfolio at the weekly frequency. Section 5 then discusses the trade-off between style drift and a lower turnover when switching to a lower rebalancing frequency.

a broadly invested portfolio, with the advantage of low turnover and it corresponds to the equilibrium portfolio (Perold, 2007). The popularity of the market capitalization portfolio originates from the CAPM, which states that the market capitalization portfolio is optimal under very strict assumptions. In practice, these assumptions are often violated, thereby leading to the rejection of the market capitalization portfolio as the mean-variance efficient choice.

Fundamental value weighting. The approach for fundamental value weighting is popularized by Arnott et al. (2005), who propose a framework where assets are selected and weighted based on fundamental metrics related to company size such as the book value of common equity and operating cash flow. Proponents of fundamental value weighting, such as Arnott et al. (2005) and Treynor (2005), argue that market capitalization portfolios overweight overvalued stocks and underweight undervalued stocks in noisy markets. If pricing errors are corrected, this leads to a drag on returns for market capitalization portfolios. Several studies report a long-run outperformance of fundamental value indices compared with the market capitalization portfolio (see, e.g., Hemminki & Puttonen, 2008; Stotz, Wanzenried, & Dohnert, 2010; Walkshäusl & Lobe, 2010). However, the apparent outperformance has been criticized by Perold (2007) and Graham (2012), who emphasize that this result is sensitive to the assumption that stock prices systematically revert to fair value, as well as by De Moor, Liu, Sercu, and Vinaimont (2013) and Chen, Dempsey, and Lajbcygier (2014), who show that the extra returns generated by the fundamental value portfolio disappear after accounting for the time-varying exposure to various risk factors.

We follow Arnott et al. (2005) in the construction of the fundamental value portfolio. First, we construct four fundamental value portfolios based on: (1) the book value of common equity, (2) dividends, (3) net operating cash flow, and (4) sales. Second, these single-metric portfolios are aggregated in equal proportions to yield a composite fundamental value portfolio. The fundamental data were retrieved from the COMPUSTAT database on an annual basis from 1984 to 2013. The metrics are lagged by one quarter to ensure data availability. The dividends, net operating cash flow, and sales measures are five-year rolling averages.

Risk-based weighting. Among others, Baker and Haugen (2012) and Dutt and Humphery-Jenner (2013) show that low volatility stocks earn higher returns compared with high volatility stocks, and they demonstrate that this phenomenon is consistent over both time and different markets. Risk-based pricing anomalies contribute to the popularity of these portfolios (see, e.g., Scherer, 2011). Non-optimized risk-based weighting approaches employ a univariate weighting scheme where the weights of the stocks are inversely related to a risk measure (Leote de Carvalho, Lu, & Moulin, 2012). The possible risk measures comprise the volatility, beta, and semi-deviation. In our application, the risk-based index is the inverse volatility portfolio. The stocks' weights are set as inversely proportional to the 252-day stock volatility.

Equal-weighting. The equally weighted portfolio represents a naively diversified portfolio. The equally weighted portfolio is highly diversified in terms of weights, but the diversification in terms of risk is often limited. DeMiguel et al. (2009) show that the naive equal-weighting strategy outperforms a set of optimized strategies, where they attribute the underperformance of more sophisticated approaches to the negative impact of estimation error. Plyakha, Uppal, and Vilkov (2014) benchmark the equal-weighting approach against value- and price-weighted strategies, and they find that the equally weighted portfolio obtains a higher monthly return than the price-weighted portfolio in approximately 65% of the periods investigated. Some of the outperformance is explained by higher exposure to the market, size, value, and momentum factors.

In our application, an equal weight is assigned to each constituent of the S&P 100 investment universe.

The list of characteristics mentioned above is not exhaustive. Examples of other characteristics used in practice include the appurtenance to a sector, the stock's momentum, the firm's profitability, and the stock's price-earnings or price-dividend ratios.

3.2. Frequency of rebalancing

The MVDS portfolio invests in the monthly rebalanced characteristic-based portfolios. The dynamic allocation of the MVDS aims to exploit the time-varying relative performance of the characteristic-based portfolios. The frequency of rebalancing the MVDS portfolio is a trade-off between timeliness and excessive portfolio turnover. We are not interested in the day-to-day changes in relative performance of the characteristic-based portfolios, since capturing this noisy signal is likely to be accompanied by high turnover and transaction costs. Also, few ETF investors have the discipline to change their allocations on a daily basis. A lower rebalancing frequency is thus more realistic. Our main results in Section 4 are obtained for the weekly rebalanced MVDS portfolio. Section 5 reports the performance of the MVDS portfolio rebalanced at the monthly and quarterly frequency.

3.3. *Estimation of the conditional mean and covariance matrix*

The weekly rebalancing of the MVDS portfolio requires to estimate the conditional mean and covariance matrix of the characteristic-based portfolios. Various estimators have been proposed in previous studies, but it is beyond the scope of this study to investigate all of them.[10] Instead, we follow investment practice and we use the weekly returns of the stocks over a three-year (156 weeks) rolling window as input data. The conditional mean and covariance matrix are estimated in two steps, where the vectors of the expected returns are estimated first using the classical sample average estimator, and the exponentially weighted moving average (EWMA) model is then used to exploit the more persistent time-variation in the conditional covariance matrix.[11]

At the stock level, the estimations of the $(N \times 1)$ vector of expected returns $\boldsymbol{\mu}_{t+1|t}$ and the $(N \times N)$ covariance matrix $\boldsymbol{\Sigma}_{t+1|t}$ are based directly on the stock return vectors $\{\mathbf{r}_s\}_{s=t-156}^{t}$. To estimate the characteristic-based portfolio (co)moments $\widetilde{\boldsymbol{\mu}}_{t+1|t}$ and $\widetilde{\boldsymbol{\Sigma}}_{t+1|t}$, we create the return series that corresponds to the composition of the characteristic-based portfolios over the investment. More precisely, we construct the $(K \times 1)$ return vectors $\{\widetilde{\mathbf{r}}_s\}_{s=t-156}^{t}$, where $\widetilde{\mathbf{r}}_s \equiv (\mathbf{r}_s' \mathbf{X}_s)'$, with $\mathbf{X}_s$ the $(N \times K)$ characteristics' weight matrix at time s. Thus, we directly consider that the time-variation in $\widetilde{\boldsymbol{\mu}}_{t+1|t}$ and $\widetilde{\boldsymbol{\Sigma}}_{t+1|t}$ is driven by two components, i.e., the changes in the variations of the stock returns and the changes in the composition of the characteristic-based portfolios.

We estimate the covariance matrix using the EWMA estimator with a decay parameter of 0.94 as advocated by RiskMetrics Group (1996). The conditional covariance matrix is initialized at the shrinkage covariance matrix of Ledoit and Wolf (2003) over the estimation window. Under the shrinkage method, the correlation matrix is estimated as a weighted average of the sample correlation and the equicorrelation matrix. In a robustness analysis, we consider the alternative of using the traditional sample covariance estimator and the shrinkage estimation approach. The sample covariance matrix is affected by the curse of dimensionality, so we expect that the shrinkage covariance estimator will be especially useful in the mean-variance allocation over N assets, whereas no shrinkage is needed for the dynamic style portfolios that allocate to the K characteristic-based portfolios.

An alternative is to estimate $\boldsymbol{\mu}_{t+1|t}$ and $\boldsymbol{\Sigma}_{t+1|t}$ using the Haugen and Baker (1996) factor model approach. The factor model approach assumes that the return generating process is defined by $r_{i,t+1} = \sum_{k=1}^{K} \delta_{k,t} z_{k,i,t} + \epsilon_{i,t+1}$, with $z_{k,i,t}$ the stock characteristic and $\delta_{k,t}$ the risk premium associated with characteristic k; see (9). First, the factor model is estimated using a pooled ordinary least squares regression and the standardized factor data. This yields $\widehat{\widehat{\boldsymbol{\delta}}}_t$, the estimated factor premiums, and $\mathbf{X}_t \widehat{\widehat{\boldsymbol{\delta}}}_t$, the estimated vector of expected returns. Under this approach $\boldsymbol{\Sigma}_{t+1|t}$ is estimated as the sample covariance of the residuals under the factor model.

[10] See, e.g., Clements and Silvennoinen (2013) for a comparison of estimation methods for covariance matrices in the context of volatility timing.

[11] Note that the EWMA process is integrated, which implies a non-mean reverting conditional expectation. As the objective of the optimization is to forecast the conditional covariance matrix over only one period, this does not pose any problem. In Section 5 we also consider the use of the Dynamic Conditional Correlation (DCC) model for forecasting the covariance matrix. The latter leads to a higher volatility in the covariance forecasts and the portfolio weights, which, in our application is not compensated by a better performance. Moreover, when estimated on one hundred stocks, the quasi-maximum likelihood estimation of the DCC parameters using the composite likelihood technique of Pakel, Shephard, Sheppard, and Engle (2014) posed several convergence problems. We therefore take the EWMA based conditional covariance estimates as the input for our main results on the MVDS portfolio performance.

3.4. Performance evaluation

We evaluate the portfolio performance on nine dimensions: the average return, the volatility, the Sharpe ratio, the value-at-risk (estimated by the 5% quantile of the out-of-sample returns), the maximum drawdown, the tracking error with respect to the market capitalization portfolio, the alpha of the Carhart four-factor model, the alpha of the four-factor model augmented with a factor capturing the low volatility anomaly, and the portfolio's turnover. As in previous studies (see, e.g., DeMiguel et al., 2009), the turnover is defined as the sum of the absolute value of the trades across the N assets:

$$\text{Turnover} \equiv \frac{1}{T-1}\sum_{t=1}^{T-1}\sum_{i=1}^{N}(|w_{i,t+1} - w_{i,t^+}|) \tag{12}$$

where $w_{i,t+1}$ is the desired portfolio weight of asset i at time $t+1$ and w_{i,t^+} is the portfolio weight before rebalancing. The turnover can be interpreted as the average percentage of wealth traded at each rebalancing date.

We compute the alpha of the portfolios using the Carhart (1997) four-factor model and a five-factor model, which includes, in addition to the traditional four factors, a low minus high risk equity factor, defined as the difference in return of a portfolio that is long in stocks with a low beta and residual volatility and a portfolio that is short in stocks with a high residual volatility and beta.[12] The returns on each of the factors are computed for the S&P 100 investment universe. The five-factor model is given by:

$$r_{p,t} - r_{f,t} = \alpha + \beta_{\text{MKT}}(MKT_t - r_{f,t}) + \beta_{\text{SMB}}\,SMB_t + \beta_{\text{HML}}\,HML_t + \beta_{\text{MOM}}\,MOM_t + \beta_{\text{RSK}}\,RSK_t + \epsilon_t \tag{13}$$

where $r_{p,t}$ is the monthly portfolio return, $r_{f,t}$ is the one-month Treasury bill rate, MKT_t is the return of the S&P 100 market capitalization weighted index, SMB_t is the size factor, HML_t is the book-to-market factor, MOM_t the momentum factor and RSK_t the composite risk factor, all at time t. The SMB_t, HML_t and MOM_t factor returns are constructed as described on the Kenneth R. French website.[13] The RSK_t factor is constructed by first estimating the stock's beta and residual volatility over a 156-week period. Then a composite risk indicator is defined as an equally weighted average of the z-scores based on the stocks' beta and residual volatility. The stocks are then sorted into quintiles based on the composite measure. RSK_t is the return on the (monthly rebalanced) portfolio that is long in the equally weighted portfolio of the stocks in the bottom quintile based on the composite risk measure and short in the equally weighted portfolio of the top quintile stocks based on the composite risk measure. The four-factor model is nested in the five-factor model and obtained as a special case by setting $\beta_{RSK} = 0$.

We test the significance of alpha with a t-test using the heteroscedasticity and autocorrelation robust (HAC) kernel estimators of Andrews (1991) and Andrews and Monahan (1992).

[12] Scherer (2011) and Leote de Carvalho et al. (2012) consider a six-factor model by including separately a low risk factor based on the stock's beta and residual volatility. In our sample, these factors have a correlation of 75%. In order to avoid issues of quasi-multicollinearity, we therefore considered the construction of a low risk factor based on a joint measure of the stock's beta and residual volatility.

[13] The SMB_t factor is constructed by first categorizing the stocks into tercile book-to-market portfolios and then sorting the stocks by market capitalization in two groups within each tercile. SMB_t is the portfolio return of the portfolio that is long in the smallest market capitalization stocks in each tercile and short in the largest market capitalization stocks in each tercile. The HML_t factor is obtained by first categorizing the stocks in two groups based on their market capitalization and then sorting the stocks by book-to-market value in two groups within each group. HML_t is then the portfolio return of the portfolio that is long in the two high book-to-market portfolios and short in the two low book-to-market portfolios. Finally, the MOM_t factor is constructed by first categorizing the stocks in two groups based on their market capitalization and then sorting the stocks by twelve-month prior returns in two groups within each group. MOM_t is the portfolio return of the portfolio that is long in the two high prior return portfolios and short in the two low prior return portfolios. The portfolios used to define the SMB_t and HML_t factors are rebalanced annually while the portfolio used to define the MOM_t factor is rebalanced monthly.

Table 2
Correlation matrix: characteristic-based portfolios.

	Market cap	Fund	Inv vol	EW
Market cap		91.7 / 97.9 / 99.3	96.1 / 98.8 / 99.5	98.0 / 98.8 / 99.5
Fund	90.2 / 97.9 / 99.3		92.7 / 97.6 / 98.9	92.4 / 97.6 / 98.6
Inv vol	93.6 / 98.4 / 99.3	88.6 / 96.9 / 98.8		98.2 / 99.4 / 99.7
EW	96.8 / 98.4 / 99.5	88.7 / 97.1 / 98.4	97.2 / 99.2 / 99.7	

Notes: Minimum, median, and maximum values (%) of 156-week rolling window correlations between the characteristic-based portfolios during 1990–2013. The upper-right part shows the standard Pearson correlations and the lower-left part shows the Spearman's rank correlations. Market cap: market capitalization portfolio. Inv vol: inverse volatility portfolio. Fund: fundamental value portfolio weighted portfolio.

For the mean-variance portfolios in (8), we follow Martellini and Ziemann (2010) by evaluating the performance for three different levels of the risk aversion parameter, $\gamma \in \{1, 5, 10\}$, which represent low, medium, and high risk aversion, respectively.

4. Results

We now present our main results for the time-varying weights and the performance of the dynamic style portfolios with the 1990–2013 S&P 100 investment universe. In Section 4.1, we report the properties of the single characteristic-based portfolios. In Section 4.2, we examine the time-varying weights of the characteristic-based portfolio in the mean-variance efficient dynamic style portfolio and the performance of these MVDS portfolios. The main question that we address in Section 4.3 is whether there is a gain in switching from mean-variance allocation at the stock level to mean-variance allocation across the characteristic-based portfolios.

4.1. Characteristic-based portfolios

A primary condition for the relevance of dynamic style allocation is that the characteristic-based portfolios differ in terms of mean return, volatility, and correlation.

First, we investigate this on a rolling basis. Fig. 1 shows the mean returns and standard deviation calculated over a 156-week rolling window, which demonstrate that the inverse volatility portfolio has the highest mean returns in down periods, whereas the market capitalization portfolio produces higher mean returns in bull markets. This variation in volatility is similar for all types of characteristic-based portfolios, but they differ in magnitude. The inverse volatility portfolio consistently has the lowest volatility, whereas the 156-week volatilities of the fundamental value portfolio are almost 5% higher during 2010–2012.

Table 2 shows the correlations between the different characteristic-based portfolio returns. The median correlation is highest between the equally weighted and inverse volatility portfolio. The correlation differs over time and the returns of the fundamental value portfolio have the lowest correlation with the returns of the other portfolios that we consider.

These differences in local performance also led to substantial differences in the overall sample. Panel A of Table 3 shows the performance characteristics for the characteristic-based portfolios. All alternative weighting approaches outperform the market capitalization portfolio in the long-run in terms of Sharpe ratio. However, there are differences in performance among the characteristic-based portfolios. The average returns of the fundamental value, inverse volatility, and equally weighted portfolios are higher compared with the market capitalization portfolio. As expected, the inverse volatility portfolio has the lowest volatility, whereas the volatilities of the equally weighted and fundamental value portfolios are higher. The inverse volatility portfolio has the lowest maximum drawdown and both the inverse volatility and fundamental value portfolio report a positive significant four-factor and five-factor alpha.

The differences in performance are not extreme, but the underlying sector bets are substantial. Panel B of Table 3 shows the average weights of the GICS sectors in the characteristic-based portfolios. We see that there are substantial differences in the sector loadings. On average, the market

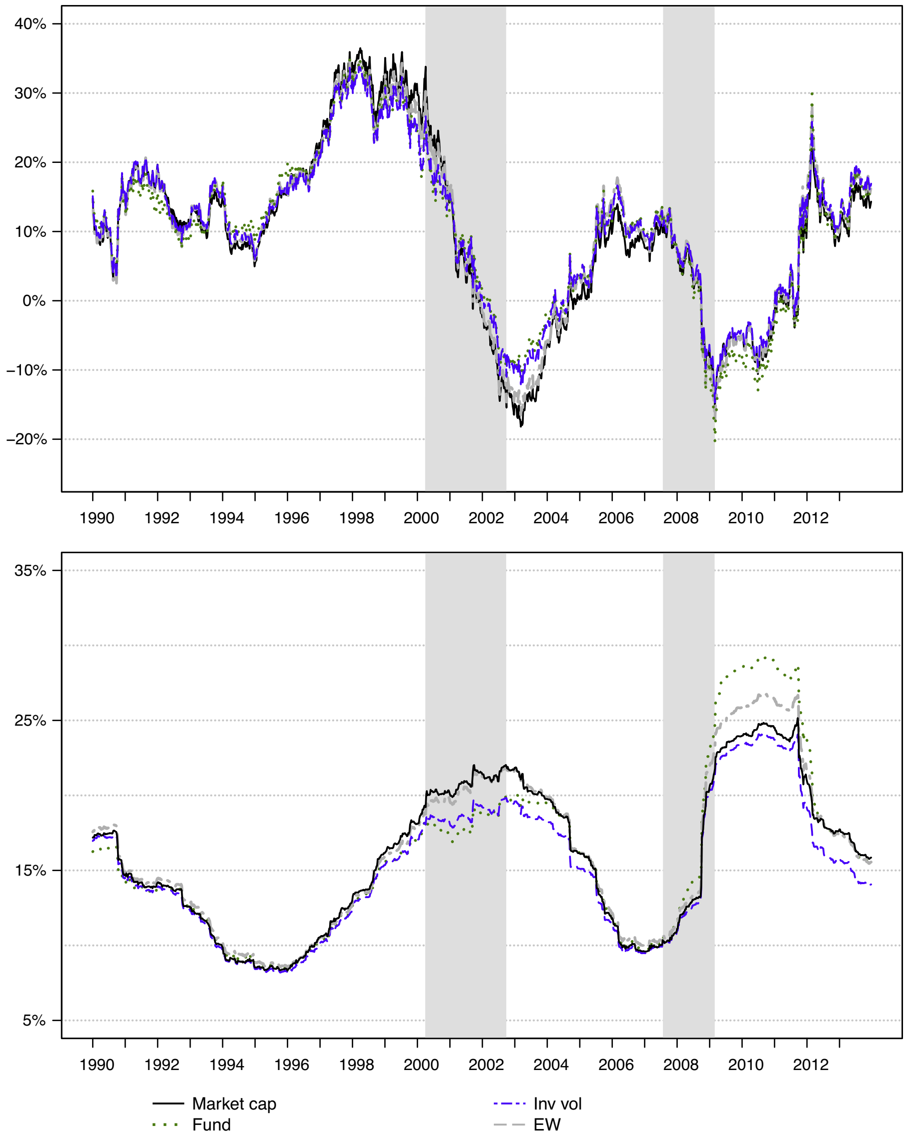

Fig. 1. Annualized average returns (top panel) and volatilities (bottom panel) for the four characteristic-based portfolios during 1990–2013. The average returns and volatilities are calculated over a 156-week rolling window. Shaded regions indicate the two largest drawdown periods of the S&P 100.

capitalization portfolio places the highest weight on the IT sector (i.e., 17%), whereas the fundamental value portfolio tends to overweight the financial (i.e., almost 18%) and energy (i.e., 14%) sectors. The inverse volatility portfolio is mostly invested in the consumer staples, healthcare, and financial sectors. The equally weighted portfolio concentrates on the financial (i.e., almost 16%), IT (i.e., almost 15%), and consumer staples (i.e., 13%) sectors.

Finally, Panel C of Table 3 shows the sector concentrations in the portfolios. We report the normalized Herfindahl index of the sector weights with respect to an equally weighted portfolio in the ten sectors (i.e., $H^*_{1/10}$), as well as the Herfindahl index of the sector weights in excess of the market

Table 3
Summary statistics of characteristic-based portfolios.

	Market cap	Fund	Inv vol	EW
Panel A: Out-of-sample performance				
Mean	9.06	9.61	10.06	9.78
Vol	14.70	14.93	13.70	14.99
SR	0.39	0.42	0.49**	0.42
VaR	−6.63	−6.81	−6.15	−6.81
MDD	50.22	56.37	46.87	51.55
TE	–	4.22	3.27	2.61
$\hat{\alpha}_4$	–	1.30*	1.23**	0.54
$\hat{\alpha}_5$	–	1.29*	1.21***	0.54
Turn	4.61	6.91	10.81	10.94
Panel B: Average weights per sector (%)				
Basic materials	2.58	3.03	3.53	3.65
Communications	7.57	9.88	6.50	6.23
Consumer goods	8.90	11.34	11.46	12.55
Consumer staples	14.21	11.70	14.85	12.38
Energy	10.93	14.31	7.93	7.90
Financial	13.86	17.86	14.98	15.94
Healthcare	13.60	9.25	13.68	13.21
Industrial	9.84	10.09	11.54	10.96
IT	17.12	10.66	11.79	14.51
Utilities	1.22	1.87	3.73	2.67
Panel C: Sector concentrations				
$H^*_{1/10}$	0.026	0.022	0.019	0.021
H^*_{mc}	–	0.011	0.001	0.004

Notes: Performance results (Panel A), average end-of-month GICS sector allocation (Panel B) and sector concentrations (Panel C) for the characteristic-based portfolios during 1990–2013. Mean: annualized average return (%). Vol: annualized volatility (%). SR: annualized Sharpe ratio. VaR: monthly value-at-risk at 95% risk level (%). MDD: maximum drawdown (%). TE: tracking error relative to market capitalization portfolio (%). $\hat{\alpha}_4$: annualized alpha of the Carhart four-factor model (%). $\hat{\alpha}_5$: annualized alpha of the five-factor model (%). Turn: average monthly turnover (%). For the risk-adjusted return measures (Sharpe ratio and alpha), the table also shows the results of significance tests, where *, **, and *** indicate that the Sharpe ratio and alpha differ significantly from the Sharpe ratio of the market capitalization portfolio and zero, respectively, at the 10%, 5%, and 1% levels based on the *t*-test with HAC standard errors. $H^*_{1/10}$: normalized Herfindahl index, defined as $H^*_{1/10} \equiv \frac{H(\mathbf{w})-1/10}{1-1/10}$ where $H(\mathbf{w}) \equiv \sum_{s=1}^{10} w_s^2$. $H^*_{1/10}$ has a value between zero and one, with a value of zero indicating perfect diversification across the sectors and a value of one indicating full concentration in one sector. H_{mc}: Herfindahl index of the average sector weights in excess of the average market capitalization sector weights, defined as $H_{\mathrm{mc}} \equiv H(\mathbf{w} - \mathbf{w}_{\mathrm{mc}})$, where $\mathbf{w}_{\mathrm{mc}}$ are the average market capitalization sector weights. H_{mc} has a value between zero and one, with a value of zero if the sector weights are identical and a value of one indicating the most extreme active sector bets with respect to the market capitalization portfolio. Market cap: market capitalization portfolio. Inv vol: inverse volatility portfolio. Fund: fundamental value portfolio. EW: equally weighted portfolio.

capitalization sector weights (i.e., H_{mc}). The first quantifies the concentration of the portfolio in a small number of sectors. The latter quantifies the concentration of the sector weights relative to the market capitalization weighted benchmark. The $H^*_{1/10}$ measure shows that all portfolios are more concentrated relative to a portfolio that puts equal bets on each sector. The market capitalization portfolio is the most concentrated. The H_{mc} measure shows that the fundamental value portfolio differs the most in terms of sector bets compared with the market capitalization portfolio.

As discussed in Section 2.2, the choice of characteristic-based portfolio also indicates the underlying expected returns for the hypothetical mean-variance efficient investor.[14] Fig. 2 plots the average implied expected returns for the stocks in the financial and IT sectors. Note that investors who believe that the market capitalization portfolio is mean-variance efficient must have a very different view of the expected returns compared with the investors who assume that the fundamental value portfolio

[14] See (7), from which the implied expected returns are computed by setting l to the risk-free rate (Ardia & Boudt, 2015; Black & Litterman, 1991).

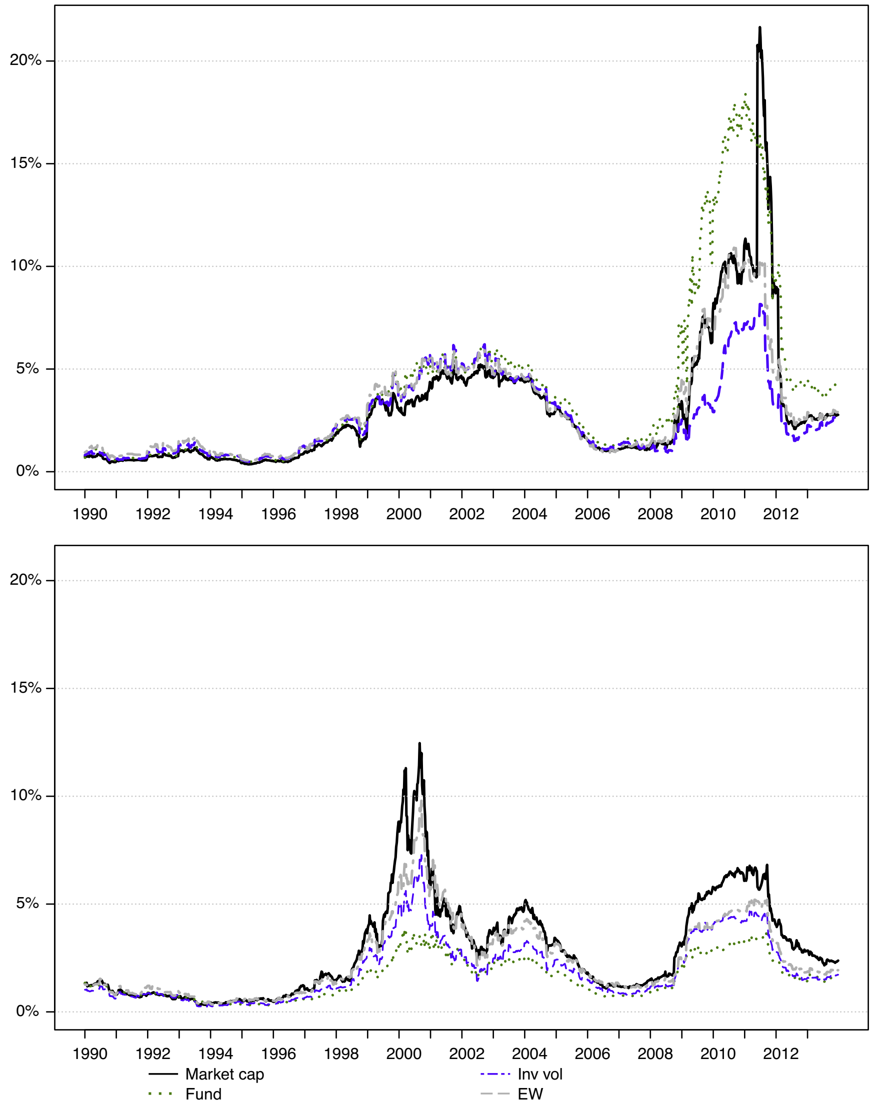

Fig. 2. Annualized average (over stocks) expected returns for financial (top panel) and IT stocks (bottom panel) according to the mean-variance efficiency condition for the four characteristic-based portfolios.

or inverse volatility portfolio are efficient. In fact, we see that the market capitalization and fundamental value portfolios tend to be more optimistic in terms of expected returns from the financial sector during 2009–2011 compared with the inverse volatility and equally weighted portfolios. The fundamental value portfolio is more pessimistic about IT stocks than the other portfolios in the early 2000s (high-tech bubble) and during the sub-prime crisis.

Thus, the alternative portfolios outperform the market capitalization portfolio in the long-run, but there are discrepancies in the results for different characteristics. The time-variation in the short-run performance gives rise to a market timing opportunity, which we aim to exploit by constructing dynamic style portfolios.

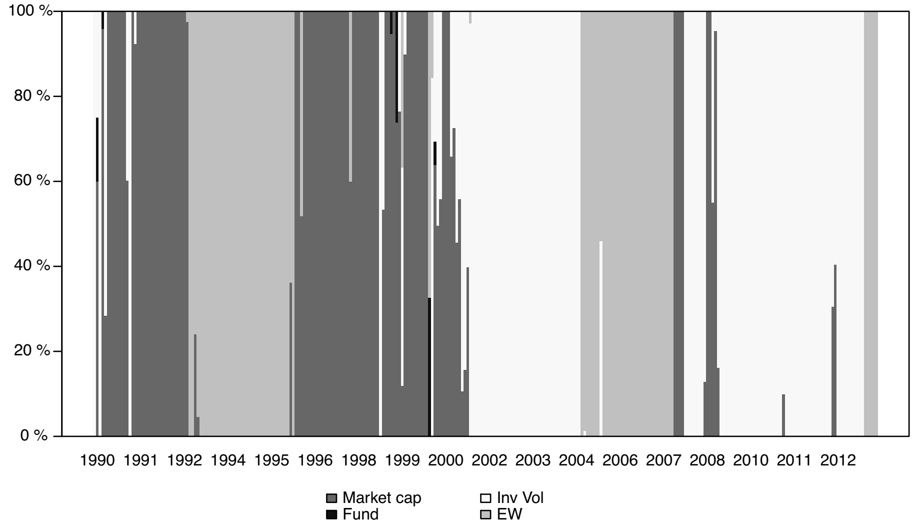

Fig. 3. Monthly rebalancing weights (%) assigned to the four characteristic-based portfolios by the mean-variance efficiency criterion (medium risk aversion case).

4.2. Dynamic style portfolios

The first question that we address is how the MVDS portfolio loads on the different characteristic-based portfolios. We focus on the results for the mean-variance efficient allocation with medium risk aversion ($\gamma = 5$), but similar results are obtained for low and high risk aversion.

Fig. 3 shows the time-varying pattern in the out-of-sample portfolio weight distribution for the different characteristics. Based on the complete sample, on average, we find that almost 31% of the portfolio value is invested in the market capitalization portfolio, 0.4% in the fundamental value portfolio, 41% in the inverse volatility portfolio, and 28% in the equally weighted portfolio.

The MVDS portfolio is usually dominated by one characteristic. This is expected as the relatively high correlation between the single characteristic-based portfolios in Table 2 already indicated a limited scope for diversification. As hypothesized, there is no all-time dominating characteristic. In fact, in up markets a large weight is invested in the market capitalization portfolio (e.g., the bullish 90s), whereas in down markets, an increasing proportion of the weights are allocated to the inverse volatility portfolio. For example, after the bursting of the high-tech bubble early in 2000 and during the aftermath of the financial crisis, the dynamic style portfolio is concentrated in the inverse volatility portfolio. Positions in the fundamental value portfolios are rare.

These portfolio weights have a large and expected effect on the performance of the dynamic style portfolio. Fig. 4 shows the relative performance chart for the MVDS and characteristic-based portfolios with respect to the market capitalization portfolio, which is the benchmark for most investors. The slope of the graph is important: a positively (negatively) sloped graph indicates outperformance (underperformance) relative to the market capitalization portfolio. The graph is flat for periods when the dynamic style portfolio is fully invested in the market capitalization portfolio. The dynamic style portfolio outperforms the market capitalization portfolio in both of the down periods indicated in the graph. In general, the dynamic style portfolio tends to outperform the market capitalization portfolio. By contrast, the alternative characteristic-based portfolios have periods of relative under- and outperformance. Overall, the dynamic style portfolio obtains more stable returns compared with the characteristic-based portfolios.

Table 4 presents the out-of-sample performance measures for the different portfolio strategies during 1990–2013. In Section 4.1, we demonstrate the outperformance of the alternative characteristic-based portfolios compared with the market capitalization portfolio in terms of Sharpe ratio. However, there are also discrepancies in the results obtained for the alternative characteristics.

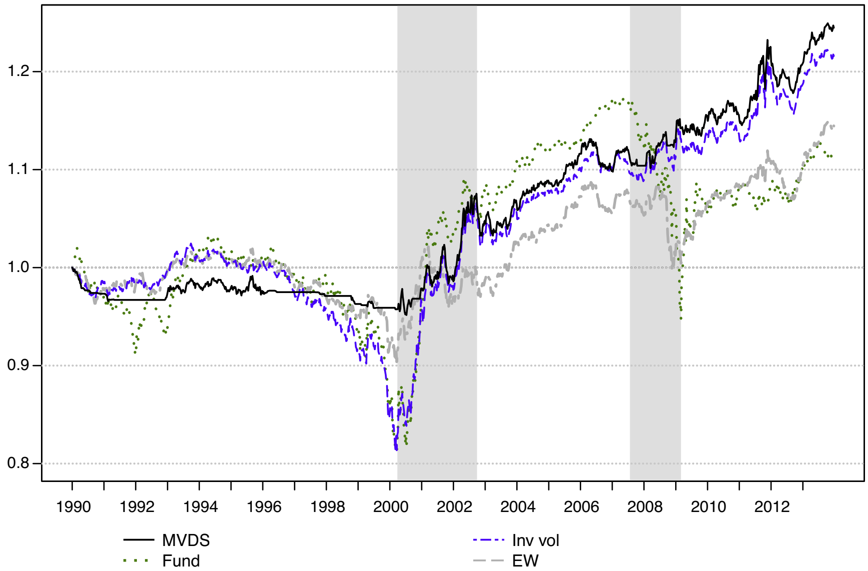

Fig. 4. Cumulative performance of the dynamic style mean-variance optimized portfolio (medium risk aversion case) and three characteristic-based portfolios relative to the cumulative performance of the market capitalization portfolio. Shaded regions indicate the two largest drawdown periods of the S&P 100.

Table 4
Performance results of mean-variance optimized portfolios during 1990–2013.

	Mean	Vol	SR	VaR	MDD	TE	$\hat{\alpha}_4$	$\hat{\alpha}_5$	Turn
Panel A: MVDS with EWMA									
$\text{MVDS}_{\gamma=1}$	9.87	14.44	0.45**	−6.53	47.62	1.78	0.66*	0.66**	37.36
$\text{MVDS}_{\gamma=5}$	10.03	14.12	0.47***	−6.36	47.29	2.27	0.87**	0.86**	43.32
$\text{MVDS}_{\gamma=10}$	9.89	14.04	0.46**	−6.32	47.33	2.85	0.86**	0.85**	79.18
Panel B: MV with EWMA									
$\text{MV}_{\gamma=1}$	3.65	35.55	0.01**	−15.22	89.57	29.86	−5.60	−5.52	94.17
$\text{MV}_{\gamma=5}$	7.95	21.58	0.21	−9.29	58.08	16.16	−1.98	−1.92	127.89
$\text{MV}_{\gamma=10}$	8.87	16.78	0.32	−7.34	49.06	12.32	−0.12	−0.12	137.13
Panel C: MV with factor model									
$\text{MV}_{\gamma=1}$	4.06	19.76	0.04***	−9.28	79.37	12.84	−4.43	−4.42	121.81
$\text{MV}_{\gamma=5}$	9.24	13.83	0.43	−6.27	53.11	9.63	1.70	1.66	85.14
$\text{MV}_{\gamma=10}$	10.31	12.75	0.54	−5.66	49.45	9.77	3.14*	3.10*	70.80

Notes: MVDS: dynamic style mean-variance optimization with low ($\gamma = 1$), medium ($\gamma = 5$) and high ($\gamma = 10$) risk aversion parameter. MV: mean-variance optimized across all stocks. For the risk-adjusted return measures (Sharpe ratio and alpha), the table shows the results of significance tests, where *, **, and *** indicate that the Sharpe ratio and alpha differ significantly from the Sharpe ratio of the market capitalization portfolio and zero, respectively, at the 10%, 5%, and 1% levels based on the *t*-test with HAC standard errors. See Table 3 for details.

Ex-post, we are able to identify the outperforming characteristic-based portfolio, but there is a selection problem ex-ante. This selection problem is addressed by the dynamic style allocation approach, which has the advantage of being purely data-driven.

Panel A of Table 4 shows the performance characteristics for the MVDS portfolio for the three levels of risk aversion. The MVDS portfolio is compared with the characteristic-based portfolios at least the second best performing portfolio for all performance measures, except turnover. In particular, the MVDS portfolios have similar average returns as the alternative characteristic-based portfolios, but also lower volatilities and thus a higher Sharpe ratio. Furthermore, the maximum drawdown is lower compared with the market capitalization, fundamental value, and equally weighted portfolio. The dynamic character of the MVDS is reflected by a higher turnover.

Table 5
Alpha and factor exposures.

	$\hat{\alpha}$	$\hat{\beta}_{MKT}$	$\hat{\beta}_{SMB}$	$\hat{\beta}_{HML}$	$\hat{\beta}_{MOM}$	$\hat{\beta}_{RSK}$	R^2
Panel A: MVDS with EWMA							
$MVDS_{\gamma=1}$	0.64*	0.9770***	0.0653***	0.0137	0.0216*		98.73
	(0.36)	(0.0106)	(0.0163)	(0.0128)	(0.0115)		
	0.66**	0.9747***	0.0652***	0.0146	0.0218*	−0.0021	98.73
	(0.36)	(0.0112)	(0.0164)	(0.0130)	(0.0115)	(0.054)	
$MVDS_{\gamma=5}$	0.87**	0.9504***	0.0930***	0.0235*	0.0272**		98.09
	(0.36)	(0.0101)	(0.0176)	(0.0120)	(0.0120)		
	0.86**	0.9666***	0.0937***	0.0171	0.0262**	0.0149*	98.14
	(0.36)	(0.0104)	(0.0196)	(0.0128)	(0.0128)	(0.0081)	
$MVDS_{\gamma=10}$	0.86**	0.9278***	0.1327***	0.0511***	0.0122		97.32
	(0.36)	(0.0121)	(0.0230)	(0.0126)	(0.0128)		
	0.85**	0.9627***	0.1343***	0.0375***	0.0100	0.0321***	97.57
	(0.36)	(0.0161)	(0.0215)	(0.0121)	(0.0128)	(0.0135)	
Panel B: MV with EWMA							
$MV_{\gamma=1}$	−5.60	1.5274***	0.1900	−0.2327	0.4908***		38.18
	(6.12)	(0.1631)	(0.3372)	(0.1674)	(0.1738)		
	−5.52	1.2549***	0.1175	−0.1260	0.5076***	−0.2511***	40.48
	(6.00)	(0.1622)	(0.3209)	(0.1549)	(0.1539)	(0.0882)	
$MV_{\gamma=5}$	−1.98	1.0578***	0.1640	−0.0239	0.3900***		53.01
	(3.24)	(0.0917)	(0.1704)	(0.0839)	(0.0915)		
	−1.92	0.9643***	0.1597	0.0127	0.3985***	−0.0861**	53.64
	(3.36)	(0.0955)	(0.1676)	(0.0792)	(0.0873)	(0.0396)	
$MV_{\gamma=10}$	−0.12	0.8423***	0.1302	0.0471	0.2777***		56.45
	(2.40)	(0.0728)	(0.1321)	(0.0592)	(0.0610)		
	−0.12	0.8449 ***	0.1303	0.0461	0.2776***	0.0024	56.30
	(2.40)	(0.0762)	(0.1339)	(0.0579)	(0.0609)	(0.0292)	
Panel C: MV with factor model							
$MV_{\gamma=1}$	−4.43	1.0498***	−0.0004	−0.0577	0.0705		57.73
	(2.76)	(0.0599)	(0.1032)	(0.0602)	(0.0484)		
	−4.42	1.0229***	−0.0017	−0.0472	0.0722	−0.0249	57.66
	(2.76)	(0.0641)	(0.1010)	(0.0626)	(0.0478)	(0.0397)	
$MV_{\gamma=5}$	1.70	0.7074***	0.0856	0.0893**	0.0328		60.78
	(2.04)	(0.0439)	(0.0707)	(0.0379)	(0.0344)		
	1.66	0.8228***	0.0908	0.0441	0.0257	0.1064***	63.65
	(1.92)	(0.0470)	(0.0673)	(0.0398)	(0.0299)	(0.0238)	
$MV_{\gamma=10}$	3.14*	0.6220***	0.0884	0.1230***	0.0191		59.40
	(1.92)	(0.0432)	(0.0672)	(0.0380)	(0.0343)		
	3.10*	0.7555***	0.0945	0.0707*	0.0108	0.1231***	63.99
	(1.80)	(0.0427)	(0.0612)	(0.0391)	(0.0266)	(0.0224)	

Notes: Estimation results for the annualized alpha (%) and monthly factor exposures obtained by regressing the monthly excess returns of mean-variance optimized portfolios on the factors of the four-factor and five-factor models over the period 1990–2013. Estimation by ordinary least squares and HAC standard errors (reported in parentheses). Per portfolio approach, four-factor estimates are first reported, five-factor estimates are reported second. R^2: adjusted R-squared (%). The table also shows the results of significance tests, where *, **, and *** indicate that the coefficients differ significantly from zero, respectively, at the 10%, 5%, and 1% levels based on the t-test with HAC standard errors. MVDS: dynamic style mean-variance optimization with low (γ = 1), medium (γ = 5) and high (γ = 10) risk aversion parameter. MV: mean-variance optimized across all stocks.

Overall, these results seem to support the benefits of the MVDS portfolio compared with the single characteristic-based portfolios. We now examine whether the performance gains of the MVDS portfolio could be replicated by a static portfolio invested in the portfolios corresponding to the Fama-French-Carhart risk factors and an additional risk factor capturing the low volatility anomaly. To do so, we regress the MVDS portfolio returns on the factor-mimicking portfolios. The intercept of the factor models (i.e., the alpha) can be interpreted as the added value of dynamic style investing compared with a buy-and-hold strategy invested in the factor-mimicking portfolios, with weights set to the coefficients of the estimated regression model. The economic benefits are thus confirmed when the estimated intercept is positive and significant. In Panel A of Table 5 we show that this is the case for the MVDS portfolios. For each level of risk aversion considered, the estimated four-factor and five-factor alpha is significant and positive. We also note that the factor exposures depend on the level of the risk aversion. The higher the risk aversion, the more the MVDS portfolio is exposed to the composite (beta and residual volatility) risk factor. This is as intuitively expected, since the higher the risk aversion, the higher is the exposure to the inverse volatility portfolio. Based on the complete sample, on average, we find that the MVDS portfolio with low risk aversion invests almost 39% of the portfolio value in the market capitalization portfolio, 20% in the inverse volatility portfolio, and 41% in the equally weighted portfolio. For high risk aversion, we find that on average 20% is invested in the market capitalization portfolio, 3% in the fundamental value portfolio, 53% in the inverse volatility portfolio and 22% in the equally weighted portfolio.

In summary, the MVDS portfolios report similar performance measures as the best performing characteristic-based portfolio. All of the performance measures are comparable with those of the superior characteristic-based portfolio, i.e., higher average return, lower volatility and maximum drawdown, as well as a significant positive four-factor and five-factor alpha. Fig. 4 also shows that the outperformance of the dynamic style portfolio over the whole period is similar to that of the best performing characteristic-based portfolio, with two advantages. First, the selection of the investment style is purely data-driven unlike the choice of one investment style. Second, there are few periods of underperformance compared with the benchmark market capitalization portfolio. This is important because investors tend to allocate their funds based on short-term relative performance (see, e.g., Barberis & Shleifer, 2003; Cahan & Luo, 2013).

4.3. Economic value of restricting portfolio weights to a combination of characteristic-based portfolios

The dynamic style allocation portfolios seek a compromise between gains from optimization and imposing structure on the optimized weights by requiring that they are a linear combination of characteristic-based portfolios. In this section, we examine the success of this trade-off compared with a portfolio allocation approach where the optimizer is free to allocate across all N stocks.

Table 4 presents the out-of-sample performance characteristics for the MVDS and the mean-variance portfolios at the stock level (MV) using EWMA estimation and the factor model.

A comparison of the performance of MVDS portfolios (Panel A of Table 4) and MV portfolios using EWMA (Panel B) clearly indicates that the performance of the MVDS portfolios is better, where they outperform MV using EWMA portfolios. In particular, the annualized Sharpe ratio increases from 0.21 to 0.47. Furthermore, the value-at-risk and maximum drawdown are substantially lower. While the MVDS portfolios report positive significant alpha, the MV portfolios using EWMA report insignificant alpha. Moreover, the average monthly turnover is almost 85% higher for the MV portfolios, thereby implying higher transaction costs.

An alternative to the MVDS portfolio is imposing structure via the factor model (Panel C of Table 4). This approach reduces the impact of estimation error and improves the performance compared with the MV using EWMA portfolios, i.e., these portfolios report lower volatility, value-at-risk, and maximum drawdown. Comparing the results of the MVDS and factor model portfolios, we find an outperformance for the MVDS portfolios in terms of alpha and turnover for low and medium risk aversion, while the performance measures are similar for a high risk aversion parameter.

Table 5 presents a detailed analysis of the factor exposures for both the Carhart four-factor model and the five-factor model. Similarly as for the MVDS portfolio, the exposure to the momentum and low risk factor are positive, and the higher the value of the risk aversion parameters is, the higher is the exposure to the low risk factor. The factor tilts depend on the approach. The MV using EWMA portfolios exhibit a negative exposure to the composite (beta and residual volatility) risk factor and tilts to momentum stocks. The factor model portfolios exhibit different factor exposures depending on the level of risk aversion. For medium and high risk aversion the portfolios are positively exposed to the composite risk factor.

Next, we estimate how much an investor is willing to pay to switch from a mean-variance efficient portfolio over all stocks to a dynamic style portfolio. We address this question by calculating the level of an out-of-sample annualized management fee, which is determined so a mean-variance utility optimizing investor is indifferent to both allocation approaches. For a certain risk aversion level γ, the fee for the realized mean-variance utility is computed so the gross returns of the mean-variance efficient dynamic style portfolio equals the realized utility computed on the mean-variance efficient portfolio over the whole universe.[15]

The annualized fee is positive and ranges from approximately 5% (for $\gamma = 10$) to almost 11.5% (for $\gamma = 1$). Thus, switching from stock-based allocation to characteristic-based allocation pays off for all types of mean-variance investors and the gains of switching tend to be higher for investors with lower risk aversion.

5. Further improvements

In this section, we discuss the results obtained after several improvements and robustness checks in the portfolio framework. First, we propose diversification-based allocation approaches for constructing dynamic style portfolios. Second, we show that imposing a no-trade zone and/or turnover constraint allows similar performance to be achieved at a significantly lower turnover. Third, we investigate the use of the dynamic style portfolio for an investor who also allocates his wealth partially to cash. Fourth, we relax the long-only constraint and report the performance of 130/30 portfolios. Finally, we demonstrate the robustness of our results to the estimation window and the choice of covariance estimator.

5.1. Diversification-based portfolios

In Section 4.3, we focus on the style allocation that is optimal according to the mean-variance efficiency criterion. In this section, we discuss the results obtained with two alternative criteria, i.e., equally weighted and equal-risk-contribution (ERC) (Maillard, Roncalli, & Teiletche, 2010) dynamic style portfolios. By construction, these diversification-based allocations are less concentrated than the mean-variance allocation. The equal-weighting approach is the most diversified in terms of weight, whereas the ERC allocation seeks diversification in terms of risk. A further advantage of diversification-based approaches compared with the mean-variance allocation methods is that they do not require an estimation of the expected returns.

Panel A of Table 6 presents the performance characteristics for the diversification-based approaches. The equally weighted and ERC portfolios have similar return and risk characteristics, but the turnover is lower for the equally weighted portfolio. A comparison of the mean-variance efficiency criterion and the diversification-based approaches indicates that the MVDS portfolio outperforms the equally weighted and ERC portfolios in terms of the realized mean returns and volatility. In addition, the maximum drawdown is lower and the alphas for the four-factor and five-factor model are higher. The largest difference is in terms of turnover, which is less than 1% for the equally weighted approach, thereby indicating the static nature of that strategy compared with the mean-variance efficient style allocation portfolios, where the turnover tends to be around 40%. In the next section, we show that this turnover can be substantially reduced with almost no effect on the gross performance of the MVDS portfolio.

5.2. Reducing turnover

Optimized portfolios are often criticized due to the additional turnover imposed by dynamic adjustments. Higher turnover leads to higher transaction costs, thereby having a negative impact on portfolio performance. In this section, we limit the turnover by: (i) lowering the rebalancing frequency to monthly and quarterly rebalancing, (ii) imposing a no-trade zone, and (iii) including a turnover constraint.

[15] The underlying calculation for the net returns is as follows. Let Φ be the annual management fee and R be the gross return at a given month. Then, the net return (after management fee) for the corresponding month is $(1+R)(1-\Phi/12)-1 \approx R-\Phi/12$. If μ denotes the average out-of-sample average return and σ^2 is the out-of-sample variance, the realized utility is given by $\mu - \frac{\gamma}{2}\sigma^2$.

Table 6
Performance results for diversification-based and mean-variance optimized dynamic style portfolios during 1990–2013.

	Mean	Vol	SR	VaR	MDD	TE	$\hat{\alpha}_4$	$\hat{\alpha}_5$	Turn
Panel A: Risk-based portfolios									
Equally weighted (1/*K*)	9.65	14.44	0.43	−6.60	51.01	2.12	0.77**	0.76**	0.78
Equal-risk-contribution	9.57	14.45	0.43	−6.59	51.14	2.15	0.76**	0.76**	135.19
Panel B: MVDS with monthly rebalancing									
$\text{MVDS}_{\gamma=1}$	9.58	14.45	0.43	−6.60	49.47	1.98	0.30	0.30	15.47
$\text{MVDS}_{\gamma=5}$	9.82	14.31	0.45*	−6.48	49.33	2.13	0.76*	0.75*	18.91
$\text{MVDS}_{\gamma=10}$	10.19	14.06	0.48**	−6.37	49.08	2.50	0.76*	0.76	21.38
Panel C: MVDS with quarterly rebalancing									
$\text{MVDS}_{\gamma=1}$	9.74	14.46	0.44*	−6.56	48.44	1.94	0.29	0.30	8.54
$\text{MVDS}_{\gamma=5}$	9.66	14.30	0.44*	−6.50	48.86	2.02	0.74*	0.73*	8.71
$\text{MVDS}_{\gamma=10}$	10.19	14.12	0.48**	−6.39	48.55	2.34	1.15**	1.15**	12.22
Panel D: MVDS with trading bounds									
$\text{MVDS}_{\gamma=1}$	9.76	14.48	0.44**	−6.56	47.69	1.72	0.55*	0.55*	31.70
$\text{MVDS}_{\gamma=5}$	9.98	14.19	0.46**	−6.42	47.09	2.26	0.87**	0.86**	23.09
$\text{MVDS}_{\gamma=10}$	9.85	13.97	0.46*	−6.31	47.26	2.89	0.86**	0.86**	41.06
Panel E: MVDS with turnover constraint									
$\text{MVDS}_{\gamma=1}$	9.24	14.69	0.40	−6.73	50.73	174.27	0.02	0.02	11.96
$\text{MVDS}_{\gamma=5}$	9.62	14.40	0.43*	−6.55	48.40	188.01	0.46	0.46	11.04
$\text{MVDS}_{\gamma=10}$	9.87	14.23	0.46**	−6.44	48.28	212.86	0.75*	0.75*	11.65
Panel F: MVDS with a risk-free asset									
$\text{MVDS}_{\gamma=1}$	8.61	13.83	0.38	−6.48	46.16	4.93	−0.30	−0.31	45.98
$\text{MVDS}_{\gamma=5}$	8.03	9.99	0.47	−4.19	31.52	8.38	0.61	0.60	44.71
$\text{MVDS}_{\gamma=10}$	6.39	6.27	0.49	−2.41	13.76	16.65	0.57	0.56	47.62
Panel G: 130/30 MVDS									
$\text{MVDS}_{\gamma=1}$	9.69	14.75	0.43	−6.66	49.70	2.55	0.36	0.38	65.50
$\text{MVDS}_{\gamma=5}$	10.07	14.35	0.47*	−6.43	48.06	2.75	1.26*	1.25*	55.18
$\text{MVDS}_{\gamma=10}$	10.13	14.07	0.48**	−6.26	43.84	3.00	1.04	1.02	75.96

Notes: MVDS: dynamic style mean-variance optimization with low ($\gamma = 1$), medium ($\gamma = 5$) and high ($\gamma = 10$) risk aversion parameter. MV: mean-variance optimized across all stocks. For the risk-adjusted return measures (Sharpe ratio and alpha), the table shows the results of significance tests, where *, **, and *** indicate that the Sharpe ratio and alpha differ significantly from the Sharpe ratio of the market capitalization portfolio and zero, respectively, at the 10%, 5%, and 1% levels based on the *t*-test with HAC standard errors. See Table 3 for details.

One approach to limit the monthly turnover is adding a no-trade zone to the portfolio allocation process. As shown by Leland (1999), this can be the optimal portfolio policy in the presence of transaction costs and estimation uncertainty for the optimal portfolio weights. We follow Brandt et al. (2009) by implementing the decision rules to trade using the mean-squared distance between the desired portfolio weights $\widetilde{\mathbf{w}}_{t+1}$ and the weights $\widetilde{\mathbf{w}}_{t^+}$ prior to the rebalancing date $t+1$, i.e., $\widetilde{\mathbf{w}}_{t+1} \equiv \widetilde{\mathbf{w}}_{t^+}$ if $\frac{1}{K}\sum_{k=1}^{K}(\widetilde{w}_{k,t+1} - \widetilde{w}_{k,t^+})^2 \le \xi^2$. The portfolio is only rebalanced if this distance is larger than a threshold ξ, which is set to 0.25. Higher values of ξ are associated with a risk of style drift between the actual portfolio weights and the desired weights, whereas a smaller value tends to lead to more turnover.

Alternatively, the monthly turnover can be limited by adding a maximum turnover constraint to (8). Starting from an initial holding $\widetilde{\mathbf{w}}_{t^+}$ immediately before the rebalancing date $t+1$, an investor buys $\widetilde{\mathbf{w}}^+_{t+1}$ and sells $\widetilde{\mathbf{w}}^-_{t+1}$ assets to achieve the desired weight $\widetilde{\mathbf{w}}_{t+1}$, i.e., $\widetilde{\mathbf{w}}_{t+1} \equiv \widetilde{\mathbf{w}}_{t^+} + \widetilde{\mathbf{w}}^+_{t+1} - \widetilde{\mathbf{w}}^-_{t+1}$. Turnover is defined as the sum of absolute trades across all assets; see (12). The optimization problem is the same as in (8) with the additional turnover constraint $\widetilde{\mathbf{w}}^+_{t+1} + \widetilde{\mathbf{w}}^-_{t+1} \le \tau$, where τ is the bound on the turnover. We set the monthly turnover bound to 10%, which is more restrictive than a bound on the average monthly turnover.

The performance results are presented in Table 6 (Panels B–E). Imposing turnover restrictions reduces the average turnover significantly but without a substantial loss in gross returns, thereby leading to higher net returns.

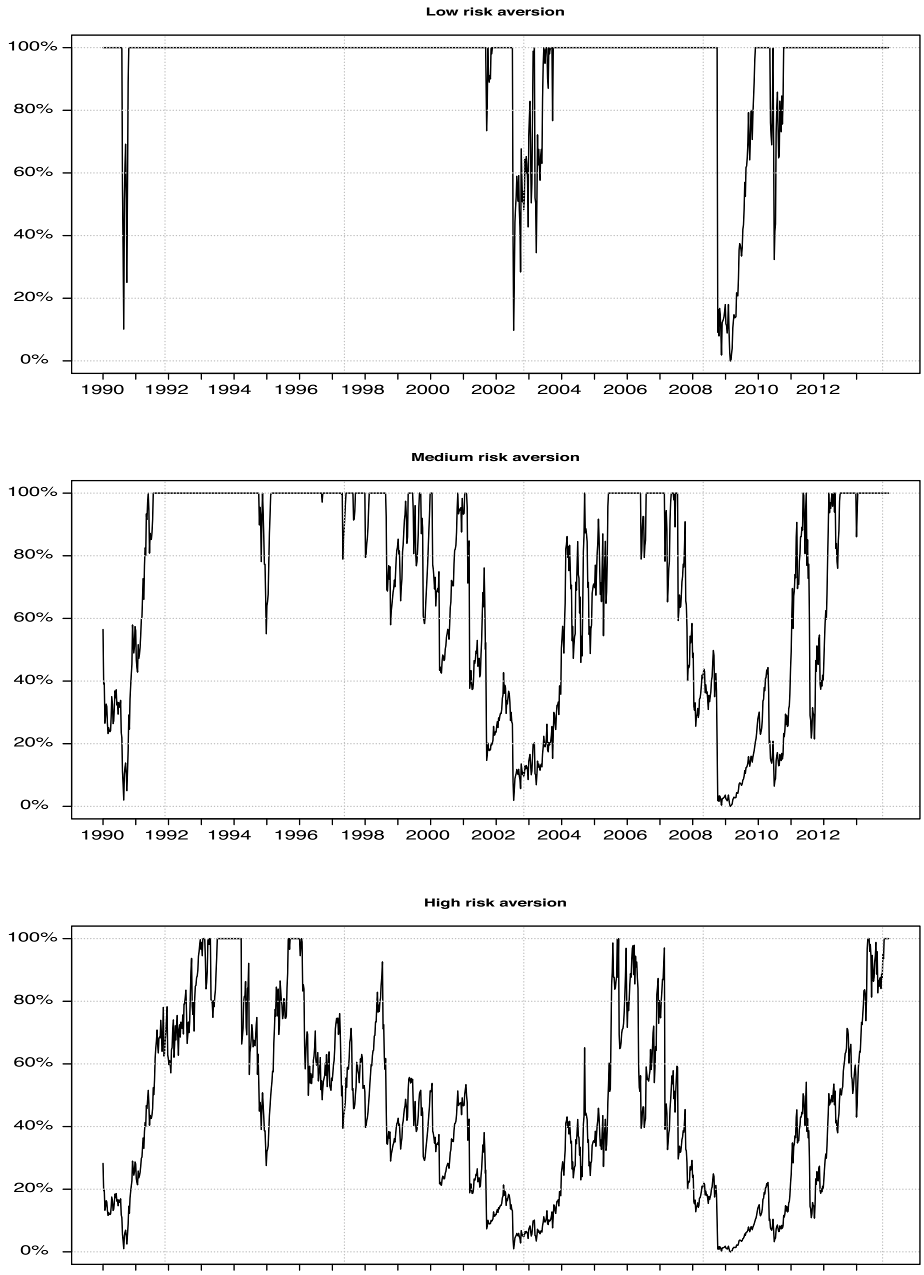

Fig. 5. Weight (%) invested in the equity portfolios for the equity-cash allocation portfolios with low (top panel), medium (middle panel) and high (bottom panel) risk aversion parameter.

5.3. Reduction of drawdown by including cash in the asset allocation

We now consider the more realistic case of an investor who invests in a risk-free asset and the characteristic-based portfolios at the same time. The risk-free rate used is the one-month

Table 7
Performance results of mean-variance optimized portfolios with alternative covariance matrix estimations.

	Mean	Vol	SR	VaR	MDD	TE	$\hat{\alpha}_4$	$\hat{\alpha}_5$	Turn
Panel A: MVDS with EWMA four-year estimation window									
$MVDS_{\gamma=1}$	9.61	14.49	0.43	−6.59	48.76	1.84	0.42	0.42	34.71
$MVDS_{\gamma=5}$	10.05	14.13	0.47**	−6.36	47.04	2.34	0.91**	0.91**	52.26
$MVDS_{\gamma=10}$	9.86	13.94	0.46*	−6.28	47.30	3.00	0.91**	0.90**	74.52
Panel B: MV with EWMA four-year estimation window									
$MV_{\gamma=1}$	1.72	36.23	−0.04**	−15.63	88.52	29.88	−7.70	−7.62	98.67
$MV_{\gamma=5}$	6.97	21.01	0.17	−9.08	61.99	15.39	−2.63	−2.61	128.35
$MV_{\gamma=10}$	6.90	16.48	0.22	−7.27	53.93	11.82	−1.86	−1.87	140.12
Panel C: MVDS with EWMA five-year estimation window									
$MVDS_{\gamma=1}$	9.63	14.67	0.43	−6.69	48.86	1.66	0.41	0.41	44.59
$MVDS_{\gamma=5}$	10.05	14.12	0.47**	−6.35	47.06	2.40	0.93**	0.92**	50.72
$MVDS_{\gamma=10}$	9.84	13.90	0.46*	−6.26	47.14	3.09	0.92**	0.90**	72.66
Panel D: MV with EWMA five-year estimation window									
$MV_{\gamma=1}$	15.04	34.09	0.34	−14.27	88.10	27.79	4.94	5.03	83.38
$MV_{\gamma=5}$	12.26	20.25	0.43	−8.77	61.79	14.44	2.51	2.53	121.45
$MV_{\gamma=10}$	10.24	16.30	0.42	−7.17	52.58	11.36	1.45	1.45	134.90
Panel E: MVDS with shrinkage covariance matrix									
$MVDS_{\gamma=1}$	9.85	14.45	0.45**	−6.54	47.76	1.77	0.64**	0.64*	37.51
$MVDS_{\gamma=5}$	10.04	14.12	0.47***	−6.35	47.17	2.25	0.88**	0.87**	42.25
$MVDS_{\gamma=10}$	9.98	14.03	0.47**	−6.31	47.30	2.95	0.94**	0.93**	76.64
Panel F: MV with shrinkage covariance matrix									
$MV_{\gamma=1}$	5.11	35.14	0.05*	−14.85	88.32	29.43	−4.49	−4.40	90.18
$MV_{\gamma=5}$	8.35	21.00	0.24	−9.08	62.18	15.62	−1.93	−1.91	107.49
$MV_{\gamma=10}$	9.60	15.96	0.39	−6.96	49.56	11.78	0.56	0.54	109.63
Panel G: MVDS with sample covariance matrix									
$MVDS_{\gamma=1}$	9.84	14.43	0.45**	−6.54	48.34	1.88	0.65*	0.65*	25.40
$MVDS_{\gamma=5}$	10.06	14.21	0.47***	−6.40	48.13	2.07	0.85**	0.85**	32.93
$MVDS_{\gamma=10}$	10.30	13.96	0.49***	−6.27	47.37	2.50	1.17***	1.17***	47.22
Panel H: MV with sample covariance matrix									
$MV_{\gamma=1}$	4.66	34.22	0.04*	−14.60	88.73	28.77	−4.83	−4.74	92.46
$MV_{\gamma=5}$	8.48	19.68	0.26	−8.59	64.47	14.69	−1.43	−1.42	93.98
$MV_{\gamma=10}$	8.32	15.38	0.32	−6.77	53.25	11.36	−0.45	−0.46	87.59
Panel I: MVDS with dynamic conditional covariance matrix									
$MVDS_{\gamma=1}$	9.78	14.47	0.44**	−6.56	47.66	1.81	0.56	0.56	60.98
$MVDS_{\gamma=5}$	9.88	14.26	0.46**	−6.46	46.60	2.36	0.67*	0.67*	120.38
$MVDS_{\gamma=10}$	10.09	14.12	0.47**	−6.32	46.76	2.93	1.04***	1.04***	176.82

Notes: MVDS: dynamic style mean-variance optimization with low ($\gamma = 1$), medium ($\gamma = 5$) and high ($\gamma = 10$) risk aversion parameter. MV: mean-variance optimized across all stocks. For the risk-adjusted return measures (Sharpe ratio and alpha), the table shows the results of significance tests, where *, **, and *** indicate that the Sharpe ratio and alpha differ significantly from the Sharpe ratio of the market capitalization portfolio and zero, respectively, at the 10%, 5%, and 1% levels based on the *t*-test with HAC standard errors. See Table 3 for details.

Treasury bill rate. The portfolio optimization problem is similar to (8), except that $\widetilde{\boldsymbol{\mu}}_{t+1|t}$ is now set as the excess return over the risk-free rate and the equity weights in $\widetilde{\mathbf{w}}_t$ are still long-only, but they no longer need to sum to unity. The remaining portfolio weight is invested in the risk-free asset.

The proportion of weights allocated to the equity portfolios depends on the level of risk aversion. The top panel of Fig. 5 shows this weight in the case of low risk aversion ($\gamma = 1$), which demonstrates that the portfolio is mainly concentrated in the equity portfolios. In the middle and bottom panels of Fig. 5, we see that the weight assigned to the equities exhibits higher variation for medium and high risk aversion. Following the financial crisis, the medium and high risk aversion portfolios are almost fully invested in the risk-free asset. On average, an investor with medium risk aversion invests 28% in the market capitalization portfolio, 15% in the inverse volatility portfolio, 27% in the equally weighted portfolio, and 29% in the risk-free asset.

Adding a risk-free investment has a positive impact on the return and risk characteristics. Panel F of Table 6 shows the performance measures in the case of and extended investment universe. The annualized average returns decreases from approximately 10–8%, but the volatility is also lower. The maximum drawdown decreases substantially from approximately 47% to 31%. The turnover is similar to the full-equity MVDS portfolio.

5.4. Relaxing the long-only assumption: 130/30 optimized portfolios

In the main applications, we study the performance of the portfolios for a long-only investor. We now consider an investor who can short up to 30% of his portfolio value. Panel G of Table 6 reports the performance measures for the 130/30 optimized MVDS portfolios. Compared with the results for the long-only investor in Table 4, we find that allowing for 30% short sales leads to similar mean and volatility and a slightly higher tracking error. The largest changes are in terms of portfolios' alpha, which are still positive, but only significant for investors with medium risk-aversion. On average, the turnover increases by more than 10%. We therefore recommend to implement the MVDS portfolio with a long-only constraint.

5.5. Sensitivity to the estimation window and covariance matrix estimate

The MVDS portfolio weights depend on the estimated covariance matrix. It is beyond the scope of our paper to report the performance results for all possible estimators. Our main finding is however that the results are robust to the choice of covariance estimation method and the choice of estimation window. We document this claim in Table 7 where we report the performance of the portfolios for several estimation setups: (i) an estimation sample of four (Panels A–B) and five years (Panels C–D) (instead of three years), (ii) the use of the shrinkage covariance estimate of Ledoit and Wolf (2003) with the equicorrelation matrix as target (Panels E–F) and sample covariance matrix (Panels G-H) and (iii) the approach of using the dynamic conditional correlation model of Engle (2002) with GARCH(1,1) dynamics for the variances (Panel I). The performance of the MVDS portfolios is qualitatively similar for all estimation approaches. For the MV portfolios optimizing at the stock level, the results are more sensitive to choice of covariance estimation method, but the general result of superiority of the MVDS portfolio with respect to the MV portfolio still holds.

6. Conclusion

What is the best way to invest in stocks? Traditionally, the choice has been between passive equity portfolios invested in a market capitalization portfolio and active portfolio management by exploiting dynamic changes in the expected returns and covariances of stocks. In this study, we investigate a middle ground by dynamically investing in ETF-mimicking portfolios that track the performance of characteristic-based portfolios, as well as exploiting the time-variation in the covariance and risk premia associated with an investment in these portfolios. This question is empirically relevant because ETFs that track alternative characteristic-based portfolios have grown in popularity in the past few years. In these characteristic-based portfolios, the weights are often set as a normalized version of the stock's characteristics such as the market capitalization, book value, or risk metrics. The characteristic-based portfolios claim to be a good proxy for the mean-variance efficient portfolios, but previous research demonstrates that their relative performance is time-dependent. Thus, to exploit the life-cycle dependency, we propose a method for dynamic style portfolio allocation and we derive the conditions under which this approach is mean-variance efficient.

We test the proposed style allocation portfolios on the investment universe of S&P 100 stocks over the period 1990–2013 on a wide set of performance measures. We find that the time-variation in the relative performance of the characteristic-based portfolios can be exploited in terms of significant stability gains. The dynamic style portfolios obtain a more smooth wealth creation path compared with the buy-and-hold investment in the characteristic-based portfolios.

The former has similar performance characteristics to the best performing single characteristic-based portfolio, but has the advantage of being less sensitive to the market regime. The market timing bets are as intuitively expected. The market capitalization criterion dominates the allocation in bullish periods (e.g., the bullish 90s), whereas the low risk criterion is the dominant investment in down-markets (e.g., following the 2008 financial crisis). The fundamental value and equal-weighting criteria have less influence on the optimized portfolio allocations. Importantly, the improvement in performance is also visible in terms of a positive and significant four-factor and five-factor model alpha in the dynamic style portfolio returns, after correcting for the risk factor exposures.

We also show that in the presence of estimation error, it is worthwhile for the investor to switch from a mean-variance allocation at the stock level to an approach where the investor only optimizes across the characteristic-based portfolios. We estimate that an investor would be willing to pay a management fee between approximately 5% and 11.5% to switch from stock-based allocation to allocation over characteristic-based portfolios. Since the average expense fee for ETFs is less than 50 basis points (The Wall Street Journal, 2015), the investor is thus better off by dynamically investing in ETFs than in stocks. As such our paper makes a strong case in favor of ETF investing and proposes an effective way to diversify investment style risk by dynamically allocating across the styles based on a conditional mean-variance optimization.

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