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Scanning near-field optical microscopy: transfer function and resolution limit

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Abstract

We present scanning near-field optical microscopy as an optical instrument characterized by a transfer function. This approach gives some theoretical guidelines for the design of near-field optical measurement systems. We emphasize that it is important to distinguish between the resolution for the optical field and the resolution for the object. In addition, to solve the general inverse diffraction problem the measurement of phase and amplitude of the electromagnetic field is necessary.

Keywords: Scanning near-field optical microscope; Rigorous diffraction theory; Sub-wavelength resolution

1. Introduction

Since the introduction of near-field optical microscopy, many different configurations of near-field measurements have been proposed [1]. Parallel to the technological advances, several efforts have been made in the theoretical modelling of near-field optical detection systems. A recent overview of near-field diffraction theories has been published by Girard et al. [2]. The paper includes different scattering theories, and theories based on the plane wave, respective multipole expansions of the electromagnetic field. The basic question to be answered by any theory is the resolution limit of near-field optical microscopy. Discussions about the resolution of near-field optical microscopes can be found in different papers [3–6]. Vigoureux [6] presents a historical overview of the resolution limit of classical microscopy (mainly the Rayleigh criterion), and analyzes the consequences for near-field microscopy. The main conclusion (which is in the meantime well known) is that information on sub-wavelength details of electromagnetic fields is contained in the non-radiative (or evanescent) components of the field. Therefore, it is important to collect the near-field information.

In our present paper, the near-field detection system is considered as an optical instrument characterized by a transfer function. For this purpose, the measurement system is separated into different parts as shown in Fig. 1.

The object is illuminated by an incident wave and generates a field distribution in the output plane. In general, the interaction of light with sub-wavelength structures has to be computed rigorously. We define a spectral response function H_{obj} as being the action of the object on the incident wave of a certain *spatial* frequency. The resulting field is detected by the scanning probe at a constant distance from the sample. The radiating part of the field propagates in free space, whereas the evanescent part is attenuated. This effect is described by the free space transfer function P. The probe itself is again characterized by a spectral response function H_{tip}. This approach can of course also be applied to the inverse case, where the tip is used to illuminate the structure.

Compared to a classical optical instrument, the term *resolution* has to be defined differently in near field optics.

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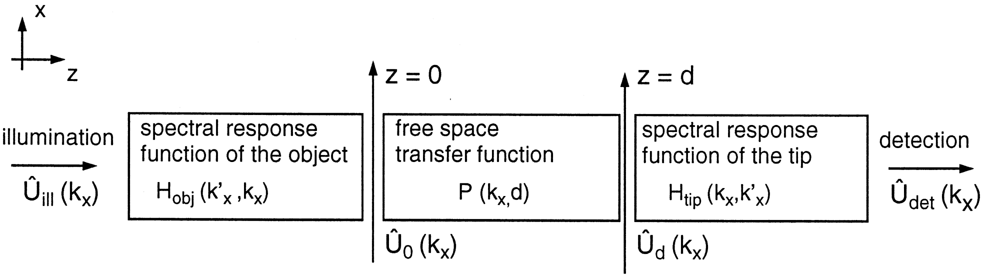

Fig. 1. The scanning near-field optical microscope is considered as an optical instrument characterized by spectral response and transfer functions.

In classical optics, the field is directly related to the object. Thus, the resolution for the field and the resolution for the object are the same. In near-field optics, due to the complicated interaction problem, the resolution is related to the ability to reconstruct the electromagnetic field in the output plane (or output space) of the object. In consequence, sub-micrometer resolution of a measured field does not imply that the object can be determined with the same accuracy.

We limit the analysis to non-elastic interaction, 2D geometry, and to TE-polarization. Furthermore, no back reflections from the tip are taken into account. This means that the theory presented here is not a self-consistent approach. However, it allows us to discuss the field resolution problem of near-field optical microscopy in a comprehensive way. Furthermore, the reconstruction of the object from the field data is not treated in this paper.

2. Spectral response function of the object

Most of the near-field papers describe the interaction problem by a macroscopic perturbation theory [7,8]. This approach writes the field as a superposition of the zero order field plus a first order perturbation term. This perturbation is proportional to the incident field and to the Fourier transform of the surface profile. However, the perturbation theory is limited to smooth surfaces, i.e. the surface roughness is typically small compared to the wave-

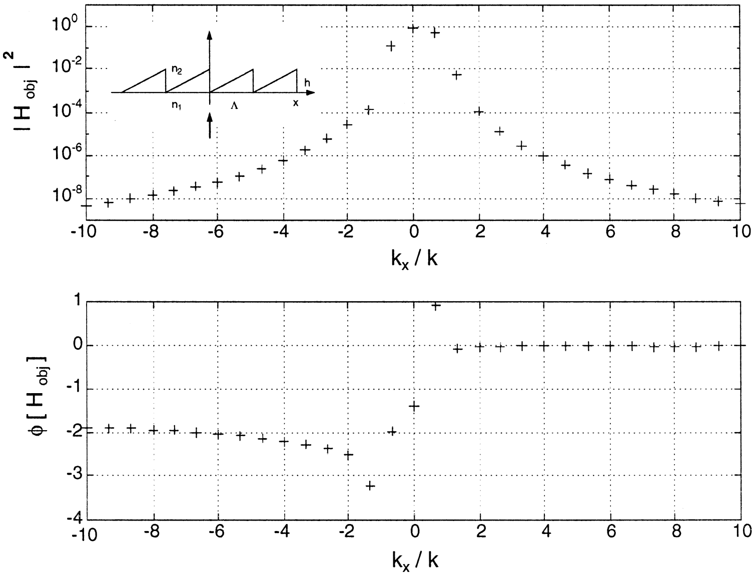

Fig. 2. Spectral reponse of a blazed grating structure (intensity and phase) in the output plane of the element for a perpendicular illumination. The spectrum has been calculated with the rigorous eigenmode method [12]. The parameters of the setup are $n_1 = 1.5$, $n_2 = 1$, grating period $\Lambda = 1.5\lambda$, height $h = \lambda$, and TE-polarization.

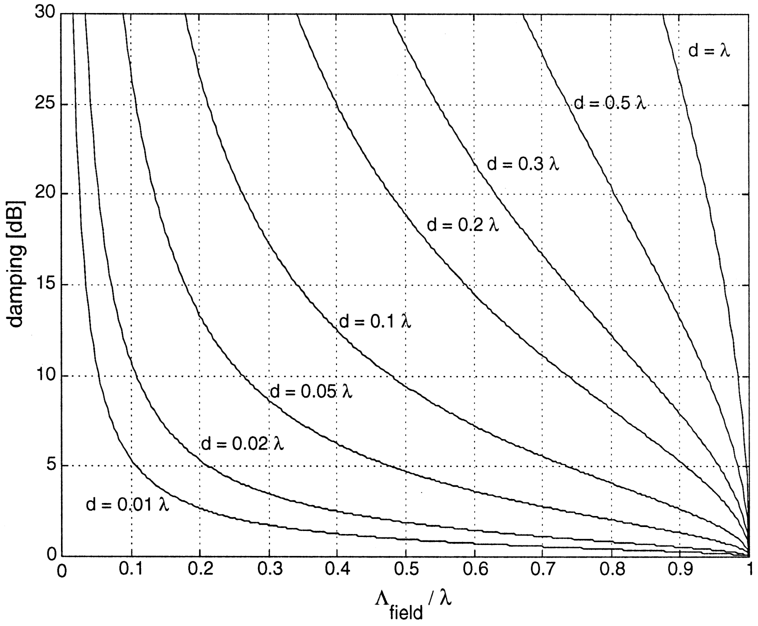

Fig. 3. Damping of evanescent waves versus the field size Λ/λ for different scanning distances.

length. For deep structures (e.g. typical surface relief gratings) this approach is not valid anymore and the solution of the interaction problem has to rigorously fulfil Maxwell's equations. In the case of periodic objects, most of the theories are based on the differential representation of the inhomogeneous wave function [9–12]. These approaches directly compute the complex amplitudes of the diffracted (propagating and evanescent) waves. Hence, it is possible to compute the light distribution in the near-field of any periodic structure [13].

For linear materials and coherent illumination, it is possible to define a spectral response function $H_{\text{obj}}(k'_x, k_x)$ which relates the output spatial frequency spectrum $\hat{U}_0(k_x)$ to the incident excitation $\hat{U}_{\text{ill}}(k'_x)$ by a linear superposition

$$\hat{U}_0(k_x) = \int_{-\infty}^{\infty} \hat{U}_{\text{ill}}(k'_x) H_{\text{obj}}(k'_x, k_x)\, \mathrm{d}k'_x . \tag{1}$$

This function describes the action of the object on an incident wave. In its most general form the entire spectrum is taken into account, i.e. $-\infty < k_x, k'_x < \infty$. In the case of infinite plane wave illumination ($\hat{U}_{\text{ill}}(k'_x) = A_0\, \delta(k'_x - k_{x0})$), Eq. (1) becomes

$$\hat{U}_0(k_x) = A_0 H_{\text{obj}}(k_x, k_{x0}) . \tag{2}$$

Eq. (2) is valid for the classical scanning near-field optical microscope (SNOM) and the photon tunneling microscope (PSTM/STOM). Fig. 2 shows the calculated plane wave spectrum of a blazed grating structure in a SNOM configuration.

The calculations were performed with the rigorous eigenmode method [12]. In general, the spectral response is a complex function including an amplitude and a phase distribution. It follows that, similar to the holographic reconstruction process, the amplitude *and* the phase have to be measured to reconstruct the original object field [2]. Different approaches for measuring the phase by near-field interferometry have already been presented in the open literature [16–20].

3. Free space transfer function

The radiating part of the generated object field is propagating in free space, whereas the evanescent part is attenuated. In the plane wave spectrum this transfer can be written as a filter function

$$U_d(k_x) = P(k_x, d) U_0(k_x), \tag{3}$$

where $P(k_x, d)$ is the free space transfer function and d is the distance from the tip to the sample. In particular, the complex free space transfer function is given by

$$P(k_x, d) = \begin{cases} \exp\left[\mathrm{i} d \sqrt{k^2 - k_x^2}\right] & \text{for} \quad |k_x| < k, \\ \exp\left[-d \sqrt{k_x^2 - k^2}\right] & \text{for} \quad |k_x| \geq k, \end{cases} \tag{4}$$

where $k = 2\pi/\lambda$ and λ is the free space wavelength. Thus, evanescent waves are affected by an amplitude

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[2] In holography, the original wave is reconstructed from the recorded phase and amplitude. Note that, for the case of weak perturbations, it is possible to reconstruct the surface profile without the necessity for phase retrieval [14,15].

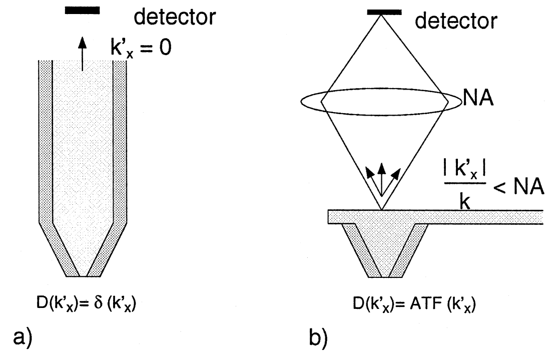

Fig. 4. Acceptance function $D(k'_x)$ for different geometries. (a) Fiber tip detection, (b) SNOM based on a cantilever tip.

change, propagating waves by a phase change. The local sub-wavelength field information is contained in the evanescent waves. The exponential damping of this evanescent waves has therefore some crucial consequences for the scanning tip distance. The damping factor is shown in Fig. 3 as a function of the field period Λ_{field} for different scanning distances. It is obvious that the scanning distance should be as small as possible.

In the complete system the damping effect of the higher diffraction orders is even increased by the low-pass filter characteristic of the probe tip.

4. Spectral response of the tip

The detection is made by scanning a tip over the sample. If the property of the detection system does not change during the scan the linear system is space invariant. Therefore, the detected signal can be expressed as a convolution

$$U_{\text{det}}(x) = \int_{-\infty}^{\infty} t(x-x')\,U_d(x')\,\mathrm{d}x' = t_{\text{tip}} * U_d\,, \tag{5}$$

where $t(x-x')$ is the impulse response of the detector. In frequency space Eq. (5) becomes a multiplication

$$\hat{U}_{\text{det}}(k_x) = T(k_x)\hat{U}_d(k_x)\,, \tag{6}$$

where $T(k_x)$ is the transfer function of the detector. For linear systems the transfer function is expressed as a linear superposition integral,

$$T(k_x) = \int_{-\infty}^{\infty} H_{\text{tip}}(k_x,k'_x)\,D(k'_x)\,\mathrm{d}k'_x\,, \tag{7}$$

where $H_{\text{tip}}(k_x,k'_x)$ is the spectral response of the tip and $D(k'_x)$ the detector acceptance function. The acceptance

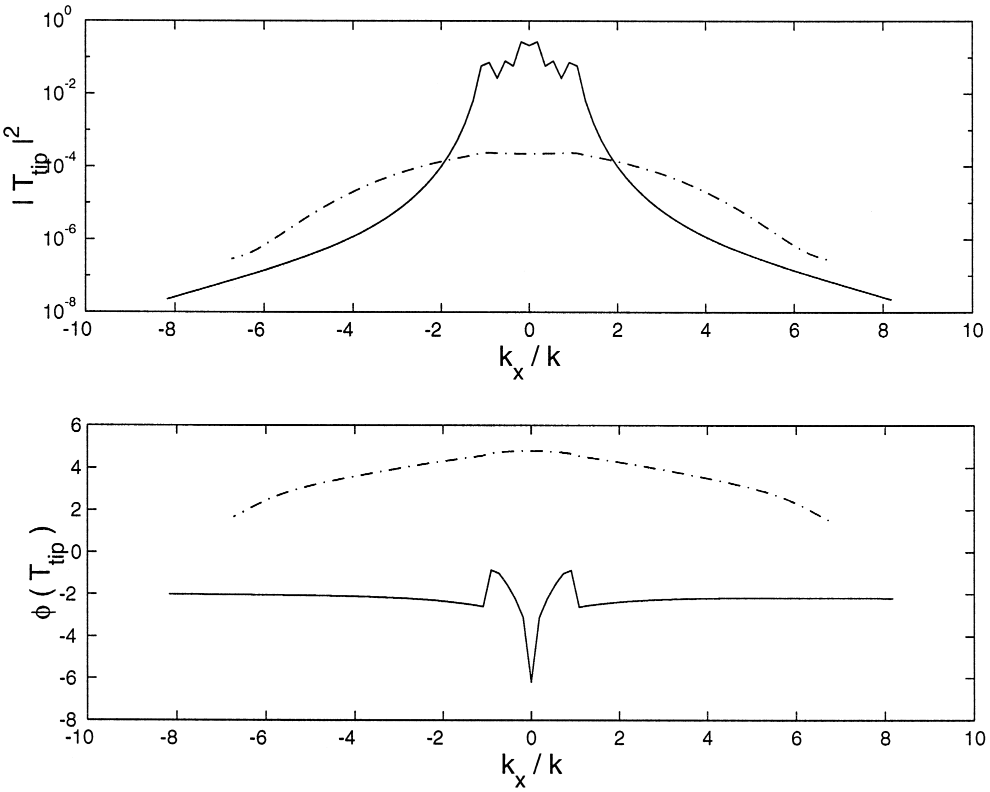

Fig. 5. Calculated intensity and phase of the complex spectral response function of a one-dimensional dielectric probe tip: uncoated (solid line) and coated (dashed line). The coated tip has a clear slit aperture of $\lambda/10$.

function of the tip depends on the chosen geometry. For a classical single-mode fiber tip SNOM (Fig. 4a) only one frequency is accepted, $D(k'_x) = \delta(k'_x)$ and the transfer function becomes particularly simple,

$$T(k_x) = H_{\mathrm{tip}}(k_x, 0)\,. \tag{8}$$

In the case of a cantilever tip based SNOM, the acceptance function of the detector is equal to the *coherent amplitude transfer function* (ATF) [21] of the detection system. The different frequencies are now integrated over the numerical aperture of the system. For an ideal imaging system the coherent amplitude transfer function is a binary function, Eq. (7) can be rewritten as

$$T(k_x) = \int_{-k\,\mathrm{NA}}^{k\,\mathrm{NA}} H_{\mathrm{tip}}(k_x, k'_x)\,\mathrm{d}k'_x \tag{9}$$

where NA is the numerical aperture of the imaging system. The aim of the tip-detector is to provide sub-wavelength resolution. For this purpose, the geometrical dimensions of the tip have to be in the sub-wavelength range. Thus, the spectral response $H_{\mathrm{tip}}(k_x, k'_x)$ also has to be calculated with a rigorous model. In Fig. 5 the spectral response of a coated and uncoated dielectric fiber tip is shown. The calculations have been made by the same algorithm as for the grating structure, by considering periodically arranged tips. The uncoated tips have higher coupling efficiencies for low spatial frequencies than the coated tips. On the other hand, the electromagnetic field is less confined, i.e. the angular spectrum is more narrow.

5. Field reconstruction, resolution

In its simplest configuration (plane wave illumination, fiber tip detection) the spectrum of the detected signal is obtained by multiplying the spectral response of the object field $H_{\mathrm{obj}}(0, k_x)$ with the free space transfer functions $P(k_x, d)$ (Eq. (4)) and spectral response of the tip $H_{\mathrm{tip}}(k_x, 0)$ (Eq. (6)),

$$\hat{U}_{\mathrm{det}}(k_x) = \hat{H}_{\mathrm{tip}}(k_x, 0)\,P(k_x, d)\,\hat{U}_0(k_x)\,, \tag{10}$$

$$\hat{U}_0(k_x) = \hat{U}_{\mathrm{ill}}(k_x)\,\hat{H}_{\mathrm{tip}}(0, k_x)\,. \tag{11}$$

Inversely, it is theoretically possible to determine the object field by deconvoluting the measured field with the impulse response of the system. For this purpose, the transfer function of the probe and the distance have to be known exactly. One the other hand, one should be aware that the process of deconvolution is in general quite sensitive to noise. A real measurement system is characterized by a certain signal to noise ratio (SNR). If the system is shot-noise limited, the SNR is proportional to the *power spectrum density* and the total number of collected photoelectrons $\bar{n}_{\mathrm{e}}$ during the scan (total integrating time). Hence

$$\mathrm{SNR}(k_x) = \frac{|\hat{U}_{\mathrm{det}}(k_x)|^2}{\sum_m |\hat{U}_{\mathrm{det}}(k_{x,m})|^2}\,\bar{n}_{\mathrm{e}}\,, \tag{12}$$

where $\hat{U}_{\mathrm{det}}(k_{x,m})$ is the sampled spectrum of the field.

As a numerical example, we consider the case of resolving a field variation at the object surface of 50 nm at a wavelength of 500 nm. For a scanning distance of 10 nm, the field is attenuated by 10 dB (Fig. 3). At a scanning distance of $d = 25$ nm the field is already damped by 25 dB. The low-pass filter characteristic of a coated tip, having an aperture of 50 nm, adds another 50 dB to the attenuation with respect to the central frequency. The situation is even worse for the uncoated tip (see Fig. 5), namely 80 dB instead of 50 dB. In this numerical examples, the signals are mainly decreased by the limited resolution of the tips. Considering a resolution of 100 nm, the response of the tips is much better. The coated tip attenuates the signal by 15 dB, the uncated by 60 dB. In practice, it is necessary to determine the spectral response of the tip experimentally.

6. Conclusions

The near field microscope can be considered as an optical instrument defined by a transfer function. In this representation, the limitation of the microscope can be clearly discussed, although backreflections from the tip are not taken into account. Sub-wavelength signals have to overcome the exponential attenuation due to propagation of the evanescent waves (Fig. 3) and the low-pass filter characteristic of the probe tip (Fig. 5). The spectral response of the tip expresses the capability of conversing evanescent to propagating waves. The system is characterized by a signal to noise ratio which depends on the spatial frequency. The signals have to be above the noise level of the instrument. This limitations are fundamental for any type of near-field microscopy. Furthermore, it is important to realize that in a general case, similar to holography, amplitude *and* phase measurements are needed in order to solve the reconstruction problem. We emphasize that there is a difference between the object resolution and the object *field* resolution. The resolution limit usually discussed in the open literature is the object *field* resolution. The statement how good the object is resolved has lost its classical meaning in sub-wavelength optics. The object reconstruction itself (its geometry and its dielectric behavior) needs some a priori information about the object, the detection system and the illumination.

Acknowledgements

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