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A Note on Definitions in Propositional Calculi [1]

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1 Implicit *versus* explicit definitions

If we call *definition* every symbolic procedure wich enables us to introduce a symbol α in a logical system and gives to the symbol an accurate and determined position in the deductive framework, then it is obvious that two main kinds of definitions occur in axiomatisations of propositional calculi: explicit definitions and implicit ones. Formaly speaking, explicit definitions are those which are constituted of two separate expressions, (respectively called *definiendum* and *definiens*) stated to be in a certain relation which underwrites their inferential equivalence and fulfil the following general conditions:

1. The *definiendum* is an expression including $\alpha(v_1 v_2 \cdots v_n)$, where α is the symbol to be introduced and $v_1, v_2, \cdots, v_n$ are variables indicating its argument places (if any).
2. The symbol α to be introduced is the only new symbol in the *definiendum* and it does not occur more than once.
3. No variable in the *definiendum* occur more than once.

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4. The *definiens* is a *wff* of the language in which the definition is stated, it contains only symbols previously introduced.

5. Every variable occuring in the *definiens* also occurs in the *definiendum.*

The aim of explicit definitions is then to introduce in a certain language L a new symbol α, by the way of an expression which is still available in L before the introduction of α. Clearly explicit definitions cannot be used for the introduction of all the symbols of the logic to be axiomatised, for the first explicit definition requiers the use of symbols which are to be introduced by a different method, namely by implicit definitions.

Implicit definitions can also be used in order to introduce non primitive symbols and, as we know (in particular from mathematical examples), implicit definitions involves a big variety of subtypes. It is not my aim to present a complete picture of them; it will suffice for the present purpose to remind that they are all based on the same idea that it is possible to characterize a symbol α by adding to the language in question, with the status of thesis, an expression (or plurality of expressions) containing α. As implicit definitions can notoriously lead to important difficulties which cannot be simply eliminate by some given general conditions on their construction (cf. e.g. Horwich [2] and Hale & Wright [1]), their use is usualy restricted to the introduction of symbols which cannot be explicitly defined.

The way primitive terms of a system are introduced is an important example of the necessity of implicit definitions. But the existence of different equaly valuable axiomatisations or formal systems for the same logic (or theory) based on different sets of primitive terms leads to questions I would like to examine and which have been often neglected in modern logic.

2 Traditional and modern conceptions

In the old tradition of the *more geometrico* method, dominated by the canonical example of Euclidian geometry, the choice of such or such set of primitive terms was conditioned by epistemic considerations: primitive terms had to be the simplest intuitively understandable ones which can be characterised by axioms true by clear evidence. The search of the smallest set of primitive terms fulfilings these conditions and adequate for the full axiomatisation of the intended theory was naturaly also a central goal.

The discovery of non-Euclidian systems of geometry was the point of departure of a complete re-examination of this traditional position: as different systems using the same vocabulary were accepted unless they were based on incompatible sets of axioms, it was no more possible to consider primitive terms to be related to already given intuitively clear notions and axioms to express true jugments concerning these notions. After this modification in the conception of the axiomatic method, primitive terms had to be considered as implicitly defined by the axiomatic basis and the traditional relationship between the axiomatic systems and natural intuition was (at least theoreticaly) cut.

One of the consequences of this revolution was that the choice between different axiomatisations of the same theory based on different repartitions of the terms into primitive and non-primitive ones was no more considered to be a matter of epistemic considerations (exepted maybe in pedagogical situations). The only still available choice criteria were the adequacy of the set of primitive terms and the search of the most economical solution. This latter criteria is of course not necessary but it has at least two important justifications. The first justification has often been seen purely aesthetic[2]; it consists in considering that the search of the smalest number of primitive terms is involved in the axiomatic method itself. The second justification is more interesting for my purpose: the choose of a smaller set of primitive terms is a way to restrict the use of implicit definitions, in favor of explicit ones.

At this point, it seems clear that the best axiomatisation of a theory will be that which can capture the whole intended theory using the smallest possible number of primitive terms. But I will show, through the example of classical propositional calculus, that the problem is far from beeing closed by this simple view.

3 Classical propositional calculus

According to the classical theory of adequate sets of propositional connectives, the following examples are perfectly possible choices for the ax-

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[2] For an example of this position, see Sobociński [21], an exellent and quite unknown paper in which a long list of criteria are examined, which can be used in comparing different axiomatic sytems.

iomatisation of the full classical calculus[3]: $\{\wedge, \vee, \sim\}, \{\wedge, \sim\}, \{\vee, \sim\}, \{\supset, \sim\}, \{|\}, \{\downarrow\}$. On the other hand, it is easy to prove that the following ones are not adequate: $\{\equiv, \sim\}, \{\supset\}, \{\supset, \vee\}, \{\equiv\}$. Several comments can be made concerning these examples: 1. the adequacy of $\{\wedge, \sim\}$ and $\{\vee, \sim\}$ shows clearly that $\{\wedge, \vee, \sim\}$ must be rejected by those who requier for their axiomatisation a set of independant connectives; 2. the only adequate sets with only one connective are $\{|\}$ and $\{\downarrow\}$[4]; 3. non-adequate sets can be used for the study of certain important fragments of the full classical calculus, as for example $\{\supset\}$-calculus or $\{\equiv\}$-calculus[5]; 4. even non-adequate, $\{\equiv, \sim\}$ gives rise to a wider fragment than $\{\equiv\}$ and $\{\supset, \vee\}$ is not stronger than $\{\supset\}$ for $\vee$ can be expressed in terms of $\supset$ alone.

According to the criteria discussed at the end of the last section the best axiomatisations of the full classical calculus have to be based either on $\{|\}$ or on $\{\downarrow\}$. Such an axiomatisation has been proposed in 1917 by Nicod [17], it consists in one single axiom and a single inference rule:

Axiom: $(P|(Q|R))|((S|(S|S))|((T|Q)|((P|T)|(P|T))))$
Rule: $\{P, P|(Q|R) \rightarrow R\}$

The other connectives of the calculus can then be introduced without difficulty by explicit definitions such as the following:

D1. $\sim P \quad =_{df} P|P$
D2. $P \vee Q =_{df} (P|P)|(Q|Q)$
D3. $P \wedge Q =_{df} (P|Q)|(P|Q)$
D4. $P \supset Q =_{df} \sim P \vee Q$
D5. $P \equiv Q =_{df} (P \supset Q) \wedge (Q \supset P)$

On a purely theoretical point of view, this kind of axiomatisation seems to be the best. Nevertheless most of modern logicians were not completly satisfied by it and did work with other adequate sets containing usualy two or even three connectives. The search for a clearer and more readable construction (two notions which are important in pedagogical presentations of logic) is not in my opinion sufficient to explain this situation. A more important reason is certainly that logicians try to find a system in which primitive terms are directly related to what is generaly conceived as central

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[3] Exepted for conjunction that I symbolise $\wedge$, I use in this paper the Peano-Russellian symbols for propositional connectives.

[4] For a proof, see for example Mendelson [13] (27-28).

[5] Classical studies of these calculi are to be found in Łukasiewicz [10] and [11].

theoretical notions of logic. In this respect, it seems to me that three connectives stand out against the whole picture, beeing in a special position: the conditional $\supset$ comes first for it is strongly related to the notion of deductibility (as deduction theorem shows); then comes the negation $\sim$, due to its central role in *reductio ad absurdum*; at last, the biconditional $\equiv$ has also to be mentioned for it is directly linked to the relation of logical equivalence. If I am right in this interpretation of logicians' inclinations, it becomes easy to understand that the most successfull set of primitive connectives is $\{\supset, \sim\}$ for none of the three above mentioned connectives is adequate alone and $\{\equiv, \sim\}$ is not adequate either (this fact giving to the biconditional the worst position).

This reason for the rejection of $\{|\}$ (or $\{\downarrow\}$) as the best choice in axiomatisation of the full propositional calculus is clearly of epistemic nature and cannot then be used officialy in an orthodox conception of pure axiomatic logic. The fact is that the use of $\{\supset, \sim\}$ gives more room to implicit definition than it is the case with $\{|\}$, with which only one connective is implicitly defined. This would be unproblematic if there would have been no difficulty in the increase of implicit definition in the case of $\{\supset, \sim\}$ comparing to $\{|\}$. Unfortunately, there is a difficulty which is quite wellknown. This difficulty is not easy to see in every axiomatisation of the $\{\supset, \sim\}$-calculus as for example the very compact Meredith's one [16]:

MerAx:
$((((p \supset q) \supset (\sim r \supset \sim s)) \supset r) \supset t) \supset ((t \supset p) \supset (s \supset p))$
Rules: *Det*, *Sub*

But it appears clearly in the following one based on three axioms and the two classical inference rules *Det* and *Sub*:

$A1 : p \supset (q \supset p)$
$A2 : (p \supset (q \supset r)) \supset ((p \supset q) \supset (p \supset r))$
$A3 : (\sim p \supset \sim q) \supset ((\sim p \supset q) \supset p)$
Rules: *Det*, *Sub*

Negation only occurs in the third axiom and is completely absent from the other axioms and from the inference rules. Thanks to this peculiarity it is possible to understand this axiomatisation either as the full $\{\supset, \sim\}$-calculus, or as the following $\{\supset\}$-calculus where the former axiom $A3$ is now an implicit definition of the non primitive symbol $\sim$:

$A1 : p \supset (q \supset p)$
$A2 : (p \supset (q \supset r)) \supset ((p \supset q) \supset (p \supset r))$
Rules: Det, Sub

$Def_{\sim} : (\sim p \supset \sim q) \supset ((\sim p \supset q) \supset p)$

These two constructions are of course perfectly equivalent in the sense that both determine the same set of theses. But the difficulty with the introduction of $\sim$ is made clear by the second one, for $\supset$-tautology (known as Peirce's law)

$Peirce$: $((p \supset q) \supset p) \supset p)$

is not a consequence of $A1$ and $A2$ by the use of the rules, but necessitates the introduction of $Def_{\sim}$ in order to be proved. This shows that $Def_{\sim}$ is a *creative* implicit definition, for it allows to prove an expression which does not include the new symbol $\sim$ and which is not provable whitout the definition. It is even *strongly creative* in the sense that the primitive notion which is modified by the addition of $Def_{\sim}$ is precisely the connective ($\supset$) which is used in the definition in order to characterise the new symbol $\sim$.

For this difficulty is a consequence of the introduction by implicit definitions of more than one connective, it shows, in this perspective, the superiority of axiomatisations based on a single term, like Nicod's one. Nevertheless, it has to be noticed that the whole picture cannot be given without paying attention to the way explicit definition is used; its importance regarding the notion of *adequacy* (a set of connectives is said to be *adequate* when every possible classical connective is *explicitly defininable* on its basis) cannot be neglegted in the present discussion.

4 Explicit definitions

A great majority of contemporary logicians seem to share the conception of explicit definition I will call here the *external* conception. In this conception the aim of a definition is the introduction of a new conventional way of designating certain expressions or range of symbols of the official object language of the system in question. In Nicod's system, for example, the introduction of $\sim$ by the way of

D1. $\sim P =_{df} P|P$

does not increase officialy the object language by the addition of the new symbol $\sim$, but gives only the possibility to designate in the metalanguage expressions of the form $P|P$ by the way of the convenient and shorter metalinguistic expression $\sim P$. It has then to be noticed that D1 can hardly be said to be a genuine definition. As a matter of fact, the metalinguistic symbol $=_{df}$ cannot state that the *definiendum* and the *definiens* are inferentialy equivalent, for the *definiendum* is not an expression of the system. Stricly speaking, it follows from this remark that no formal system (in the usual sense of these words) can be said to constitute the full classical propositional calculus, for such a system would never include the tautologies involving non primitive connectives. With the external conception of explicit definitions, saying that $\{|\}$ is *adequate* signifies that, in an ideal system containing *all* the classical connectives as primitive terms, every expression E would be provably equivalent with an expression E^* in which $|$ occurs as the only connective.

At first sight, these considerations seems to be of purely proof-theoretical interest, with no consequence on semantical or structural conceptions of logic. In the rest of this paper, I will show however that the adoption of a different conception of explicit definition, that I will call the *internal* conception, leads to an important re-evaluation of the notion of *adequate set of connectives.*

5 Internal explicit definitions

Originated in the logical work of S. Leśniewski, the central idea of the internal conception is to consider definitions as theses of the system[6]. As every thesis, they should be introduced on the basis of axioms and inference rules. Definitions are then to be *wff* of the object language. Aside from the symbol to be defined, they should not involve symbols which do not belong to the system, as for example $=_{df}$. So as to state that *definiendum* and *definiens* are inferentialy equivalent expressions, it is then only possible to use the primitive terms of the system. For that purpose the most natural choice is no doubt the biconditional $\equiv$. The choice of $\equiv$ is nevertheless only possible in systems including it as one of the primitive connectives.

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[6] Cf. Lesńiewski [8] and Rickey [18].

In the twenties, S. Leśniewski and A. Tarski showed that it is possible to obtain a extended propositional system (called Protothetic[7]), from wich full classical calculus is a part, on the basis of $\{\equiv, \forall_s, \forall_{s/s}\}$: biconditional and the universal quantifier binding propositional and unary connective variables (variables of category s and s/s)[8]. Apart from (more or less) usual inference rules, Protothetic includes also a special rule for the introduction of definition-theses; depending on what the system already contains, this rule allows to consider as theses closed biconditional expressions of the form

$$Def_\alpha : (\forall v_1 v_2 \cdots v_n)(\alpha(v_1 v_2 \cdots v_n) \equiv E_{v_1, v_2, \cdots, v_n})$$

where the first argument of $\equiv$ is the *definiendum* and the second one the *definiens*, both fulfiling the general conditions presented at the begining of this paper.

Of course this solution requiers the use of a quantified propositional logic, but Leśniewski and Tarski also examined simpler possibilities in the framework of classical unquantified systems. Using, for example a complete $\{\equiv, \sim, \vee\}$-calculus, a definitional rule can be added which is based on the definitional frame

$$\text{(DF1)}: \; D_{um} \equiv D_{ens}$$

the following thesis beeing an example:

$$Def_\supset : (p \supset q) \equiv (\sim p \vee q)$$

Even definitions stated with $\equiv$ are the most natural, the authors of the Warsaw School examined also possibilities using other primitive connectives. On the basis of the provable inferential equivalence of the following expressions in the complete $\{\equiv, \sim, \supset, |\}$-calculus

$$p \equiv q$$
$$\sim ((p \supset q) \supset \sim (q \supset p))$$
$$(p|q)|((p|p)|(q|q))$$

the authors showed that a definitional $\{\supset, \sim\}$-calculus is possible with the definitional frame

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[7] For a modern presentation of Lesniewski's Protothetic, see Miéville [14] and [15].

[8] Definition of $\sim$ and $\wedge$ on the basis of $\{\equiv, \forall_s, \forall_{s/s}\}$ is to be found in Tarski [22]; see also Miéville [14] (164-174). Other interseting solutions based on $\{\equiv, \forall_s, \forall_{s/ss}\}$ are presented in Sobociński [20].

(DF2): $\sim((D_{um} \supset D_{ens}) \supset \sim(D_{ens} \supset D_{um}))$

and also a definitional $\{|\}$-calculus with the frame

(DF3): $(D_{um}|D_{ens})|((D_{um}|D_{um})|(D_{ens}|D_{ens}))$

At this point a particular attention must be paid to the $\{\supset\}$- and $\{\supset,\equiv\}$-fragments of the full classical calculus, for it can be shown in the latter that the following expressions are inferentialy equivalent provided v is a propositional variable which occurs neither in A nor in B:

$A \equiv B$

$((A \supset B) \supset ((B \supset A) \supset v)) \supset v$

This result shows that a definitional $\{\supset\}$-calculus is possible with the following definitional frame

(DF4): $((D_{um} \supset D_{ens}) \supset ((D_{ens} \supset D_{um}) \supset v)) \supset v$

where v is any variable wich does not occur either in D_{um} or in D_{ens}.

6 Lejewski's $\supset$-definitions

As in the classical calculi using external explicit definitions stated with $=_{df}$, the difference between $\{|\}$ and $\{\supset\}$ still remains here, for it is still impossible in the definitional $\{\supset\}$-calculus to define all the classical connectives, at least without introducing quantification[9].

Nevertheless, Lejewski [7] had the idea to use definitional frame (DF4) with an other kind of *definiendum*. As it is illustrated by several mathematical accepted definitions, like for example the following one, which introduces the new symbol $\circ$ on the basis of the primitive functor S (successor):

$Def_{\circ} : \circ(n) = m \quad =_{df} \quad S(S(n)) = m$

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[9] As it was already shown by Russell [19], $\sim P$ can be defined in $\{\supset, \forall_s\}$-calculus using the *definiens*: $P \supset (\forall v)v$.

the notion of explicit definition does not requier the *definiendum* (here: $\circ(n) = m$) to be identical with the function to be defined ($\circ(n)$). Using this possibility for the *definiendum* to be different from the *function to be defined* (F_{tbd}), Lejewski proposed to replace in (DF4) both occurences of D_{um} (up to then identical with F_{tbd}) by a *definiendum* of the form

$$D_{um} : F_{tbd} \supset w$$

where w is a propositional variable which does not occur in F_{tbd}. Apart from this peculiarity of Lejewski's $\supset$-defininitions, the general conditions concerning the constitution of the *definiendum* and the *definiens* are still the usual ones.

The two following $\supset$-definitions using (DF4) and the new sort of D_{um} are examples given by Lejewski. The first example is a definition of classical negation:

$$Def_{\sim} : (((\sim p \supset w) \supset ((p \supset w) \supset w)) \supset ((((p \supset w) \supset w) \supset (\sim p \supset w)) \supset v)) \supset v$$

where

$$D_{um} = \sim p \supset w \quad ; \quad F_{tbd} = \sim p$$
$$D_{ens} = (p \supset w) \supset w$$

Second example (Sheffer's stroke):

$$Def_{|} : ((((p|q) \supset w) \supset ((p \supset (q \supset w)) \supset w)) \supset ((((p \supset (q \supset w)) \supset w) \supset ((p|q) \supset w)) \supset v)) \supset v$$

where

$$D_{um} = (p|q) \supset w \quad ; \quad F_{tbd} = p|q$$
$$D_{ens} = (p \supset (q \supset w)) \supset w$$

It must be precised that these definitions are stated in a definitional $\{\supset\}$-system called by Lejewski S_1 which is a version of a Łukasiewicz's complete $\{\supset\}$-calculus including a special rule for the use of definitions[10].

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[10] For a description of S_1 rules, see Lejewski [7], 195.

System S_1:
ŁukAx: $((p \supset q) \supset r) \supset ((r \supset p) \supset (s \supset p))$
Rules: Det, Sub, Def

At last, Lejewski [7] proved that the above written examples of $\supset$-definitions are adequate for the introduction of the classical connectives $\sim$ and $|$ and also that there exist in his system S_1 adequate similar $\supset$-definitions for every classical connective[11]. S_1 is then a complete axiomatisation of the full classical propositional calculus.

7 Conclusion

Due to a long tradition which culminated in Whitehead and Russell's *Principia Mathematica*, it has been widely thought in logic that definitions are only to introduce "mere typographical conveniences (...) theoretically superfluous" ([24], I.11). This view is essentialy related to the external conception of explicit definitions. Russell was nevertheless one of the first to underline the strange status of external definitions:

> It is a curious paradox, puzzling to the symbolic mind, that definitions, theoretically, are nothing but statments of symbolic abreviations, irrelevant to the reasoning and inserted only for practical convenience, while yet, in the development of a subject, they always require a very large amount of thought, and often embody some of the greater achievements of analysis ([19], 63).

On the other hand, it is perfectly possible to reject this standard position and the view that definitions are "irrelevant to the reasoning". This leads to the adoption of an internal conception and the idea that definitions have to be considered as theses of the formal system, whose introduction is justified by the use of a special inference rule. This, of course, requiers that we change our idea of what a formal system is: due to the fact that new symbols are officially introduced in the object language, the system can no

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[11] In the case of negation, Lejewski shows that ŁukAx + $Def_{\sim}$ associated with Det and Sub constitute a complete classical $\{\supset, \sim\}$-calculus, ([7], 197-198).

more be viewed as a (closed) set of theses. It is rather a formal machinery in which theses have to be ordered, following step by step the inscriptions of definitions.

My aim in this note was not to argue in favor or against this non-standard approach of formal systems, but only to show that certain central notions of logic are dependant on our view on explicit definitions. Lejewski's important (and quite unknown) result that $\{\supset\}$ is adequate for the construction of the full classical propositional calculus goes clearly in that direction. It shows that the notion of *adequate set of connectives* is wider in the internal conception than it is in the standard external one[12].

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[12] For a more complete discussion on internal definitions, see also my [4] and [5].

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