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Design of Lattice Wave Digital Filters in SPW Environment

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ABSTRACT

Lattice Wave Digital Filters (LWDFs) are known to be insensitive to finite precision arithmetic effects and can readily be designed for standard filtering applications using a set of explicit formulas elaborated by Gazsi.

This paper presents the necessary elements to develop an LWDF design package within the Signal Processing Workstation (SPW) design framework. This package, based on Gazsi's formulas and additional frequency transformations, supports the realisation of lowpass, highpass, bandpass and band-reject filters using either Butterworth, Chebyshev, or Elliptic approximations. Ideal and finite precision arithmetic models of the basic LWDF components are also provided to perform detailed simulations.

1. INTRODUCTION

The *Signal Processing Worksystem* (SPW/COMDISCO) is an efficient top-down design framework for the development, simulation, and implementation of Digital Signal Processing (DSP) algorithms. Among other facilities, this environment provides the necessary tools for the design of conventional FIR filters (equiripple, windowed) and IIR filters (Butterworth, Chebyshev, Elliptic, Bessel).

However, the design of IIR filters is only supported in form of a cascade of first and second-order sections, which - under certain circumstances - show to be sensitive to finite arithmetic effects. Therefore, it can be useful to enhance the available library with complementary filters, such as Wave Digital Filters, which are suitable for more critical applications.

Wave Digital Filters (WDFs) are obtained from analogue reference filters and are characterised by excellent numerical properties with respect to coefficient accuracy requirements, dynamic range, and all aspects of stability under finite-arithmetic conditions [1]. Although WDFs can be designed using many different filter structures, the *lattice* configuration shows to be one of the most flexible and popular, in particular due to Gazsi's straightforward design procedure [2].

To complete the SPW framework with a design package for Lattice Wave Digital Filters (LWDFs), it is necessary to develop specific synthesis methods for the *Filter Design System* (FDS), and a particular library for the *Block Diagram Editor* (BDE), where FDS and BDE are both internal tools of SPW.

The synthesis methods are directly based on Gazsi's design procedure, while the BDE library holds the necessary blocks to generate the filter signal flow graph in either ideal or finite precision arithmetic.

Three amplitude approximation functions are available for the design of LWDFs, namely Butterworth, Chebyshev, and Elliptic approximations, and they can be used for the realization of lowpass, highpass, bandpass, and band-reject filters.

The remainder of this paper is organised as follows. Chapter 2 introduces the basic properties of WDFs and LWDFs, while chapter 3 briefly presents the SPW environment. Next, chapter 4 will discuss the FDS synthesis methods previewed for the design of LWDFs. The necessary blocks for ideal and finite precision modelling and simulation of LWDFs are then presented in chapter 5. Finally, an example illustrating each design step is given in chapter 6.

2. INTRODUCTION TO WDFs

This chapter briefly reviews the essential properties of Lattice Wave Digital Filters. The interested reader is invited to refer to [1, 2] for a more detailed discussion.

2.1 Basic concepts

Three complex frequency domains are traditionally considered in the field of Wave Digital Filters [1, 10], namely :

- the p-domain, where p is the actual complex frequency;
- the ψ-domain, where ψ corresponds to Richards' variable used for the design of the analogue reference filter;
- the z-domain, where z is the common complex frequency variable used for digital signal processing.

The introduction of Richards' frequency variable ψ is motivated by the fact that WDFs are basically derived from analog reference filters with either a lumped or a commensurate distributed network, or even a mixture of both filter structures. As such, the variable ψ provides a common filter description for the analogue reference filters.

The frequency variables are defined as follows :

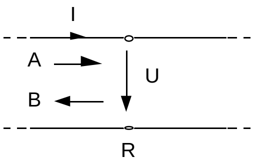
$$\psi = \frac{z-1}{z+1} \quad ; \qquad \psi = \tanh(pT_S/2) \quad ; \qquad z = e^{pT_s} \qquad \text{(2.1a, b, c)}$$

where T_s is the sampling period.

According to (2.1a), one notices that the frequency variables ψ and z are related by the normalized bilinear transform. Also, one observes that real frequencies ω in the p-domain are mapped onto real frequencies in the ψ-domain, and reciprocally :

$$\varphi = \tan(\omega T_S/2) \quad ; \qquad \text{for :} \quad p = j\omega \quad , \qquad \psi = j\varphi \qquad (2.2)$$

The correspondence between a WDF and the related reference filter is established in the frequency domain, and the relevant signal quantities are defined with respect to each *port* of the reference network (Figure 1). One distinguishes two kind of signal quantities, namely so-called *incident waves A* and *reflected waves B*, which are related to the port voltage, current, and resistance according to (2.3) [1].



$$A = U + RI \qquad B = U - RI \tag{2.3}$$

Fig. 1: Port representation and related wave quantities

2.2 Lattice Wave Digital Filters

Lattice Wave Digital Filters (LWDFs) are derived from doubly terminated lossless and symmetrical lattice filters as shown in Figure 2. These reference filters are assumed to be terminated on equally-valued resistances, and contain two canonic reactances Z_1 and Z_2.

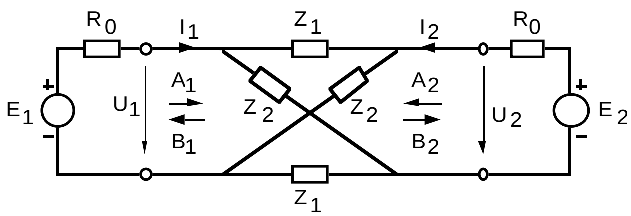


Fig. 2: Symmetrical lattice reference filter

According to the classical scattering parameter theory [9, 10], the scattering matrix **S** of a two-port network is defined as :

$$\begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} \tag{2.4}$$

Taking into account the symmetry of the lattice filter (i.e. $S_{21} = S_{12}$; $S_{11} = S_{22}$), and applying the Bartlett-Brune bisection theorem [9] to determine the input impedances as seen from each network port, one can readily determine the scattering coefficients [1] :

$$S_{21} = S_{12} = (S_2 - S_1)/2 \tag{2.5a}$$

$$S_{11} = S_{22} = (S_2 + S_1)/2 \tag{2.5b}$$

where S_1 and S_2 correspond to the reflectances of impedances Z_1 and Z_2, respectively :

$$S_i = \frac{Z_i - R_0}{Z_i + R_0} \tag{2.6}$$

The reflectances S_1 and S_2 take the form of all-pass functions, since Z_1 and Z_2 are pure reactances for lossless two-ports [1, 9].

Finally, rearranging and factoring the achieved expressions, one obtains the following relation :

$$\begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} S_1 & 0 \\ 0 & S_2 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} \tag{2.7}$$

For the sequel of this paper, we will consider the case of a single input network with $A_2 = E_2 = 0$, and the corresponding wave flow graph is given in Figure 3. Also, Gazsi's notation will be adopted [2], where B_1 is assumed to be the normal output signal while B_2 is defined as the complementary output signal.

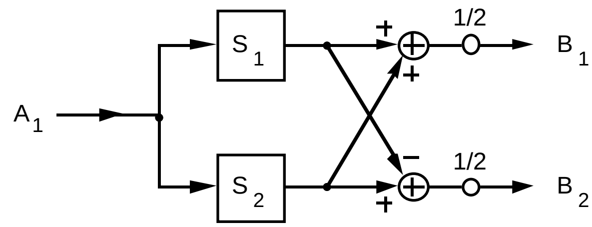


Fig. 3: Wave flow graph of a symmetrical lattice two port

2.2.1 First and second-order all-pass functions

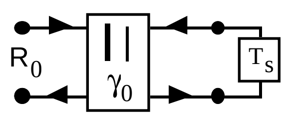
The reflectances S_1 and S_2 can be realised using many different canonic decomposition schemes for the design of Lattice Wave Digital Filters, including the Foster and Cauer structures, Richards' structure, the cascade of all-pass sections, or any combination of these possibilities [1].

However, one of the most appealing and used structure is the canonic cascade of elementary all-pass sections. This is motivated by the simplicity and regularity of the resulting filter structure, and by the availability of the efficient design procedure provided by Gazsi's [2]. Moreover, this implementation structure is well-adapted to the realisation of half-band (or bireciprocal) and branching filters [1, 2].

The reflectance of a first-order all-pass section is specified by :

$$S_0 = \frac{-\psi + B_0}{\psi + B_0} \tag{2.8}$$

and the related wave flow graph, containing a so-called two-port adaptor [1, 2], is given in Figure 4.



where $\gamma_0 = \dfrac{1 - B_0}{1 + B_0}$ (2.9)

Fig. 4: Wave flow graph of a first-order all-pass section

Similarly, the reflectance of a second-order all-pass section is defined by:

$$S_i = \frac{\psi^2 - A_i \psi + B_i}{\psi^2 + A_i \psi + B_i} \tag{2.10}$$

and the corresponding WDF realisation is shown in Figure 5 :

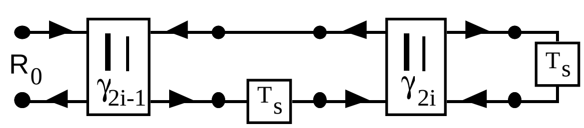


Fig. 5: Wave flow graph of a second-order all-pass section

with $\gamma_{2i-1} = \dfrac{A_i - B_i - 1}{A_i + B_i + 1}$ and $\gamma_{2i} = \dfrac{1 - B_i}{1 + B_i}$ (2.11a, b)

2.2.2 Design of LWDFs

The block diagram of a *low pass* (or *high pass*) LWDF is shown in Figure 6. For lowpass and highpass filters, it can be shown that the reflectance S_1 has an odd degree, while S_2 has an even degree. Consequently, as shown in the upper branch of Figure 6, the reflectance S_1 contains a first-order all-pass section in addition to the required second-order sections, while S_2 is made of second-order sections only (lower branch in Figure 6).

Fig. 6: Block diagram of a LWDF low pass (high pass) filter for N=9,13... (N=11,15,..)

The design of a lowpass LWDF is merely reduced to applying Gazsi's formulas for the chosen approximation function. Taking into account the given filter specifications (sampling frequency, passband / stopband edges, passband / stopband attenuation), one obtains directly the corresponding adaptor coefficients γ_i .

2.2.3 Two-port adaptors

In order to achieve an optimal dynamic range within the lattice reactances, the implementation of the two-port adaptors has to be adjusted in function of the coefficient value γ_i. It can be shown that an almost optimal $L\infty$ scaling is obtained using the adaptor structures defined in Figure 7, except for the case of very narrow and very wide band filters [2, 8].

2.2.4 Frequency transformations

Strictly speaking, Gazsi's formulas support the design of lowpass filters only. Hence, frequency transformations should be introduced for the realisation of highpass, bandpass and band-reject filters.

However, these transformations can be simplified using the power complementarity property of lossless two-ports, which can be expressed as follows for real frequencies (i.e. $p = j\omega$) :

$$\left|S_{11}(j\omega)\right|^2 + \left|S_{21}(j\omega)\right|^2 = 1 \qquad (2.12)$$

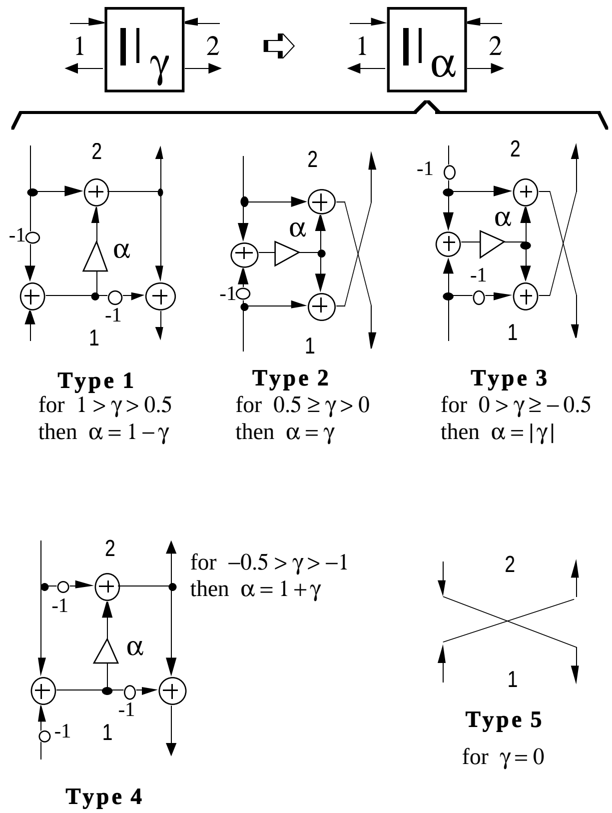


Fig. 7: Wave flow graph of a two-port adaptor

Equation (2.12), also known as Feldkeller's relation, implies that for any frequency where S_{11} is operating in the passband, S_{21} is working in the stopband, and vice versa. Consequently, when S_{11} defines a lowpass (bandpass) filter, S_{21} corresponds to a highpass (band-reject) filter.

Also, when S_{11} is maximally flat (equiripple) in the passband, S_{21} will be maximally flat (equiripple) in the stopband. The same conditions apply to the stopband of S_{11} (passband of S_{21}). Hence, choosing a Butterworth or an Elliptic function for S_{11} will result in the same approximation function for S_{21}. In contrast, choosing a Chebyshev function for S_{11} leads to an Inverse Chebyshev function for S_{21} , and vice versa.

Finally, assuming that the filter specifications are properly adjusted, the global design problem reduces to the realisation of lowpass and bandpass filters, and can be stated as follows:

- transform the desired specifications into specifications for the lowpass reference filter;
- design the lowpass reference filter;
- transform the synthesised lowpass into the desired filter.

For *highpass* filters, one assigns the desired filter transfer function to S_{21} and derives the specifications for the complementary lowpass function S_{11} using Feldkeller's relation (2.12). This operation is illustrated in Figure 8.

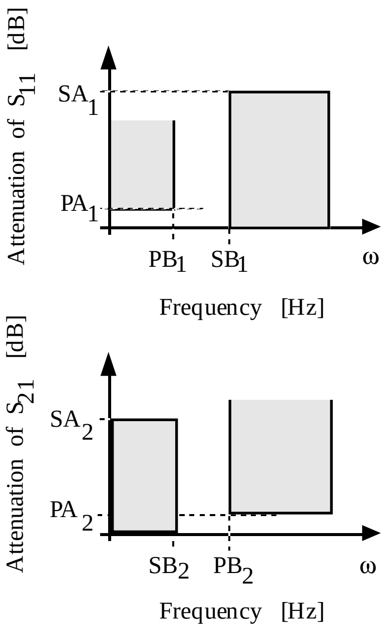


Fig. 8: Complementary specifications

The specifications of the complementary lowpass filter are achieved from the highpass specifications using:

$$SA_1 = -10 \cdot \log\left(1 - 10^{PA_2/10}\right) \qquad SB_1 = PB_2 \qquad (2.13a)$$

$$PA_1 = -10 \cdot \log\left(1 - 10^{SA_2/10}\right) \qquad PB_1 = SB_2 \qquad (2.13b)$$

Bandpass filters are obtained using a frequency transformation, first to determine the specifications of the lowpass reference filter, and second to transform the obtained reference filter into a bandpass [3, 4]. The second step is discussed hereafter.

Each elementary all-pass section of the lowpass reference filter is transformed separately. A first-order all-pass section is transformed into a second-order section, and an original second-order section into a fourth-order section, which can be factorised into two second-order sections. The final filter structure is therefore a cascade of second-order sections for each lattice reactance.

• First-order all-pass section :

$$\left.\frac{-\psi + B_0}{\psi + B_0}\right|_{LP} \longrightarrow \quad - \left.\frac{\psi^2 - (B_0/\beta_{0x})\,\psi + \varphi_{0x}{}^2}{\psi^2 + (B_0/\beta_{0x})\,\psi + \varphi_{0x}{}^2}\right|_{BP} \qquad (2.14)$$

• Second-order all-pass sections :

$$\left.\frac{\psi^2 - A_i\psi + B_i}{\psi^2 + A_i\psi + B_i}\right|_{LP} \longrightarrow$$

$$\left.\frac{\psi^4 - \dfrac{A_i}{\beta_{0x}}\psi^3 + (2\varphi_{0x}{}^2 + \dfrac{B_i}{\beta_{0x}{}^2})\psi^2 - (A_i\dfrac{\varphi_{0x}}{\beta_{0x}})\psi + \varphi_{0x}{}^2}{\psi^4 + \dfrac{A_i}{\beta_{0x}}\psi^3 + (2\varphi_{0x}{}^2 + \dfrac{B_i}{\beta_{0x}{}^2})\psi^2 + (A_i\dfrac{\varphi_{0x}}{\beta_{0x}})\psi + \varphi_{0x}{}^2}\right|_{BP} \qquad (2.15)$$

where:

$$\beta_{0x} = \frac{1}{1 + \varphi_{0x}{}^2} \qquad (2.16a)$$

$$\varphi_{0x}{}^2 = \tan\left(\frac{1stPB \cdot T_s}{2}\right) \cdot \tan\left(\frac{2ndPB \cdot T_s}{2}\right) \qquad (2.16b)$$

The frequency specifications 1stPB and 2ndPB refer to Figure 9.

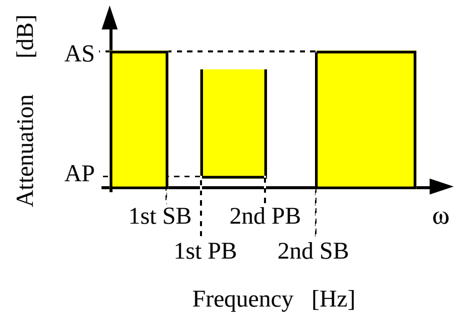


Fig. 9: Bandpass filter specifications

Finally, *band-reject* filters are readily designed by transforming the original specifications into complementary bandpass specifications using Feldkeller's relation.

3. INTRODUCTION TO SPW ENVIRONMENT

Signal Processing Worksystem (SPW) is the foundation of the DSP Framework, COMDISCO's family of software products for the development and implementation of DSP and communication systems. A brief description of some particular tools is given hereafter.

3.1 Block Diagram Editor and Signal Calculator

The *Block Diagram Editor* (BDE) of SPW provides a pictorial description of DSP algorithms, and supports the placement and interconnection of signal processing blocks stored in libraries (e.g. adders, unit delays, FFTs, etc.) [5]. Hierarchical designs are supported, and a block can represent a single function or an entire sub-system.

Once the design is captured, it can be tested using the simulator. The simulation results are displayed in the *Signal Calculator* (Sig_Cal), a tool provided for editing and analysing real and complex signals [5]. Mathematical operations can also be applied to the signals.

3.2 Filter Design System

The *Filter Design System* (FDS) is a software tool for the design and analysis of conventional IIR and FIR filters [6].

IIR filters are considered either in the direct form or as a cascade of elementary sections. The user has access to the filter coefficients which can be quantized individually. The frequency response, impulse response and pole/zero configuration are then processed accordingly.

The results provided by FDS can be interfaced to BDE for simulation purposes via a special block entitled "FILTER". One parameter of this block specifies the FDS file containing the filter coefficients.

3.3 Hardware Design System

The *Hardware Design System* (HDS) is an optional tool of SPW for the design, modelling and hardware implementation of systems with fixed point data representation [7]. Only part of HDS, namely *HDS Analyser* is considered here.

HDS Analyser includes a set of parameterised fixed point signal processing modules for the design of bit-accurate block diagrams. These blocks support common *methods* of overflow handling (wrap-around, saturation, etc.) and data quantization handling (two's complement truncation, rounding, etc.).

The finite precision effects can be readily studied by changing the number representation attributes such as the data wordlength or the location of the binary point, to mention a few. Once a design is completed, it can be simulated, and the achieved results can be analysed using Sig_Cal.

3.4 Including an LWDF design package into SPW

To include an LWDF design package into SPW, specific FDS design methods should be developed together with an additional BDE-HDS library. This is illustrated in Figure 10 and will be discussed in detail in the next two chapters

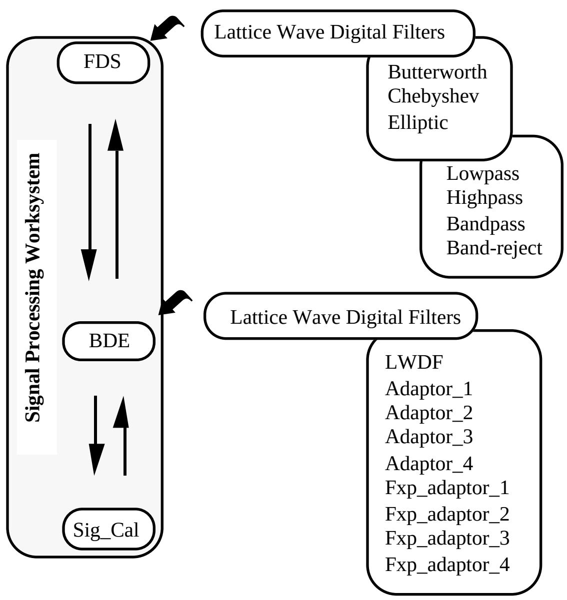


Fig. 10: LWDF design package

4. FDS DESIGN METHODS FOR LWDFs

The required design methods are directly based on Gazsi's explicit formulas for Butterworth, Chebyshev and Elliptic amplitude approximations, including the two-port adaptor transformation described in section 2.2.3.

Moreover, the filter transfer function is also provided in the direct form to allow a rough analysis using the Sig_Cal environment.

4.1 LWDF parameters

All LWDF design methods require 9 parameters listed below and defined according to Figure 9 :

Smpl. Frq:	Sampling frequency of the filter	[Hz]
Filter's type:	approximation function to synthesise (lp : lowpass; hp : highpass; bp : bandpass ; br : band-reject)	
Min Att Stop B:	minimum attenuation in the stopband	[dB]
Max. Att Pass B:	maximum attenuation in the passband	[dB]
1st SB edge Frq:	first stopband frequency edge	[Hz]
1st PB edge Frq:	first passband frequency edge	[Hz]
2nd SB edge Frq:	second stopband frequency edge	[Hz]
2nd PB edge Frq:	second passband frequency edge	[Hz]
FDS plot:	selection flag. In the *first mode* ("Yes"), the direct form of the filter is processed, followed by a filter analysis (frequency and impulse responses, pole/zero configuration). In the *second mode* ("No"), the adaptor coefficients and adaptor types are evaluated. In this case, no further analysis is performed.	

For lowpass and highpass filters, the frequency parameters *2ndPB* and *2ndSB* are simply set to zero.

It should be noticed that the design margin distribution between the passband, transition band and stopband is set by default and is not accessible to the user in the current version of the LWDF design methods.

As an example, the parameter set of an Elliptic highpass filter is shown in Figure 14.

4.2 LWDF Synthesis

Once the parameters have been edited, the selected design method can be invoked, and the results appear in the "Coefficient Display" area of SPW. Two cases must be considered.

4.2.1 FDS plot = Yes

In this mode, the design method first computes the γ_i coefficients as defined by Gazsi [2]. Second, using equations 4.1, 4.2, 2.8 and 2.10, the transfer function of each constituent reflectance is determined, and finally the direct form of the LWDF is derived. Finite precision effects cannot be studied in this mode.

• First-order all-pass section (cf Figure 4) :

$$B_0 = \frac{1-\gamma_0}{1+\gamma_0} \tag{4.1}$$

• Second-order all-pass section (cf Figure 5) :

$$A_i = \frac{2(1+\gamma_{2i-1})}{(1-\gamma_{2i-1})(1+\gamma_{2i})} \qquad B_i = \frac{1-\gamma_{2i}}{1+\gamma_{2i}} \tag{4.2}$$

4.2.2 FDS plot = No

This operational mode of FDS provides the α_i coefficients and related adaptor types. The α_i coefficients are displayed in the upper part of the "Coefficients" area and are labelled b[i], while the corresponding adaptor types are given in the lower part and are noted a[i], see Figure 14.

The indexation scheme used for the α_i coefficients and adaptor types depends on the kind of filter to be synthesised.

The indexation used for lowpass and highpass filters is specified in Figure 6. The zero index corresponds to the first-order section of reflectance S_1 and is always located at the beginning of the upper branch.

The indexation scheme previewed for bandpass and band-reject filters is defined in Figure 11.

Fig. 11: Indexation for bandpass and band-reject filters for N=10,17, ... (N=14, 22, ...)

5. LWDF DESIGN IN BDE

Once the filter has been synthesised in FDS, it can be simulated in BDE in three different ways.

5.1 Double precision design

The first possibility is to enter the flow graph of the LWDF into the BDE tool according to Figure 6 or 11, using the adaptor coefficients α_i and adaptor types obtained from FDS. The different adaptor models are available in a library to simplify the edition of the flow graph, as shown in Figure 12 for the case of an adaptor of type 1.

The designed LWDF can readily be simulated by computing the frequency and time responses. Also, the adaptor coefficients can easily be tuned if required.

5.2 Finite precision design

The second possibility is to describe the LWDF flow graph with fixed point blocks (HDS blocks) in order to study the finite precision arithmetic effects. Accordingly, fixed point adaptors are provided as shown in Figure 13.

This kind of simulation is especially useful to study the hardware implementation of LWDFs on specific processor architectures.

Fig. 12: Adaptor of type 1: a) instance
b) internal structure

Fig. 13: Fixed point adapt. of type 1: a) instance
b) internal structure

5.3 Design using the "FILTER" block

Finally, the LWDFs can be roughly simulated using the "FILTER" block (cf section 3.2). To obtain a valid simulation, the designer must be careful to save the direct form of the LWDF filter within the FDS tool.

6. APPLICATION EXAMPLE

The synthesis and simulation of a particular Elliptic highpass LWDF are discussed hereafter.

Figure 14 shows the filter synthesis step using FDS : the desired specifications are given in the "Parameters" area, and the results are displayed in the "Coefficients" area. For the given example, the adaptor coefficients α_i and adaptor types have been computed and are displayed according to the convention adopted in chapter 4. Note that the texts "Poly Coefficients", "Number of Numerators", and "Number of Denominators" could not be changed in the SPW framework and have no sense in the current context (the number seven mentioned beside these texts corresponds to the filter order).

Figure 15 presents the double precision flow graph of the filter. The signal referred to by the "SIGNAL SOURCE" block corresponds to an impulse (sampling frequency : 2000 Hz).

The simulation results are shown in Figure 16, including the impulse and the frequency responses of the filter.

Finally, the special purpose block entitled "FILTER" is shown in Figure 17.

7. CONCLUSION

This paper presents a design package for Lattice Wave Digital Filters integrated into the Signal Processing Worksystem environment.

This package contains FDS synthesis methods based on Gazsi's explicit formulas and frequency transformations. Also, new BDE models for the design of LWDF signal flow graphs are provided.

In the future, it is previewed to refine the developed synthesis methods by extending them to bireciprocal filters. Also, it will be useful to provide the user with a better control of the design margin allocation.

Acknowledgement

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Fig. 17: Flow graph of the highpass filter using the "FILTER" block

Fig. 14: Elliptic highpass filter synthesis in FDS

Fig 15: Double precision flow graph of the highpass filter synthesised in FDS (cf Fig. 14)

Fig. 16: Simulation results for the Elliptic highpass LWDF : Impulse response (left), and frequency response (right)

Fig. 17: Flow graph of the highpass filter using the "FILTER" block