Published in Proceedings of the Third International Bi-Conference Workshop
on Agent-Oriented Information Systems (AOIS-2001), 33-40, 2001
which should be used for any reference to this work

Multiagent auctions for multiple items

Houssein Ben Ameur
Département d'Informatique
Université Laval
Québec, Canada
benameur@ift.ulaval.ca

Brahim Chaib-draa
Département d'Informatique
Université Laval
Québec, Canada
chaib@ift.ulaval.ca

Peter Kropf
Département d'Informatique
Université de Montréal
Montréal, Canada
kropf@iro.umontreal.ca

ABSTRACT

Available resources can often be limited with regard to the number of demands. In this paper we propose an approach for solving this problem using the mechanisms of multi-item auctions for allocating the resources to a set of software agents. We consider the resource allocation problem as a market with vendor and buyer agents participating in a multi-item auction. The agents exhibit different acquisition capabilities which let them act differently depending on the current context or situation of the market. We present a model for this approach based on the English auction, and discuss experimental evidence of such a model.

1. INTRODUCTION

Agent technology is becoming one of the most important and exciting areas of research and development in computer science today. This technology is a significant area of interest for such applications as telecommunications, information management and the Internet, electronic commerce, computer games, interactive cinema, information retrieval and filtering, user interface design, industrial process control, open systems, etc. The successful adoption of this technology in all these areas will have a profound impact both on industry, and also on the way in which future computer systems will be conceptualized and implemented.

Many applications, if not most of them, require multiple agents, called also *multiagent systems* (MAS). In such systems, knowledge, action and control are distributed among the agents, which may cooperate, compete or coexist depending on the context. MAS have shown to be relevant [5] for understanding, implementing and operating complex socio-technical systems as represented for example by e-business systems. In this paper we propose a model for multi-item auctions which has a strong relationship to the efforts in building multiagent systems.

Auctions have always been an important market mechanism. The work presented in this paper focuses on the approach where auctions are considered as a process of automatic negotiation applying the multiagent paradigm. Negotiation is central to any commerce and market. It can be defined as a *Mechanism that allows a recursive interaction between a principal and a respondent in the resolution of a good deal* [2].

An auction commonly restricts the negotiation variables essentially to the price and the quantity in case of multiple items. An open auction allows the agent to review his offers, and if the auction is public, to refine them by analyzing the offers of other participants and by considering the auction's evolution. The negotiation strategy may thus be adapted according to the rules of the market. Finally, an auction negotiates a mutually acceptable solution for both the vendor and the buyer while the market forces alone decide on the negotiation termination. The use of auctions as a mechanism for automatic negotiation and for resource re-allocation in multiagent systems was demonstrated by T. Sandholm [4, 3].

This paper is organized as follows: Section 2 presents a model for multi-item auctions. The strategy and equilibrium conditions of the auction process relying on this model are presented and analyzed in Section 3. Following that, Section 4 presents an implementation of the model. The simulation results of three typical cases are presented and discussed in Section 5. Finally, we conclude the paper in Section 6.

2. A MULTI-ITEM AUCTION MODEL

2.1 Multi-item auctions

In may cases, auctions include multiple non-identical items. The price that a bidder may offer for one item may depend in complicated ways on what other items it can get. In such cases, it is often preferable to allow bids on *combinations of items*, as opposed to bids on only single items. Such an auction is called ”combinatorial auction”. Bidding in combinatorial auctions gives the possibility to bidders to submit offers on a package of objects that interest them. For example, in a auction where computer components are sold, a buyer may be interested in having a computer screen with a central unit, but he is not interested in winning only the screen. This type of auction is preferable here, since it allows the bidders to express their true preferences, and may thus lead to better allocations. However, the exponential number of possible combinations usually results in computational intractability in dealing with such auctions.

Most work on multi-item auctions suppose two simplifying conditions: the quantity of items to sell is fixed as well as the quantities requested by the buyers. These two hypothesis do not meet the requirements of many situations where auctions are used. Lengweiler [1] for example proposes an auction model, where the available quantity is not fixed. It can therefore change during the auction as it is for example the case for stock values. The approach proposed in this paper is inspired from Lengweiler's model, and it is based on an English auction with multiple items, private evaluations and variable requested quantities. In a simple English auction, bidders submit increasing public offers to the auctioneer until no more bidder is able or willing to submit a better bid. The winner is the last participant who submitted the best bid.

2.2 Model

Consider a multi-item auction with one single vendor and a finite number n of buyers or *agents* $A_1, \ldots, A_n$. A quantity Q of identical items is available to be sold. Each buyer A_i wishes to acquire a quantity q_i of items with:

$$\sum_{i=1}^{n} q_i > Q$$

The evaluations $V_i, i = 1, \ldots, n$ of the n buyers are extracted from a rectangular uniform distribution $F(V)$ on the interval $[V_{\min}, V_{\max}]$ where

$$V_{\max} - V_{\min} > 1$$

This defines a model of private, independent evaluations. Given that F is rectangular and uniform, the following holds

$$F(V_i) = \frac{1}{V_{\max} - V_{\min}}$$

Because the items are identical, each buyer A_i has the same evaluation v_i for all the items it wants to acquire such that

$$V_i = (v_i \times q_i)$$

The participants in the auction submit their offers as if they would desire to acquire just one single item. Their respective desired quantities q_i are unknown to the vendor. Let b_i the function of submission offers describing the auction strategy of buyer A_i depending on its evaluation V_i and the desired quantity q_i of the item i:

$$b_i = b(V_i, q_i), \quad i = 1, \ldots, n$$

Note that the buyer may decide to decrease the desired quantity during an auction in order to increase his evaluation of the item i according to

$$v_i' = \frac{V_i}{q_i'}$$

where v_i' is the new evaluation of item i for buyer A_i, and q_i' the new quantity desired with $q_i' < q_i$.

Suppose for example that buyer A_i has a global evaluation $V_i = 100$ for the desired items. If this buyer asks for a quantity of items $q_i = 10$, his evaluation for each of them would be

$$v_i = \frac{V_i}{q_i} = \frac{100}{10} = 10$$

If the buyer decides to decrease the quantity asked for to $q_i' = 5$ items, the evaluation v_i' is calculated as follows:

$$v_i' = \frac{V_i}{q_i'} = \frac{100}{5} = 20$$

At the end of the auction, buyer A_i receives $\bar{q}_i$ items such that:

$$\begin{array}{lcll} \bar{q}_i & = & q_i & \text{if} \quad Q - \sum_{b_j > b_i} q_j \geq q_i \\ \bar{q}_i & = & Q - \sum_{b_j > b_i} q_j & \text{if} \quad 0 < Q - \sum_{b_j > b_i} q_j < q_i \\ \bar{q}_i & = & 0 & \text{if} \quad Q - \sum_{b_j > b_i} q_j \leq 0 \end{array}$$

In fact, winning bidders will obtain the quantities they bid for, while the last winning bidder will obtain just the remaining quantity. All loosing participants will obtain a null quantity.

Indeed, the quantity $\bar{q}_i$ obtained by buyer A_i depends on the demands of the other buyers A_j having submitted offers b_j superior to his offer b_i. The quantities of those buyers is thus

$$\sum_{b_j > b_i} q_j$$

The last winning buyer A_i may thus have his demand partially satisfied by the remaining quantity not sold to the others. For example, suppose that 10 items are for sale. The best offer for 8 of the items he desires is 15. The second best offer is 10 for 5 items. The first buyer will receive the 8 items he asked for, while the second buyer will only receive $10 - 8 = 2$ items.

The winning buyers pay the amount of their bids multiplied by the quantity obtained: $b_i \times \bar{q}_i$. Their respective gains are:

$$U_i = (\bar{v}_i - b_i) \times \bar{q}_i = V_i - b_i \bar{q}_i$$

If the evaluation of the items to buy in the previous example is 18 for the first winning buyer A_1 and 14 for the second one A_2, their respective gains are:

$$A_1 : U_1 = (18 - 15) \times 8 = 24$$

$$A_2 : U_2 = (14 - 10) \times 2 = 8$$

The function of gain U_i is called the *utility* of buyer A_i. Each buyer A_i tries to maximize its utility U_i.

3. STRATEGY AND EQUILIBRIUM

Every participant in the auction tries to win as if it were a simple English auction. Because a participant does not know which are the quantities asked for by the other participants, it may happen that his bid will be out-done by other buyers. It is thus faced with the risk that the demanded quantity q_i will be entirely allocated to another buyer offering a higher price. The buyer is always faced with the dilemma where he wants to minimise his bid b_i to maximize his gains, but where on the other hand it must take care that his bid b_i has the best chances to win. A winner's course strategy taking into account these two contradicting constraint must therefor be defined.

Assume a buyer A_i submits an offer b_i. For this bid to win, it is necessary that all the other $n - 1$ bidders submit inferior offers with regard to b_i. The probability that any offer b_j is inferior to the offer b_i knowing that A_i demands the quantity q_i is:

$$P(b_j < b_i | q_i) = \int_{V_{\min}}^{q_i, b_i} F(V) dV$$

$$\text{with } F(x) = \frac{1}{V_{\max} - V_{\min}}$$

resulting in:

$$P(b_j < b_i | q_i) = \frac{q_i b_i - V_{\min}}{V_{\max} - V_{\min}}$$

The probability that all offers of the other $n - 1$ buyers are inferior to b_i is thus:

$$\prod_1^{n-1} \frac{q_i b_i - V_{\min}}{V_{\max} - V_{\min}} = \left[\frac{q_i b_i - V_{\min}}{V_{\max} - V_{\min}} \right]^{n-1}$$

All the buyers A_i try to optimise their winning course by maximizing the probability to win the auction. The buyer therefor maximizes the following expression:

$$\prod_i (V_i - q_i b_i) \left[\frac{q_i b_i - V_{\min}}{V_{\max} - V_{\min}} \right]^{n-1}$$

The following expression is now resolved:

$$\partial \prod_i / \partial b_i = 0$$

This results in

$$-q_i (q_i b_i - V_{\min})^{n-1} +$$
$$(V_i - q_i b_i)(n-1) q_i (q_i b_i - V_{\min})^{n-2} = 0$$

which yields after factorisation in

$$q_i (q_i b_i - V_{\min})^{n-2} \times$$
$$[-(q_i b_i - V_{\min}) + (V_i - q_i b_i) \cdot (n-1)] = 0$$

Keeping only the solutions which maximize $\prod_i$:

$$(q_i b_i - V_{\min}) + (q_i b_i - V_i)(n-1) = 0$$

the result is

$$\hat{b}_i = \frac{V_{\min} + (n-1) V_i}{n q_i}$$

The expression $\hat{b}_i$ represents the optimal offer of buyer A_i in the sense that it is the minimal offer maximizing the probability to win the auction, while assuming that the evaluations of the other bidders are uniformly distributed.

4. SIMULATION

The model presented in the previous section has been implemented as a simulation platform based on the multiagent paradigm. There are multiple buyer agents and a vendor agent. The application was entirely coded in Java, because of the portability, robustness and simplicity of this programming language. The agents were implemented as Java threads running on a local machine. The program offers the possibility to define the number of items to sell by the vendor agent, the minimum bid increment and the reservation price as well. Then, we have the possibility to characterise the buyer agents through the definition of their number and the amount of money allowed to them. We may choose to randomly generate buyer agents bidding strategies, or fix them manually. Finally, the auction may be run. All the agent characteristics may be loaded from/saved to a file, and similarly for the bids generated by the buyers through the auction execution. Those results are then inspected and analysed in order to induce some empirical conclusions. The simulations were made on a Pentium III PC with 128 Meg of RAM. The auction simulation process execution time depends essentially on the number of buyer agents and the minimum bid increment value. Nevertheless, an auction simulation doesn't take more than approximately 5 to 10 seconds on such a machine. This auction duration is a mean estimation based on the simulations we made. Those simulations included from 5 to 30 buyer agents.

4.1 Vendor agent

The vendor agent supervises the auction process in a central manner. It engages the auction by announcing its start to the buyers and then expects offers from the buyer agents. Each time the vendor receives an offer, it announces it to the buyers as the current best offer without revealing the buyer's identity nor the quantity asked for. The vendor agent knows the number n of buyers participating in the auction. If a buyer leaves the auction, it decrements the number n of participating buyer agents. When only one buyer is left, the vendor agent announces the end of the auction.

The vendor agent can be considered as a reactive agent which reacts to exterior stimuli, i.e. the messages arriving from the buyer agents A_i. It interacts with the environment by sending messages to the buyers as described above.

4.2 Buyer agent

The behavior of the buyer agent is defined by his winning course strategy as discussed in Section 3.. The strategy for maximizing the buyer's gain is fixed in advance.

4.2.1 Enrichment strategy

While bidding, the buyer agent must take a decision on: (1) the offer to submit and (2) the decrease of the quantity demanded. There are thus two parameters to be set for choosing the enrichment strategy: (1) how and when to place an offer, and (2) how and when to decrease the quantity asked for.

The enrichment strategy of the buyer agent is modelled with two linear functions, which represent a more or a less aggressive behavior:

1. price offered as a function of time, during the auction progress, as illustrated in Figure 1.
2. quantity asked for as a function of time, during the auction progress as illustrated in Figure 2.

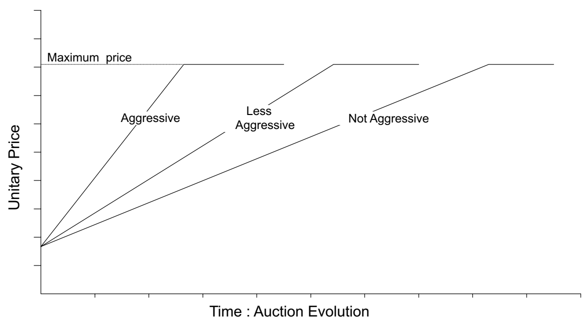

Figure 1: Price offered during the auction's progress.

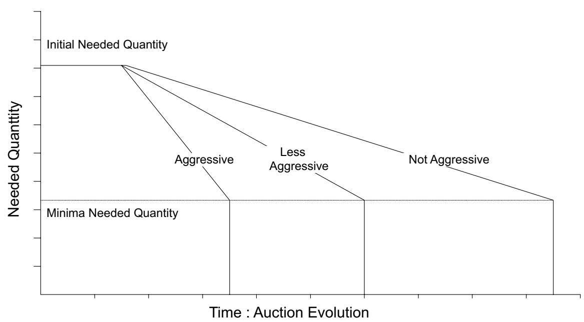

Figure 2: Quantity demanded during the auction's progress.

4.2.2 *Behavior of the buyer agent*

When a buyer agent starts to participate in an auction, it first determines the time interval of inactivity according to its enrichment strategy. At the end of this waiting period, the agent verifies the current price. If his offer is still the winning one, i.e. no other buyer agent has submitted a superior offer during its time of inactivity, he determines his new waiting time according to his enrichment strategy. The buyer agent continues this process until another buyer has submitted a better offer. At this time, the buyer agent starts a process of enrichment in order to submit a superior offer. If it can submit a better offer, the buyer agent sends a message to the vendor agent with his price offer and the quantity desired.

In case the buyer cannot submit a better offer, he tries to decrease his quantity. The buyer agent seeks to diminish the quantity asked for according to his strategy, i.e. more or less aggressively. As long as he cannot make a better offer, he continues to decrease the quantity. If the agent achieves his goal, it submits the new offer, otherwise it abandons the auction.

Finally, when the auction will be terminated, the buyer agent receives a signal from the vendor agent and then clears the auction. The winner will receive a message from the vendor indicating the price and quantity won. It has to be noted here, that there may be more than one winner, each acquiring a different quantity as illustrated in the example in Section 2.2.

5. EMPIRICAL EVALUATION

This section presents and discusses empirical results obtained by the simulation of the model described in this paper. Three different cases are evaluated.

Case 1: Same needs and same evaluations

Suppose there are 10 items to sell and $n = 5$ buyer agents. All the buyers $A_{1\ldots5}$ dispose of the same amount of money to spend ($V_i = V_{\min} = V_{\max}, 1 \leq i \leq 5$). The initial quantity each buyer A_i asks for is equal to 10 items ($q_i = 10, 1 \leq i \leq 5$), and the minimal quantity accepted by the vendor agent shall be 6 items. The strategies of the buyers are randomly generated.

This scenario represent a case of aggressive competition because all the buyers show the same (or similar) needs for the quantities desired and their evaluations of the items are the same as well. One can thus expect that the buyers will heavily decrease their quantity demands in order to win the auction. The revenue of the vendor is in this case not affected by the number of buyer agents. Indeed, the following equation holds:

$$\hat{b}_i = \frac{V + (n-1)V}{nq_i} = \frac{V}{q_i} = v_i$$

The optimal offer for each buyer agent is therefore his own evaluation v_i of the item.

Figure 3 first shows the revenue of the vendor depending on the means the buyers dispose, i.e. the buyers' evaluations, without any decrease of the requested quantities. Second, the figure presents the vendor's revenue in case the desired quantities are decremented down to a minimum of 6 items.

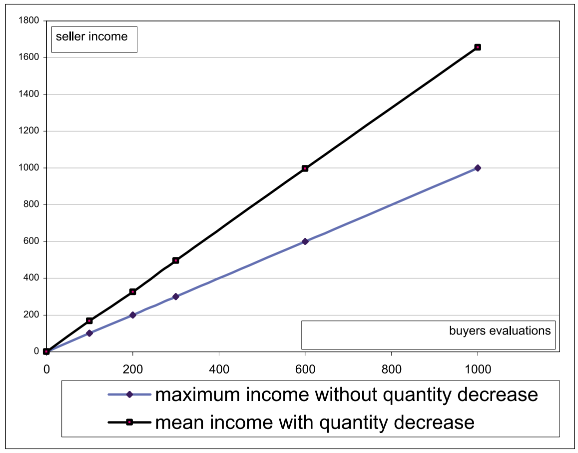

Figure 3: Vendor revenue in case of heavy competition.

Case 2: Same needs but different evaluations: influence of the number of buyer agents

Suppose the same parameters as in *Case 1* except that the buyer agents have different evaluations, i.e. they dispose of different amounts of money to spend. The evaluations of the 5 buyers are determined according to a uniform distribution with $V_{\min} = 100$ and $V_{\max} = 300$.

The competition is not as aggressive as in *Case 1*. Along the auction, the agents with smaller evaluations will abandon. In the end only two agents will stay and compete to win the auction.

Figure 4 shows the influence of the number of buyer agents on the revenue of the vendor. The model presented in this paper allows to considerably increase the mean revenue of the vendor. In order to maximize his revenue, the vendor should attract a large number of buyers. In case there is no decrease in the quantity requested, the revenue of the vendor stabilized with 20 buyers and more. The revenue stabilisation occurs later with the possibility to decrement the desired quantity. Figure 4 shows that this happens only with 40 and more buyers. This difference can be explained by the competition which is always more accentuated compared to a model where the requested quantities do not decrease.

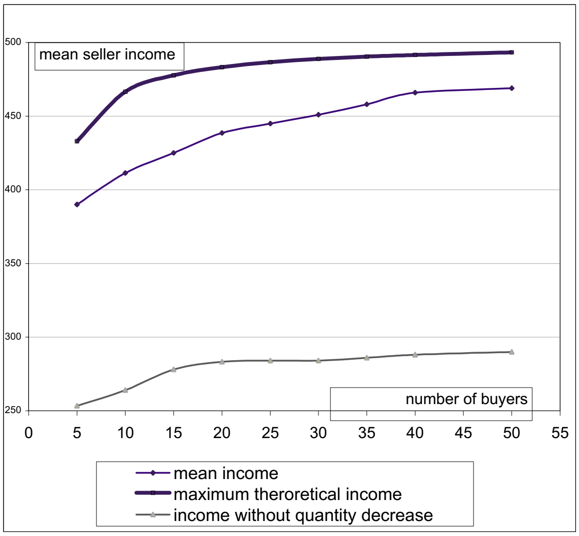

Figure 4: Vendor revenue in relation to the number of buyers.

Case 3: Same needs but different evaluations: influence of the minimal offer

The parameters are the same as in *Case 2* except that the number of buyer agents is fixed to 10. The vendor may define a minimal incremental value for an offer. While increasing this minimal incrementation value, the number of offers is decreased, which in turn minimises the communications between the buyers and the vendor. The simulation experiments have shown that a large incrementation value results in a globally decreasing mean revenue for the vendor. Figure 5 presents this correlation. The curve representing this relation shows however, that there are certain incrementation values allowing better revenues than smaller ones. Indeed, the curve has a wave form with increasing period. For example, the minimal incrementation value of 10 generates superior revenues compared to the incrementation values of 8 and 9. This behavior depends of the function determining the evaluations of the buyers. Given that $V_{\max} = 300$ in the current example, the maximal offer a buyer with an evaluation of $v_i = 300$ can make to obtain the 6 items (minimal number of requested items) will be 50 per item. If the minimal incrementation is set to 9, the maximal offer per item will be 45. On the other hand, if the minimal increment value is 10, the maximal value of 50 to offer per item can be achieved. This explains the wave form of the curve in Figure 5.

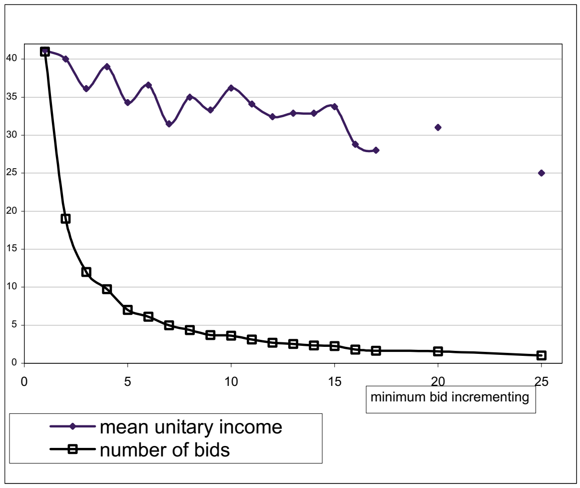

Figure 5: Vendor revenue and number of bids in relation to the minimal bid increment value.

6. CONCLUSION

In this paper we proposed a formal model for auction based automatic negotiations. This model has been implemented using MAS and was tested and evaluated with experiments. Recently, most work on multi-item auctions have addressed the combinatorial issue that allows bids on combinations of items as opposed to only single items. These approaches suppose however, two simplifying conditions: the quantity of items to sell is fixed as well as the quantities requested by the buyers. These two hypothesis do not meet the requirements of many situations where auctions are used. In some auctions, it is more desirable to not fix the available quantity. It this way, quantities can change during the auction, as it is for example the case for stock values. The approach that we have proposed here, follows this road. To achieve it, we presented a model based on an English auction with multiple items with private evaluations and variable quantities requested. With such a model, we succeed in characterizing:

1. How a large incrementation value results in a globally decreasing mean revenue for the vendor;

2. How an augmentation of the incrementation value (by the vendor) decreases the number of offers, which in turn minimises the communications between buyers and the vendor;

3. How the vendor's revenue stabilizes with 20 buyers and more in the case where there is no decrease in the quantity requested, and with 40 buyers or more when a decrease of the requested quantity is considered;

4. How the vendor's revenue increases with large quantity variations in the quantities requested;

5. Finally, the implementation using multiagent systems showed the suitability of this paradigm for this application.

In conclusion, we believe that our model for automatic negotiations is a suitable approach to enhance the capabilities of auction systems while imposing less constraints. In future work, we will investigate other enrichment strategies as well as experiment with other types of auctions. Given the first experimental results of this paper we expect to successfully demonstrate the power of this model with such variations as well.

ACKNOWLEDGEMENTS

This work was partially supported by the Canadian Natural Sciences and Engineering Research Council (NSERC).

References

[1] Y. Lengwiler. The multiple unit auction with variable supply. *Economic Theory Journal*, 14(2):373–392, 1999.

[2] OMG. Negotiation facility final revised submission. Technical report, March 1999. http://www.oms.net.

[3] T.W. Sandholm. Automated negotiation. *Communications of the ACM*, 42(3):84–85, March 1999.

[4] T.W. Sandholm. *Multiagent Systems A Modern Approach to Distributed Artificial Intelligence*, chapter Distributed Rational Decision Making. MIT Press, 1999.

[5] G. Weiss. *Multiagent Systems*, chapter Prologue, pages 1–23. MIT Press, 1999.