

On weird numbers

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This is a joint work with Gianluca Amato, Maximilian Hasler and Maurizio Parton

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- $\sigma(n) > 2n$
- For every subset $D \subset \{d \in \mathbb{N}, d|n\}$ one has always
$$\sum_{d \in D} d \neq n$$

The definition of weird number is due to Stan Benkoski (1972)

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- *Primitive weird numbers* are weird numbers that are not a multiple of a weird number.

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Erdős offered \$10 for an example of odd weird number, and \$25 for a proof that none can exist.