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Université de Neuchâtel

Institut de Physique

Quantum corrections to effective field theories of superstrings:

a study of the Green-Schwarz mechanism in Calabi-Yau compactification.

Thèse

présentée à la Faculté des Sciences

pour l'obtention du titre de

Docteur ès Sciences

par

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Neuchâtel, july 1995

IMPRIMATUR POUR LA THÈSE

Corrections quantiques aux théories effectives des supercordes.

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UNIVERSITÉ DE NEUCHÂTEL

FACULTÉ DES SCIENCES

La Faculté des sciences de l'Université de Neuchâtel sur le rapport des membres du jury,

Messieurs J.-P. Derendinger (directeur de thèse),
J.-P. Amiet, F. Quevedo
et H.-P. Nilles (Münich).

autorise l'impression de la présente thèse.

Neuchâtel, le 4 juillet 1996

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Preface

This thesis presents a part of the work I have done at the University of Neuchâtel in the high energy theory group. It addresses the field of phenomenological string theory from the effective field theory point of view. This field bloomed in the mid eighties after the discovery of the heterotic superstrings and compactifications which would lead to phenomenologically interesting models. Intensive efforts were then made to find *the* string vacuum which would lead to a string theory containing the Standard Model in its low-energy limit. It was rapidly realized that no known principles would pick out the correct vacuum but this search lead to a better understanding of string theory itself and also of supersymmetric field theories.

My interest in string phenomenology was sparked by the publication of Dixon, Kaplunovsky and Louis [17] where the threshold corrections to gauge coupling constant arising from string 1-loop were computed. The problem of the non-harmonic threshold corrections raised at that time lead to a clarification of the status of quantum corrections in supersymmetric effective field theories of strings. The understanding of these corrections in terms of anomaly cancellation of a physical string symmetry (target-space duality) by Derendinger et al. [15], in particular, showed how these symmetries could be a tool in the construction of low-energy effective theories.

Quantum corrections to string effective field theories is a difficult subject since one has to disentangle string effects from quantum field effects and at the same time keep only relevant terms in the low-energy approximation. This can be illustrated for example by the debate on the use of the linear multiplet L or the chiral multiplet S. The first one appears to be natural from the string theory point of view since it contains the curl of the antisymmetric tensor field $\partial_\mu B_{\nu\rho}$ and the dilaton C as its lowest components. These fields are always present in the massless spectrum of the heterotic superstring and moreover, the vacuum expectation value of the dilaton is, in the string frame, the string loop-counting parameter. On the other hand, from the supersymmetric field theory point of view, the chiral multiplet formulation is better suited for the analysis of its holomorphic properties.

The quantum corrections are determined by computing various scattering processes at the string level and, from the low-energy limit of these amplitudes, determine the effective theory. Going beyond the tree level allows the effective theory to be sensitive to the massive string states which can enter string loops. The use of the Wilson approach to effective theories permits to unambiguously define the order of the quantum corrections and therefore to extricate the string loop effects from the genuine field theory loops. The Wilson action is not a physical object and can suffer from anomalies. These anomalies can be separated in two classes. There are the non-local anomalies which arises through massless states circulating in fermions loops and local anomalies which are anomalies of the Kähler symmetry of the effective action and can be traced back to target-space duality. The latter can be cancelled, in some cases, by a four-dimensional Green-Schwarz mechanism. This leads us to the work of this thesis.

The idea behind my work is the following. The computation of quantum corrections at the string level is difficult and up to now a limited amount of data is available. For example, the work of Dixon et al. cited earlier only includes moduli dependent threshold corrections to gauge couplings leaving out any contribution of the matter fields. If exact results are difficult to derive, one can try to extract hints from a limiting case which would direct our search of string quantum corrections. This philosophy has motivated the study of the reduction of the Green-Schwarz counterterm in Calabi-Yau compactification of the heterotic superstring.

We work in the zero-slope limit of the superstring and start our analysis with the ten-dimensional supergravity lagrangian of the $E_8 \times E_8'$ heterotic superstring. This theory contains higher derivatives counterterms which cancel gauge, gravitational and mixed anomalies; the so called Green-Schwarz counterterms. The compactification scheme which we will consider is the $(2,2)$ Calabi-Yau compactification of the heterotic superstring and leads to an $N = 1$ supergravity in four dimensions. In the limit where the compactification scale is much smaller than the Planck scale, it amounts to a harmonic expansion of the ten-dimensional fields over the Calabi-Yau manifold. The reduction of the Green-Schwarz counterterm to four dimension will then give us informations on the effective theory in the large radius of compactification limit by fitting the terms obtained by reduction with the result of $N = 1$ supersymmetric field theory.

The text of this thesis is organized as follow. The first chapter is a general introduction to the problem of quantum corrections in effective theories and presents the superconformal approach to supergravity. The second chapter is the reduction of the Green-Schwarz counterterm itself while the analysis is done separately in chapter 3. These three chapters can be read independently since they refer only to results and not to the details of each other.

Chapter 1

Quantum corrections to effective field theories of superstrings

Since the advent of string theory, high-energy particle theorists have embarked on two opposite research directions. One approch starts on the solid ground of the Standard Model of elementary particles and goes up in energy to describe physics beyond the electroweak scale while the other starts at the Planck scale with the new concept of extended objects at the fundamental level and goes down in energy. Today the gap between superstrings and experimental physics is bridged in part by effective quantum field theory. We know that physics up to the Tev scale is well described by quantum field theory. This means that in its low-energy limit, string theory must be described by an effective field theory. Gaining control of the passage from high to low-energy is a crucial step for string theories since it contains the seeds of its first experimental checks by introducing new effects which will have no counterpart in ordinary field theory.

In this chapter we will briefly describe how string theory determines the form of this effective theory and the different roles played by massless and massive string states. The accent will be put on the role played by string symmetries which introduce very restrictive constraints on the form of the effective theories. We will also introduce the basics of the superconformal formalism, one of the formalisms used to describe the low-energy effective theory. As an example of the interplay between string theories and four-dimensional field theories, we will end this chapter by expeditiously going through the problem of loop corrections to gauge coupling, a problem that has fuelled intensive work in the past few years. This will allow us, to introduce the concept of moduli fields and their links to field-dependent couplings.

1.1 Effective field theories as a low-energy limit of superstrings

The low-energy limit of string theories is a rather clear concept. It is the limit in which the physics appears to be described by point-like objects. This corresponds to $\lim_{\alpha' \to 0}$ or equivalently to $\lim_{M_P \to \infty}$. Technically, the situation is more intricate since there are various ways to obtain an effective quantum field theory from strings. They all have some pro and con as I will briefly show below.

The effective field theory describes the dynamics of the massless string degrees of freedom. The heavy states, being of order of the Planck mass, play only an indirect role when quantum loops are taken into account. At the beginning of the nineties, some one-loop string effects to effective fields theories were computed [17, 3, 4, 5] in the form of threshold corrections to gauge couplings. At this point, the interpretation of these corrections as an anomaly cancellation process was not clear. This confusion can be traced back to the distinction between the notions of Wilson effective actions and the usual one particle ireducible (1PI) effective action of field theory. Let us recall that the Wilson action is an effective action in which all the modes above a certain scale μ have been integrated out. This means that not only the heavy states have to be integrated but also light states with large momenta. The Wilson action is not a physical object since it does not take into account the running of the physical quantities. It can therefore be anomalous under some physical symmetries of the theory. The physical quantities have to be extracted out of the 1PI effective action. In the case of effective field theory of strings, this 1PI effective action corresponds to the generating functional of the one-particle irreducible Feynman diagram of the theory defined by the Wilson action. This justifies the following nomenclature. The *effective theory* is the quantum field theory defined in the Wilson sense and the *effective action* refers to the usual 1PI.

If at tree-level the two notions are equivalent, it is not the case anymore when loop corrections are taken into account. At tree-level, there is basically one scale in the problem given by the momenta of the external states or more precisely by the ratio $\frac{E}{M_H}$ where E is the energy of the process and M_H the mass of the heavy particles. If quantum effects are taken into account, the situation is more complicated since it involves two scales. First there is an ultra-violet scale M_{UV} which cuts off loops above a certain scale where the effective theory description is not valid anymore. Second there is an infra-red scale M_{IR} which has to be introduced in order to regularize the divergences introduced by massless loops. The physical observables depend on M_{IR} through the renormalization-group equations. With this in mind, we can now more precisely state what is meant by one-loop corrections to string effective field theory. Generically, the effective field theory can be written as a supergravity lagrangian $\mathcal{L}$ whose form is determined order by order in string loops. The one-loop corrected $\mathcal{L}$ is

$$\mathcal{L} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)}.$$

$\mathcal{L}^{(0)}$ is the tree-level (classical) contribution and the one-loop corrections $\mathcal{L}^{(1)}$. When computing physical quantities, the one-loop corrections are of two types. First there are the genuine quantum field corrections which arise from loops constructed with vertices of $\mathcal{L}^{(0)}$. These vertices have themselves been constructed from string tree-level expressions. Second we have new tree-diagram arising from $\mathcal{L}^{(1)}$. Their role is to cancel anomalies of string physical symmetries (like target space duality) which arise from $\mathcal{L}^{(0)}$. This reasoning can be carried to n-loop corrections. Note that this interplay between field and string corrections can be described in the framework of the Wilson action.

One way to obtain the effective theory is to compute scattering amplitudes of massless states at the string level and then construct a four-dimensional field theory reproducing the same amplitudes in the low energy limit. In the conformal field theory language this amount to compute N-point correlation functions

$$A \sim g^{N-2} \langle V_1(z,\ \bar{z},\ k), \ldots, V_N(z,\ \bar{z},\ k) \rangle \tag{1.1}$$

where the V_i are vertex operators and g is the string coupling constant. z and $\bar{z}$ are the insertion points on the world-sheet and k is the momentum. For example, in the case of the universal sector of the heterotic superstring, the low-energy supergravity lagrangian up to two derivatives is determined by computing 2, 3 and 4 point functions at string tree-level [21, 22, 23, 8]. This leads to the following lagrangian density for the metric ($g_{\mu\nu}$), the antisymmetric tensor ($B_{\mu\nu}$), the dilaton (φ) and the gauge fields

$$e^{-1}L_{tree} = \frac{1}{2\kappa^2}R - \frac{1}{6}e^{-2\sqrt{2}\kappa\varphi}H_{\mu\nu\rho}H^{\mu\nu\rho} - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{4}e^{-\sqrt{2}\kappa\varphi}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}$$

where $H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}$ and $F_{\mu\nu}$ is the gauge field strength. By associating strings modes to fields in the above lagrangian, one relates the string coupling constant g to the four-dimensional gravitational constant κ and the gauge coupling constant g_4,

$$g_4 = \frac{1}{2\sqrt{\kappa}}g = \sqrt{\frac{2}{k\alpha'}}\kappa.$$

k is the level of the Kac-Moody algebra of world-sheet currents. Higher derivative terms are also introduced see for example Gross and Sloan [24]

This method has the advantage of providing relations between the string parameters and field theory parameters in a rather simple way. Its disadvantages are obvious. Computations become difficult when one goes to higher orders and up to now few string loop calculations have been done.

An alternative way to proceed is to use the sigma-model approach. This method is also called the background field method since it describes the propagation of the string in a given background. With the low-energy effective field theory in mind one can see these background fields as the result of the integration of heavy modes. The equations of motions of these background fields are a consequence of the superconformal invariance of the world-sheet.

If the effects of string tree-level are well known, going beyond one-loop is a difficult task and the use of exact string symmetries is a help in determining the possible forms of the effective theory, as will be shown in the next section.

1.2 Beyond the classical level

Introducing quantum corrections in effective field theories of string is a difficult problem. In the high-energy regime we are faced with string quantum effects. These induce modifications to the effective field theory. In the low-energy regime, as physics is described by a field theory, we have to take into account the usual quantum effects.

Analysis done at the string tree-level in the low-energy limit is easier since it does not require the introduction of the massive string modes. Quantum corrections to string scattering are described by world-sheet of genus greater than one. For example, one-loop corrections are determined by world-sheet with the topology of a torus. At tree-level, low momenta in the external lines imply low momentum in the internal lines which in turns means that massive string modes play no role in the computation of the scattering amplitude. At the quantum level, there is the possibility of having massive string states circulating in the internal loops even for low external momenta. We have to take the full string spectrum into account in the computation, the massless states as well as the infinite tower of massive excitations. This will be reflected in the low-energy theory by the introduction of new terms in the lagrangian.

On top of these, we have also to take into account the quantum corrections in the effective field theory. These appear as standard loop effects of field theory. Both the Wilson and the 1PIeffective actions enter are needed.

1.3 Superconformal formalism

As seen above, the effective field theory of low-energy superstring can be described by an $N = 1$ supergravity. Over the years, many supergravity theories have been constructed. They differ mainly by their auxiliary fields. All these theories can be derived from a theory with a larger symmetry algebra, the *conformal supergravity*. In order to go from the superconformal theory down to the super-Poincaré theory a new multiplet has to be introduced. This multiplet is called the compensator. Fixing the compensator allows us to go from conformal to the Poincaré theory. Various choices of the compensator will lead to different formulations of supergravity. The superconformal tensor calculus is particulary well suited for our study and we will recall some facts about it below. A complete review of tensor calculus can be found in [30].

1.3.1 Supermultiplets

Conformal supergravity is the gauge theory of the $N = 1$ superconformal algebra $su(2,2|1)$. Superconformal multiplets are characterized by two weights w and n which determine how the lowest component c of the multiplet transforms under dilatation and chiral $U(1)$ rotation,

$$\begin{aligned} \delta_{\lambda_D} c &= w\lambda_D c, \\ \delta_\theta c &= \tfrac{i}{2} n\theta c. \end{aligned} \tag{1.2}$$

λ_D parametrize the dilatations and θ the chiral rotation. The most general multiplet one can construct is a vector multiplet $\mathbf{V}$ and the others are obtained for special value of w and n. The different types of multiplets of use here are grouped in the table on top of the next page.

Multiplet	Symbol	w	n	Components
Vector (complex)	$\mathbf{V}$	w	n	$[c, \psi, m, n, b_\mu, \Lambda, d]$
Vector (real)	V	w	0	$[C, \psi, M, N, B_\mu, \lambda, D]$
Chiral (right)	Σ	n	n	$[z, \psi_R, f_\Sigma]$
Chiral (left)	$\bar{\Sigma}$	$-n$	n	$\left[\bar{z}, \psi_L, \bar{f}_{\bar{\Sigma}}\right]$
Linear (real)	L	2	0	$[c, \psi, B_\mu]$

Table 1.1: Supermultiplets

In order to do some calculations, the following formulas giving the embeddings of the chiral Σ, anti-chiral $\hat{\Sigma}$ and linear multiplet L into a vector multiplet will be useful,

$$\begin{aligned} \mathbf{V}(\Sigma) &= \left[z, -i\psi_R, -f, if, iD^c_\mu z, 0, 0\right], \\ \mathbf{V}(\bar{\Sigma}) &= \left[\bar{z}, i\psi_L, -\bar{f}, -i\bar{f}, -iD^c_\mu \bar{z}, 0, 0\right], \\ V(L) &= [c, \psi, 0, 0, B_\mu, \not{D}^c\psi, -\Box^c C]. \end{aligned} \tag{1.3}$$

In the case of the linear multiplet, we have a constraint on the B_μ field,

$$D^c_\mu B^\mu = 0$$

D^c_μ is the superconformal covariant derivative and $\Box^c$ the superconformal d'Alembertian operator. Once the symmetry has been fixed down to Poincaré, only the auxiliary field left is the gauge field A_μ of the chiral $U(1)$ transformation. The covariant derivative reduces then to,

$$D^c_\mu = \partial_\mu - \frac{i}{2} A_\mu n \tag{1.4}$$

where n is the chiral weight.

In this work we will be mostly interested in the bosonic components of the real vector multiplet (V), the chiral multiplets $(\Sigma, \bar{\Sigma})$ and the real linear multiplet (L).

$$\begin{aligned} &\Sigma: \ w = n = 0 \quad [z, 0, f_\Sigma] \quad [z, 0, -f_\Sigma, if_\Sigma, i\partial_\mu z, 0, 0] \\ &\bar{\Sigma}: \ w = n = 0 \quad \left[\bar{z}, 0, \bar{f}_{\bar{\Sigma}}\right] \quad \left[\bar{z}, 0, -\bar{f}_{\bar{\Sigma}}, -i\bar{f}_{\bar{\Sigma}}, -i\partial_\mu \bar{z}, 0, 0\right] \\ &L: \ w = 2; n = 0 \quad [C, 0, v_\mu] \quad [C, 0, 0, 0, v_\mu, 0, -\Box^c C] \end{aligned} \tag{1.5}$$

where $\Box^c C$ is [32].

$$\Box^c C = \Box C + \frac{1}{3} RC \tag{1.6}$$

since the dilaton C has chiral weight 0. The multiplication law for the bosonic components reduces to

$$\begin{array}{ll} V_i = \left[\, z_i \,,\, 0 \,,\, m_i \,,\, n_i \,,\, b_{i\mu} \,,\, 0 \,,\, d_i \right] & \\ \Sigma_i = \left[z_i, 0, f_{\Sigma i}\right] & ;\ i = 1,2 \\ \bar{\Sigma}_i = \left[\bar{z}_i, 0, \bar{f}_{\Sigma\, i}\right] & \end{array}$$

$$\begin{aligned} V_3 &= V_1 \times V_2 \\ &= \Big[z_1 z_2 \,,\, 0 \,,\, z_1 m_2 + z_2 m_1 \,,\, z_1 n_2 + z_2 n_1 ,\, z_1 b_{2\mu} + z_2 b_{1\mu} \,,\, 0 \,, \\ &\quad z_1 d_2 + z_2 d_1 + m_1 m_2 + n_1 n_2 - b_{1\mu} b_2{}^{\mu} - D^c_\mu z_1 D^{c\,\mu} z^2\Big] \\ \Sigma_3 &= \Sigma_1 \times \Sigma_2 \\ &= \left[z_1 z_2, 0, z_1 f_{\Sigma\, 2} + z_2 f_{\Sigma\, 1}\right] \\ V &= \Sigma \times \bar{\Sigma} \\ &= \Big[z\bar{z} \,,\, 0 \,,\, -(z\bar{f}_\Sigma + \bar{z} f_\Sigma) \,,\, -i(z\bar{f}_\Sigma - \bar{z} f_\Sigma) \,,\, -i(z\partial_\mu \bar{z} - \bar{z}\partial_\mu z) \\ &\quad ,\, 0 \,,\, 2(f_\Sigma \bar{f}_\Sigma - \partial_\mu \partial^\mu \bar{z})\Big] \end{aligned}$$

with

$$w_3 = w_1 + w_2 \qquad , \qquad n_3 = n_1 + n_2 .$$

Powers of bosonic multiplets are given by

$$\begin{aligned} V^k &= \Big[C^k \,,\, 0 \,,\, kC^{k-1}M \,,\, kC^{k-1}N \,,\, kC^{k-1}B_\mu \,,\, 0 \\ &\quad kC^{k-1}D + \tfrac{1}{2}k(k-1)C^{k-2}(M^2 + N^2 - B_\mu^2 - (D^c_\mu C)(D^{c\,\mu} C))\Big] \\ \Sigma^k &= \left[z^k \,,\, 0 \,,\, kz^{k-1} f_\Sigma\right] \\ \bar{\Sigma}^k &= \left[\bar{z}^k \,,\, 0 \,,\, k\bar{z}^{k-1} \bar{f}_{\Sigma}\right] \end{aligned}$$

In the case $w = n = 0$ one has the following expansion for functions of chiral fields

$$\begin{aligned} \Phi\,(\Sigma) &= \left[\phi(z) \,,\, 0 \,,\, \phi_z f_\Sigma\right] \\ \Phi\left(\Sigma, \bar{\Sigma}\right) &= \Big[\phi(z) \,,\, 0 \,,\, -\left(\phi_z f_\Sigma + \phi_{\bar{z}} \bar{f}_\Sigma\right) \\ &\quad ,\, i\left(\phi_z f_\Sigma - \phi_{\bar{z}} \bar{f}_\Sigma\right) \\ &\quad ,\, i\left(\phi_z D^c_\mu z - \phi_{\bar{z}} D^c_\mu \bar{z}\right) \,,\, 0 \\ &\quad ,\, -2\phi_{z\bar{z}}\left(\left|D^c_\mu z\right|^2 - |f_\Sigma|^2\right)\Big] \end{aligned}$$

The Chern-Simon multiplet is determined from the gauge kinetic terms. The gauge field strength W is normalized such that the highest component of WW contains the Yang-Mills action,

$$WW = \left[\ldots, \ldots, -\frac{1}{2} F^a_{\mu\nu} F^{a\,\mu\nu} + D^2 - \frac{i}{4}\epsilon^{\mu\nu\rho\sigma} F^a_{\mu\nu} F^a_{\rho\sigma}\right].$$

The gauge kinetic terms are then given by

$$\frac{1}{4}\left[SWW\right]_F + \text{h.c} \tag{1.7}$$

with S a chiral multiplet. The gauge coupling is

$$\frac{1}{g^2} = \text{Re}\left[S\right].$$

The non-abelian Chern-Simon multiplet is defined in terms of W by

$$\left[(S+\bar{S})\Omega\right]_D = -\frac{1}{2}(\left[SWW\right]_F + \text{h.c}). \tag{1.8}$$

In the Wess-Zumino gauge, its bosonic components are then

$$\left[0,0,0,0,\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\omega^{\nu\rho\sigma},0,\frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} - \frac{1}{2}D^2\right].$$

where $\omega^{\nu\rho\sigma}$ is the Chern-Simon form.

1.3.2 Tree-level lagrangian

The tree-level supergravity lagrangian will be described by the following multiplets. The compensator field S_0 will be chosen to be a chiral multiplet with $w = 1 = n$. This is the simplest choice and leads to a minimal Poincaré supergravity. Matter will be put in chiral multiplets with vanishing Weyl and chiral weight. The dilaton and the antisymmetric tensor fields will appear in a linear multiplet L. This determines the weights to be $w = 2$, $n = 0$. The lagrangian becomes a superconformal D density,

$$\mathcal{L} = \left[S_0\bar{S}_0\Phi\left(\frac{\hat{L}}{S_0\bar{S}_0},\Sigma,\bar{\Sigma}e^V\right)\right]_D + \left[S_0^3 w\left(\Sigma\right)\right]_F + \text{h.c.} \tag{1.9}$$

In the case of superstring effective theories, at tree-level, the function Φ is given by,

$$\Phi = \frac{\hat{L}}{S_0\bar{S}_0}F\left(e^{K/3}\frac{\hat{L}}{S_0\bar{S}_0}\right)$$

with the function $F(x)$ is

$$F = -\frac{1}{\sqrt{2}}\left(x\right)^{-\frac{3}{2}}.$$

The function K appearing in the argument of F is a real function of the scalars z^i and $\bar{z}_i$ called the Kähler potential while $w\left(\Sigma\right)$ is the superpotential. Considering only the bosonic sector, the field content of the various multiplets is,

$$\begin{aligned}
\Sigma^i &= \left[z^i,0,f^i\right] \\
L &= \left[C,0,V_\mu\right] \\
V &= \left[0,0,0,0,A_\mu,0,0\right] \\
S_0 &= \left[z_0,0,f_0\right].
\end{aligned}$$

Here Σ^i represents either the matter multiplets or the chiral moduli. The component expression for the bosonic kinetic terms is [16],

$$\begin{aligned} e^{-1}\mathcal{L} &= -\frac{1}{2\kappa^2}R_4 + \frac{1}{4}\frac{1}{\kappa^2 C^2}\partial^\mu C \partial_\mu C + \frac{1}{\kappa^2}K_{z^i\bar{z}_j}\partial_\mu z^i \partial^\mu \bar{z}_j \\ & -\frac{1}{4}\kappa^2 \frac{1}{(\kappa^2 C)^2}V^\mu V_\mu - \frac{1}{4}\frac{1}{\kappa^2 C}\mathrm{Tr}\,[F^{\mu\nu}F_{\mu\nu}]. \end{aligned} \tag{1.10}$$

In (1.10), the last gauge symmetry, the dilatation symmetry which remained, has been fixed by imposing that the Einstein term,

$$\frac{1}{3}z_0\bar{z}_0\left[\Phi - C\frac{\partial\Phi}{\partial C}\right]eR,$$

has the canonical form,

$$-\frac{3}{2}\frac{1}{\kappa^2}.$$

A much more detailed analysis of the supergravity lagrangian with a linear multiplet can be found in [16]. In this reference, it is also shown how one goes from the above formulation to the more familiar formulation of supergravities using chiral multiplets only.

1.4 Field-dependent gauge couplings

As briefly sketched in section (1.1), one knows in principle how to compute an effective quantum field theory representing the low-energy limit of a superstring theory. Practically, it is wiser to use some physical arguments to determine the form of the effective theory than to attempt a brute force calculation. The strategy is rather simple. One starts with a quantity which is well defined both at the field and string theory level. This allows the comparison of results obtained by using field theoretical technics with direct calculations from strings.

In recent years, much work in this spirit has been devoted to the computation of quantum corrections to gauge couplings or more precisely to field-dependent gauge couplings. In string theory, the gauge couplings g_a depends on the vacuum expectation values of moduli fields. The indice a labels the factor of the gauge group when $G = \Pi G_a$. These fields are called moduli since they parametrize the string vacuum. The value of the gauge couplings in the effective theory cannot yet be determined since we have no mechanism to fix the expectation values of these moduli. In the past few years, the dependence of g_a on these moduli fields has been intensively studied but many questions still remain open [28]. One field always present in the gravitational sector of string theory is the dilaton field. This field can be embedded into either a chiral or a linear multiplet. Even if the two choices are physically equivalent, they differ conceptually. From the supersymmetric field theory point of view it is more natural to use, as for the other moduli and matter fields, a chiral multiplet. This multiplet allow a clearer analytic analysis of the theory and general results exist for it. The problem with the embedding of the dilaton field in a chiral multiplet is that its relation to the string vertex operator can not be determined at tree-level like for all the other fields. It has to be corrected order by order in perturbation theory. This shows that from the string point of view, the situation is completely different. It is the linear multiplet which is more natural to use for the dilaton fields. Moreover the dilaton always comes with an antisymmetric tensor field which only appears through its curl in the lagrangian and becomes naturally one of the components of the linear multiplet.

The interests of the gauge couplings is that not only its field theoretical aspects are well known but also that it has been computed in string theory by various authors [17, 3, 4, 5] beyond the tree-level. Recall that at tree-level, g_a is universally given by [21],

$$\frac{1}{g_a^2} = k_a ReS, \tag{1.11}$$

where S is the chiral multiplet for the dilaton and k_a is the Kac-Moody level.

Chapter 2

The Green-Schwarz mechanism in Calabi-Yau compactification

2.1 Motivations

The Green-Schwarz counterterm ΔS is expected to yield information on quantum corrections to string effective field theories in the following way. Assuming that the compactification scale M_C is much smaller than the Planck scale M_P, one can first take the field theory (zero-slope) limit of the string and then do a compactification of the extra dimensions. In this work, we will be concerned only with the heterotic superstring. Its field theory limit is a supergravity super-Yang-Mills ten-dimensional field theory with stringy corrections. The Green-Schwarz counterterm is introduced to cancel gauge, gravitational and mixed anomalies arising through an hexagon diagram. ΔS is composed of higher dimensional operators. The reduction of this theory to four dimensions will lead to an $N = 1$ supergravity, super Yang-Mills theory. In the superconformal formalism, these theories can be characterized by the Kähler potential K, a real function of the supersymmetric multiplets and a holomorphic superpotential w. The anomalies arising in the Wilson action can be cancelled by a four-dimensional Green-Schwarz mechanism. Calculations in orbifolds have shown that these corrections to the tree level lagrangian take the form

$$-\frac{3}{32\pi^2}C(E_8)\left[\hat{L}K\right]_D.$$

One should therefore expect to see some of the components of the above expression arising in the reduction of ΔS. This "*fitting procedure*" between four-dimensional supergravity and the reduction of ΔS will be giving us informations on the large T limit of the Kähler potential K where T is the overall moduli (radius of compactification).

This chapter is organized as follow. First I recall how the Green-Schwarz counterterm is constructed in the ten-dimensional field theory limit of the $E_8 \times E_8'$ heterotic string. This construction will be done using the language of differential forms since it will be the one used later on in this work. After a brief introduction to the spin connection and some general facts about dimensional reduction, we will concentrate on our specific problem; the reduction of the Green-Schwarz counterterm. The spectrum of the Calabi-Yau compactification will be explicitly written and the harmonic expansion over K of ΔS presented. A special part of ΔS, the one involving the antisymmetric tensor, will be considered in more details since it was the subject of some analysis in the litterature. We will therefore end with some general features of the part of the Green-Schwarz counterterm involving the anti-symmetric tensor field and compare with some published results.

2.2 The Green Schwarz counterterm in 10 dimensions

The Green-Schwarz counterterm, ΔS, in ten dimensions cancels gauge, gravitational and mixed anomalies. General methods have been found to compute anomalies in various space-time dimensions. In order to define the notation, we will briefly describe the construction of ΔS using the language of differential forms developed by Zumino et al. [39, 40] and [2].

The zero-slope limit of the heterotic string theory based on $E_8 \times E_8'$ is a ten-dimensional $N = 1$ supergravity-superYang-Mills effective field theory. Even if the following results are more general, we will concentrate on this model in this thesis. Let us represent the effective action of this theory by Γ. The existence of a continuous symmetry at the classical level implies the existence of a conserved current, $\partial_\mu J^\mu = 0$. An anomaly is the breakdown of this symmetry at the quantum level. This means that the vacuum expectation value of $\partial_\mu J^\mu$ does not vanish and therefore that $\delta\Gamma \neq 0$[1]. The bosonic terms of anomalies can be characterized as follow.

Consider an infinitesimal gauge or coordinate transformation parametrized by Λ. The corresponding anomaly is described by the variation of an effective action Γ,

$$G(\Lambda) = \frac{\delta\Gamma}{\delta\Lambda}. \tag{2.1}$$

The anomaly G is subject to the Wess-Zumino consistency condition which relates two succesive transformations,

$$\delta_{\Lambda_1} G(\Lambda_2) - \delta_{\Lambda_2} G(\Lambda_1) = G(\Lambda) \tag{2.2}$$

where

$$\Lambda = [\Lambda_1, \Lambda_2]$$

In our case, the space M is ten-dimensional and mathematically G is an integral over M of a ten form. Formally, this ten form is determined by a gauge invariant 12 form $I_{12}(F, R)$ where F and R are the gauge and gravitational curvature. By using the expression of the curvatures in term of their respective generalized Chern-Simons form,

$$\mathrm{Tr}[F^n] = \mathrm{d}\Omega^{(G)}_{2n-1} \quad ; \quad \mathrm{Tr}[R^n] = \mathrm{d}\Omega^{(L)}_{2n-1},$$

it follows that

$$\mathrm{dTr}[F^m] = \mathrm{d}\,\mathrm{tr}[R^n] = 0$$

and

$$\mathrm{d}I_{12} = 0.$$

I_{12} is closed and can be written locally,

$$I_{12}(F,R) = \mathrm{d}I_{11}(A,\omega)$$

where A and ω are the gauge and spin connection. These matrix valued 1-forms are related to the curvatures by

$$\begin{aligned} F &= \mathrm{d}A + A^2 = \tfrac{1}{2} i F^B_{\hat{\mu}\hat{\nu}}\, dx^{\hat{\mu}} \wedge dx^{\hat{\nu}} T^B, \\ R &= \mathrm{d}\omega + \omega^2 = \tfrac{1}{4} R_{\hat{\mu}\hat{\nu}\hat{p}\hat{q}}\, dx^{\hat{\mu}} \wedge dx^{\hat{\nu}} t^{\hat{p}\hat{q}}, \end{aligned} \tag{2.3}$$

where $t^{\hat{p}\hat{q}} = -t^{\hat{q}\hat{p}}$ denotes generators for the vector, (**10**), representation of the Lorentz group $SO(1,9)$ and T^B are generators for the adjoint representation of $E_8 \times E_8$. Since I_{12} is gauge invariant, there exists an 11 form depending on the gauge or spin connection such that,

$$\mathrm{d}\delta_\Lambda I_{11} = 0.$$

In the above equation δ_Λ denotes an infinitesimal gauge or Lorentz transformation with parameter Λ (a zero form). Then, locally

$$\delta_\Lambda I_{11} = \mathrm{d}I^1_{10}(\Lambda, A, \omega),$$

where the superscript 1 indicates that I_{10} is linear in Λ. The consistent anomaly G can then be written as

$$G(\Lambda, A, \omega) = C \int I^1_{10}(\Lambda, A, \omega). \tag{2.4}$$

C is a constant defined by the normalization of the anomaly (see [40]). The 12-form characterizing gravitational, Yang-Mills and mixed anomalies arising in hexagon 1-loop diagram is determined by the chiral fermions of the theory and is given by [2]

$$\begin{aligned} I_{12} = & -\tfrac{1}{15}\mathrm{Tr}\,[F^6] + \tfrac{1}{24}\mathrm{Tr}\,[F^4]\,\mathrm{tr}\,[R^2] \\ & -\tfrac{1}{960}\mathrm{Tr}\,[F^2]\left(4\,\mathrm{tr}\,[R^4] + 5\,(\,\mathrm{tr}\,[R^2])^2\right) \\ & +\left(\tfrac{1}{32} + \tfrac{n-496}{13824}\right)(\,\mathrm{tr}\,[R^2])^3 \\ & +\left(\tfrac{1}{8} + \tfrac{n-496}{5760}\right)\mathrm{tr}\,[R^2]\,\mathrm{tr}\,[R^4] \\ & +\tfrac{n-496}{7560}\,\mathrm{tr}\,[R^6]. \end{aligned} \tag{2.5}$$

I adopt the convention where Tr refers to traces in the adjoint representation and tr to traces in the fundamental representation of the various groups. The Green-Schwarz anomaly cancellation can take place only if I_{12} factorizes as,

$$I_{12} = I_4 \wedge I_8. \tag{2.6}$$

This allows the possibility of cancelling anomalies using the antisymmetric tensor field $B_{\mu\nu}$.The factorization requires then that $\mathrm{tr}R^6$ is absent and, second, that $\mathrm{Tr}F^6$ factorizes. These requirements imply that $n = 496$ and that the gauge group cannot contain an independent 6 order Casimir, conditions which are far from trivial. It is amazing that in the case of the $E_8 \times E_8'$ heterotic string, both of them are realized. First the dimension of $E_8 \times E_8'$ is $248+248 = 496$ and second,

$$\mathrm{Tr}F^6 = \frac{1}{48}\mathrm{Tr}F^2\mathrm{Tr}F^4 - \frac{1}{14400}(\mathrm{Tr}F^2)^3, \tag{2.7}$$

which is specific to $E_8 \times E_8'$. This choice is not the only solution but it will be the only one studied here. From this, one can read off the 4 and 8-form of (2.6),

$$\begin{aligned} I_8(F,R) &= \tfrac{1}{24}\mathrm{Tr}F^4 - \tfrac{1}{7200}(\mathrm{Tr}F^2)^2 + \tfrac{1}{8}\,\mathrm{tr}R^4 + \tfrac{1}{32}(\,\mathrm{tr}R^2)^2 - \tfrac{1}{240}\mathrm{Tr}F^2\,\mathrm{tr}R^2, \\ I_4(F,R) &= \mathrm{tr}R^2 - \tfrac{1}{30}\mathrm{Tr}F^2. \end{aligned} \tag{2.8}$$

In this case, G takes the form,

$$\begin{aligned} G(\Lambda) &= C\left(\tfrac{2}{3}+\alpha\right)\int(\,\mathrm{tr}R^2 - \mathrm{Tr}F^2)\,I_6^1(F,R,\Lambda) \\ &+ C\left(\tfrac{1}{3}-\alpha\right)\int\left(\omega_2'^{(L)}(A,\omega,\Lambda) - \omega_2'^{(G)}(A,\omega,\Lambda)\right)I_8(F,R) \end{aligned}$$

where I_6' is defined by $\mathrm{d}I_6' = \delta_\Lambda I_7$ with $I_8 = \mathrm{d}I_7$. $\omega_2'^{(L)}$ and $\omega_2'^{(G)}$ are two 2-forms whose infinitesimal transformation gives the Lorentz and Chern-Simons form. α is an integration constant. This anomaly can be compensated by introducing a counterterm ΔS such that

$$G + \delta_\Lambda(\Delta S) = 0.$$

A crucial element which permits the construction of ΔS is the presence of the antisymmetric tensor field $B_{\hat\mu\hat\nu}$ whose gauge transformation is

$$\delta B = \omega_2^{1\,(G)} - \omega_2^{1\,(L)}.$$

ΔS is given by

$$\Delta S = -C\int\left[B\wedge I_8(F,R) - (\frac{2}{3}+\alpha)I_7(A,\omega)I_3(A,\omega)\right], \tag{2.9}$$

where B is the 2-form $\frac{1}{2}B_{\mu\nu}\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu$. I_3 and I_7 are respectively 3 and 7-forms polynomial in A and ω. I_7 is defined above and I_3 is determined by $I_4 = \mathrm{d}I_3$. The explicit form for the counterterm (2.9) is then given in terms of the connections and curvatures by introducing in the above expression

$$\begin{aligned} I_3(A,\omega) &= \Omega_3^L(\omega) - \tfrac{1}{30}\Omega_3^G(A), \\ I_7(A,\omega) &= \tfrac{1}{24}\Omega_7^G(A) + \tfrac{1}{8}\Omega_7^L(\omega) - \tfrac{1}{7200}\mathrm{Tr}F^2\Omega_3^G(A) + \tfrac{1}{32}\,\mathrm{tr}R^2\Omega_3^L(\omega) \\ &\quad -\tfrac{1}{480}\left[\Omega_3^G(A)\,\mathrm{tr}R^2 + \Omega_3^L(\omega)\mathrm{Tr}F^2\right]. \end{aligned} \tag{2.10}$$

The relevant Chern-Simons forms are [1]:

$$\begin{aligned}
\Omega_3^G(A) &= \mathrm{Tr}\left[A(dA) + \tfrac{2}{3}A^3\right], \\
\Omega_3^L(\omega) &= \mathrm{Tr}\left[\omega(d\omega) + \tfrac{2}{3}\omega^3\right], \\
\Omega_7^G(A) &= \mathrm{Tr}\left[A(dA)^3 + \tfrac{8}{5}A^3(dA)^2 + \tfrac{4}{5}A(dA)A^2(dA)\right. \\
&\quad \left. +2A^5(dA) + \tfrac{4}{7}A^7\right], \\
\Omega_7^L(\omega) &= \mathrm{Tr}\left[\omega(d\omega)^3 + \tfrac{8}{5}\omega^3(d\omega)^2 + \tfrac{4}{5}\omega(d\omega)\omega^2(d\omega)\right. \\
&\quad \left. +2\omega^5(d\omega) + \tfrac{4}{7}\omega^7\right].
\end{aligned} \tag{2.11}$$

2.3 Dimensional reduction

Before doing the actual compactification on a Calabi-Yau manifold, let us review the process of dimensional reduction and introduce some notation.

We start by introducing a notation convinient for problems involving dimensional reduction in general. The conventions are analogous to ref. [35] (see also [34] for a derivation of the effective theory of $N = 4$ superstrings using similar methods). It will be assumed that we are reducing a theory from ten dimension to four but most of the formulas can be generalized to other cases. Ten-dimensional indices, with values $0, 1, \ldots, 9$ will be hatted:

$$\begin{aligned} \text{Curved indices}: &\quad \hat{\mu}, \hat{\nu}, \hat{\rho}, \hat{\sigma}, \\ \text{Flat indices}: &\quad \hat{m}, \hat{n}, \hat{p}, \hat{s}. \end{aligned}$$

Four-dimensional space-time indices ($= 0, 1, 2, 3$) will be:

$$\begin{aligned} \text{Curved indices}: &\quad \mu, \nu, \rho, \sigma, \\ \text{Flat indices}: &\quad m, n, p, s, \end{aligned}$$

and finally indices for the six compact dimensions will read

$$\begin{aligned} \text{Curved indices}: &\quad \alpha, \beta, \gamma &= 1, 2, \ldots, 6, \\ \text{Flat indices}: &\quad a, b, c &= 1, 2, \ldots, 6. \end{aligned}$$

As for indices, fields defined in ten dimensions will be hatted when used in expressions involving reduced fields. The ten-dimensional metric $\hat{g}_{\hat{\mu}\hat{\nu}}$ and the flat metric

$$\hat{\eta}_{\hat{m}\hat{n}} = \mathrm{diag}(1, -1, \ldots, -1) \tag{2.12}$$

are related by the zehnbein $\hat{V}_{\hat{\mu}}^{\hat{m}}$,

$$\hat{g}_{\hat{\mu}\hat{\nu}} = \hat{V}_{\hat{\mu}}^{\hat{m}} \hat{V}_{\hat{\nu}}^{\hat{n}} \eta_{\hat{m}\hat{n}}. \tag{2.13}$$

Dimensional reduction proceeds in two steps. The theory is first rewritten as a 4+6 dimensional theory, with $SO(1,3)\times SO(6)$ Lorentz symmetry. Symmetries in $SO(1,9)/SO(1,3)\times SO(6)$ allow to choose a convenient form for the zehnbein, which displays the particle content of the reduced theory. Then, the fields are assumed not to depend on the internal compact coordinates.

Using ten-dimensional Lorentz symmetry $SO(1,9)$ acting on indices $\hat{m},\hat{n}\ldots$, the zehnbein can be taken as

$$\hat{V}_{\hat{\mu}}^{\hat{m}} = \begin{pmatrix} \varphi^k V_\mu^m & 2\kappa A_\mu^\beta \Phi_\beta^a \\ 0 & \Phi_\alpha^a \end{pmatrix}. \tag{2.14}$$

In this expression, the scalar fields Φ_α^a correspond to the internal vierbein, $\varphi = \det \Phi_\alpha^a$ and k is a parameter which will be determined for convenience later on. This choice for $\hat{V}_{\hat{\mu}}^{\hat{m}}$ is invariant under the $SO(1,3)\times SO(6)$ subgroup of $SO(1,9)$. The 36 components of Φ_α^a contain 21 independent scalar degrees of freedom since $SO(6)$ symmetry can be used to remove 15 components. Using eq. (2.13), the metric tensor becomes

$$\hat{g}_{\hat{\mu}\hat{\nu}} = \begin{pmatrix} \varphi^{2k} g_{\mu\nu} - 4\kappa^2 A_\mu^\gamma h_{\gamma\delta} A_\nu^\delta & -2\kappa A_\mu^\gamma h_{\gamma\beta} \\ -2\kappa A_\nu^\gamma h_{\alpha\gamma} & -h_{\alpha\beta} \end{pmatrix}, \tag{2.15}$$

where the internal metric is

$$h_{\alpha\beta} = \Phi_\alpha^a \Phi_\beta^b \delta_{ab}. \tag{2.16}$$

The inverse zehnbein $\hat{V}^{\hat{\mu}\hat{m}}$ is defined by

$$\hat{V}^{\hat{\mu}\hat{m}} \hat{V}_{\hat{\mu}}^{\hat{n}} = \hat{\eta}^{\hat{m}\hat{n}}.$$

it reads

$$\hat{V}_{\hat{m}}^{\hat{\mu}} = \begin{pmatrix} \varphi^{-k} V_m^\mu & -2\kappa\varphi^{-k} A_m^\alpha \\ 0 & \Phi_a^\alpha \end{pmatrix}, \tag{2.17}$$

with

$$A_m^\alpha = V_m^\mu A_\mu^\alpha, \qquad V_m^\mu V_\mu^n = \delta_m^n, \tag{2.18}$$

and Φ_b^α is the inverse internal vierbein, $\Phi_b^\alpha \Phi_\alpha^a = \delta_b^a$. This leads to the following inverse metric tensor,

$$\hat{g}^{\hat{\mu}\hat{\nu}} = \begin{pmatrix} \varphi^{-2k} g^{\mu\nu} & -2\kappa A^{\mu\beta} \\ -2\kappa A^{\mu\alpha} & -h^{\alpha\beta} + 4\kappa^2 \varphi{-2k} A^{\mu\,\alpha} A_\mu^\beta \end{pmatrix} \tag{2.19}$$

In ten dimensions, flat indices $\hat{m},\hat{n},\ldots$ are lowered using $\hat{\eta}_{\hat{m}\hat{n}}$ and raised with its inverse $\hat{\eta}^{\hat{m}\hat{n}}$ while in the four dimensional space-time, curved (flat) indices are now lowered with $g_{\mu\nu}$ ($\eta_{\mu\nu}$) and raised with $g^{\mu\nu}$ ($\eta^{\mu\nu}$).

As an illustration let's derive the ordinary reduction of the Einstein action. It involves the curvature tensor defined by

$$\hat{R}_{\hat{\mu}\hat{\nu}\hat{p}\hat{q}}(\hat{\omega}) = \partial_{\hat{\mu}} \hat{\omega}_{\hat{\nu}\hat{p}\hat{q}} - \partial_{\hat{\nu}} \hat{\omega}_{\hat{\mu}\hat{p}\hat{q}} + \hat{\omega}_{\hat{\mu}\hat{p}}{}^{\hat{t}} \hat{\omega}_{\hat{\nu}\hat{t}\hat{q}} - \hat{\omega}_{\hat{\nu}\hat{p}}{}^{\hat{t}} \hat{\omega}_{\hat{\mu}\hat{t}\hat{q}}, \tag{2.20}$$

in terms of the connection $\hat{\omega}_{\hat{\mu}\hat{p}\hat{q}} = -\hat{\omega}_{\hat{\mu}\hat{q}\hat{p}}$. It is useful to work with flat indices only and to introduce

$$\hat{\omega}_{\hat{m}\hat{p}\hat{q}} = \hat{V}^{\hat{\mu}}_{\hat{m}}\hat{\omega}_{\hat{\mu}\hat{p}\hat{q}}. \tag{2.21}$$

and recall that $\hat{\omega}_{\hat{m}\hat{n}\hat{p}}$ is antisymmetric under the exchange of the *last two* indices. The curvature scalar is then

$$\hat{R}(\hat{\omega}) = \hat{V}^{\hat{\mu}\hat{p}}\hat{V}^{\hat{\nu}\hat{q}}\hat{R}_{\hat{\mu}\hat{\nu}\hat{p}\hat{q}}(\hat{\omega}), \tag{2.22}$$

and Einstein action is

$$S_{grav.} = -\frac{1}{4\hat{\kappa}^2}\int d^{10}x\,\hat{V}\hat{R}(\hat{\omega}), \tag{2.23}$$

where $\hat{V} = \det \hat{V}^{\hat{m}}_{\hat{\mu}}$. The connection $\hat{\omega}_{\hat{m}\hat{n}\hat{p}}$ is a scalar under general ten dimensional coordinate transformations. It is a function of the zehnbein, its inverse and its first derivative:

$$\hat{\omega}_{\hat{m}\hat{n}\hat{p}} = -\hat{\Omega}_{\hat{m}\hat{n},\hat{p}} + \hat{\Omega}_{\hat{n}\hat{p},\hat{m}} - \hat{\Omega}_{\hat{p}\hat{m},\hat{n}}, \tag{2.24}$$

where

$$\hat{\Omega}_{\hat{m}\hat{n},\hat{p}} = \frac{1}{2}\left(\hat{V}^{\hat{\mu}}_{\hat{m}}\hat{V}^{\hat{\nu}}_{\hat{n}} - \hat{V}^{\hat{\mu}}_{\hat{n}}\hat{V}^{\hat{\nu}}_{\hat{m}}\right)\partial_{\hat{\nu}}\hat{V}_{\hat{\mu}\hat{p}} = -\hat{\Omega}_{\hat{n}\hat{m},\hat{p}}. \tag{2.25}$$

The curvature scalar can then easily be computed:

$$R(\omega) = \hat{\omega}^{\hat{m}}{}_{\hat{n}\hat{p}}\hat{\omega}^{\hat{n}\hat{p}}{}_{\hat{m}} - \hat{\omega}^{\hat{m}}{}_{\hat{m}\hat{n}}\hat{\omega}^{\hat{p}}{}_{\hat{p}}{}^{\hat{n}} + 2\hat{V}^{\hat{\mu}\hat{p}}\partial_{\hat{\mu}}\hat{\omega}_{\hat{n}\hat{p}}{}^{\hat{n}}. \tag{2.26}$$

In the gravity action (2.23), terms involving derivatives of connections can be eliminated by partial integration, to obtain

$$S_{grav.} = -\frac{1}{2\hat{\kappa}^2}\int d^{10}x\,\hat{V}\left(\hat{\omega}^{\hat{m}}{}_{\hat{n}\hat{p}}\hat{\omega}^{\hat{n}\hat{p}}{}_{\hat{m}} + \hat{\omega}^{\hat{m}}{}_{\hat{m}\hat{n}}\hat{\omega}^{\hat{p}}{}_{\hat{p}}{}^{\hat{n}}\right). \tag{2.27}$$

This equation will be useful to normalise correctly the lower dimensional theories.

eq. (2.24), (2.25), (2.14) and (2.17) can then be used to compute the connections $\hat{\omega}_{\hat{m}\hat{n}\hat{p}}$ for the case of dimensional reduction,

$$\partial_\alpha(\text{any field}) = 0.$$

This results in

$$\begin{aligned}
\hat{\omega}_{mnp} &= \varphi^{-k}\left[\omega_{mnp} + k\partial_\mu(\log\varphi)(\eta_{mn}V^\mu_p - \eta_{mp}V^\mu_n)\right],\\
\hat{\omega}_{mna} &= +\kappa\varphi^{-2k}F^\alpha_{mn}\Phi_{\alpha\,a},\\
\hat{\omega}_{mab} &= +\tfrac{1}{2}\varphi^{-k}V^\mu_m\left[\Phi^\alpha_a(\partial_\mu\Phi_{\alpha\,b}) - \Phi^\alpha_b(\partial_\mu\Phi_{\alpha\,a})\right],\\
\hat{\omega}_{amn} &= -\kappa\varphi^{-2k}F^\alpha_{mn}\Phi_{\alpha\,a},\\
\hat{\omega}_{amb} &= +\tfrac{1}{2}\varphi^{-k}V^\mu_m\left[\Phi^\alpha_a(\partial_\mu\Phi_{\alpha\,b}) + \Phi^\alpha_b(\partial_\mu\Phi_{\alpha\,a})\right],\\
\hat{\omega}_{abc} &= 0.
\end{aligned} \tag{2.28}$$

Notice that $\hat{\omega}_{amb}$ can also be written as, $\hat{\omega}_{amb} = \frac{1}{2}\varphi^{-k}V^\mu_m \Phi^\alpha_a \Phi^\beta_b (\partial_\mu h_{\alpha\beta})$. In these expressions, the abelian gauge curvatures are

$$F^\alpha_{mn} = V^\mu_m V^\nu_n F^\alpha_{\mu\nu}, \qquad F^\alpha_{\mu\nu} = \partial_\mu A^\alpha_\nu - \partial_\nu A^\alpha_\mu, \tag{2.29}$$

and the space-time connections ω_{mnp} are defined using eq. (2.24) and (2.25) for four-dimensional quantities without hat. [Also, $\Phi^a_\alpha = -\Phi_{\alpha a}$ due to our flat metric, eq. (2.12)].

The dimensionally reduced connections, eq. (2.28), always contain one space-time derivative, appearing in the form $V^\mu_m \partial_\mu$ since connections $\hat{\omega}_{\hat{m}\hat{n}\hat{p}}$ have only flat indices. The connection without space-time flat indices, $\hat{\omega}_{abc}$ must then vanish.

Inserting (2.28) into (2.27) leads to the following reduced action,

$$\begin{aligned} S_{grav} = \int d^D x e \quad & \Big\{ -\tfrac{1}{4\kappa^2}\varphi^{(D-2)k+1}\left(\omega^m{}_{np}\omega^{np}{}_m + \omega^m{}_n{}^p \omega^n{}_{np}\right) \\ & -\tfrac{1}{4}\varphi^{(D-4)k+1} h_{\alpha\beta} F^\alpha{}_{\mu\nu} F^{\beta\,\mu\nu} \\ & -\tfrac{1}{16\kappa^2}\varphi^{(D-2)K+1} g^{\mu\nu}\partial_\mu h_{\alpha\beta}\partial_\nu h^{\alpha\beta} \\ & -\tfrac{k^2(D-2)(D-1)+2k(D-1)+1}{4\kappa^2}\varphi^{(D-2)K+1} g^{\mu\nu}\partial_\mu(\log\varphi)\partial_\nu(\log\varphi) \\ & -\tfrac{2k(D-2)+2}{4\kappa^2}\varphi^{(D-2)K+1}\partial_\mu(\log\varphi) E^\mu_\nu \omega^p{}_p{}^q \Big\} \end{aligned} \tag{2.30}$$

where the explicit space-time dimension, D, dependence is shown. The free parameter k is then obtained by requiring that the dimensionally reduced theory contains a canonically normalised Einstein term. In the case $D = 4$, the zehnbein determinant is

$$\hat{V} = \varphi^{4k+1} V, \tag{2.31}$$

where $V = \det V^m_\mu$ is the vierbein determinant. With eq. (2.27) and (2.28), it is clear that $S_{grav.}$ contains the four-dimensional Einstein action multiplied by a factor φ^{2k+1}. Choosing

$$k = -\frac{1}{2} \tag{2.32}$$

normalises correctly the Einstein action. and (2.30) becomes,

$$\begin{aligned} S_{grav} = \int d^4 x e \quad & \Big\{ -\tfrac{1}{4\kappa^2} R \\ & -\tfrac{1}{4}\varphi h_{\alpha\beta} F^\alpha{}_{\mu\nu} F^{\beta\,\mu\nu} \\ & -\tfrac{1}{16\kappa^2} g^{\mu\nu}\partial_\mu h_{\alpha\beta}\partial_\nu h^{\alpha\beta} \\ & +\tfrac{1}{4\kappa^2} g^{\mu\nu}\partial_\mu(\log\varphi)\partial_\nu(\log\varphi) \Big\} \end{aligned} \tag{2.33}$$

This terminates the reduction of the gravitational sector. The dimensional reduction of the gauge sector is, on the other hand, very simple. The gauge bosons $A^B_{\hat\mu}$ are written as

$$A^B_{\hat\mu} = \begin{pmatrix} A^B_\mu & z^B_\alpha \end{pmatrix}, \tag{2.34}$$

where A^B_μ are four-dimensional gauge potentials and z^B_α are real scalar fields in the adjoint representation of the gauge group, $\alpha = 1, \ldots, 6$. Gauge curvatures $\hat F^B_{\hat\mu\hat\nu}$ read:

$$\begin{aligned} \hat F^B_{\mu\nu} &= \partial_\mu A^B_\nu - \partial_\nu A^B_\mu + f^{BCD} A^C_\mu A^D_\nu, \\ \hat F^B_{\mu\alpha} &= \partial_\mu z^B_\alpha + f^{BCD} A^C_\mu z^D_\alpha \equiv (D_\mu z_\alpha)^B, \\ \hat F^B_{\alpha\beta} &= f^{BCD} z^C_\alpha z^D_\beta, \end{aligned} \tag{2.35}$$

in terms of the structure constants f^{BCD} of the gauge group.

In order to correctly normalise four-dimensional states, one starts with the gauge kinetic terms in ten-dimensional $N = 1$ supergravity coupled to super-Yang-Mills :

$$S_{gauge} = \int d^{10}x \left[-\frac{1}{4} \phi^{-3/4} \hat V \, \hat g^{\hat\mu\hat\rho} \hat g^{\hat\nu\hat\sigma} \hat F^B_{\hat\mu\hat\nu} \hat F^B_{\hat\rho\hat\sigma} \right], \tag{2.36}$$

where ϕ is the real scalar field contained in the supergravity multiplet. The ten-dimensional gauge coupling constant is then $g_{10} = \phi^{3/8}$. After dimensional reduction, using eq. (2.31, 2.34), one obtains

$$-\frac{1}{4} \phi^{-3/4} \hat V \, \hat g^{\hat\mu\hat\rho} \hat g^{\hat\nu\hat\sigma} \hat F^B_{\hat\mu\hat\nu} \hat F^B_{\hat\rho\hat\sigma} = -\frac{1}{4} \phi^{-3/4} \varphi F^B_{\mu\nu} F^{B\,\mu\nu} + \text{other terms},$$

which indicates that the four-dimensional gauge coupling is

$$g = \phi^{3/8} \varphi^{-1/2} = g_{10} \varphi^{-1/2}. \tag{2.37}$$

2.4 Spectrum of Calabi-Yau compactification

We consider the $N = 1, d = 10$ supergravity super Yang-Mills theory obtained in the zero slope limit of the $E_8 \times E_8'$ heterotic superstring compactified on a Calabi-Yau manifold K with the holonomy group embedded in the gauge group [11]. The spectrum of fields in ten dimensions is the supergravity multiplet containing the vielbein $\hat e^{\hat m}_{\hat\mu}$, the antisymmetric tensor field $\hat B_{\hat\mu\hat\nu}$ and a scalar field $\hat\varphi$ and their fermionic partners in addition to the super Yang-Mills multiplet containing the gauge degrees of freedom $\hat A^a_{\hat\mu}$ and the gauginos $\hat\chi^a$. In this particular calculation, only the bosonic degrees of freedom are needed since (2.9) is bosonic. The next sections are of a rather technical nature. Before entering the core of the subject, we will introduce all the necessary notations and conventions. We will then write down the massless spectrum (zero-modes) of the charged fields before going to the supergravity multiplet.

2.4.1 Conventions and normalizations

As will be shown below, many terms in ΔS are directly related to $(\mathrm{Tr}F^2)^m \wedge (\mathrm{tr}R^2)^n$ for various power m and n. These terms can be easily computed in terms of the four-dimensional fields.

Let us first define the normalization of the various generators appearing in this work. As a starting point we take the standard normalization of $SU(N)$ generators for the fundamental representation $(\mathbf{N})$,

$$\mathrm{tr}[T^a, T^b] = \frac{1}{2}\delta^{ab}. \tag{2.38}$$

tr[...] means a trace in the fundamental representation $\mathbf{N}$ and Tr[...] is a trace in the adjoint representation. For a specific representation $\mathcal{R}$ the trace is written $\mathrm{Tr}_{\mathcal{R}}[...]$. For any group G and representation $\mathcal{R}$ we define $T_G(\mathcal{R})$ and $C(G)$ as

$$\begin{aligned}\mathrm{Tr}_{\mathcal{R}}[T^aT^b] &= T_G(\mathcal{R})\delta^{ab} \\ \mathrm{Tr}[T^aT^b] &= C(G)\delta^{ab}\end{aligned}$$

$C(G)$ is the quadratic Casimir of the group G. Once the convention for the $SU(N)$ generators is given, through,

$$T_{SU(N)}(N) = \frac{1}{2} \quad , \quad C(SU(N)) = N,$$

the normalization of the other generators are determined through the branching rules.

E_8 generators

The 248 hermitian generators are written T^α with $\alpha = 1, \ldots, 248$. They obey the commutation relations

$$\left[T^\alpha, T^\beta\right] = i f^{\alpha\beta\gamma} T^\gamma$$

where $f^{\alpha\beta\gamma}$ are the structure constants of E_8. For the adjoint representation,

$$(T^\alpha)^{\beta\gamma} = i f^{\alpha\beta\gamma}.$$

They are normalized

$$\mathrm{Tr}\left[T^\alpha T^\beta\right] = 30\delta^{\alpha\beta}.$$

to be consistent with (2.38) The branching rule for the adjoint of E_8 to $SU(3) \times E_6$ is

$$\mathbf{248} = (\mathbf{1}, \mathbf{78}) \oplus (\mathbf{8}, \mathbf{1}) \oplus (\mathbf{3}, \mathbf{27}) \oplus (\bar{\mathbf{3}}, \overline{\mathbf{27}}).$$

and the choice of basis for the generators T^α is made accordingly,

$$T^\alpha = \begin{cases} T^A & A = 1, \ldots, 78 \\ T^a & a = 1, \ldots, 8 \\ T^{iI} & i = 1, 2, 3 \\ \bar{T}_{iI} & I = 1, \ldots, 27 \end{cases} \tag{2.39}$$

The explicit forms are,

$$
\begin{aligned}
\left(T^A\right)^{BC} &= if^{ABC} \\
\left(T^A\right)^{iI}{}_{jJ} &= i\,f^{A\,iI}{}_{jJ} &\equiv \left(C^A\right)^I{}_J\,\delta^i_j \\
\left(T^a\right)^{bc} &= if^{abc} \\
\left(T^a\right)^{iI}{}_{jJ} &= i\,f^{a\,iI}{}_{jJ} &= \tfrac{1}{2}\left(\lambda^a\right)^i{}_j\,\delta^I_J \\
\left(T^{iI}\right)^{jJ\,kK} &= i\,f^{iI\,jJ\,kK} &= i\epsilon^{ijk}\lambda^{IJK} \\
\left(\bar{T}_{iI}\right)_{jJ\,kK} &= i\,f_{iI\,jJ\,kK} &= i\epsilon_{ijk}\bar{\lambda}_{IJK}.
\end{aligned}
$$

The other components are obtained by antisymmetry. $(C^A)^I{}_J$ are generators in the **27** and $(C^A)_J{}^I$ are generators in the $\overline{\mathbf{27}}$ of E_6 (note the order of indices !). They are related by

$$(C^A)^I{}_J = -(C^A)_J{}^I.$$

λ^a are the Gell-Mann matrices of $SU(3)$. ϵ^{ijk} is the $SU(3)$ totally antisymmetric tensor and λ^{IJK} is the E_6 symmetric tensor of order 3. Since the T^{iI} are imaginary,

$$\lambda^{IJK} = \bar{\lambda}^*_{IJK}.$$

Moreover, the normalization of E_8 generators imposes that

$$\lambda^{ILM}\bar{\lambda}_{JML} = 5\delta^I_J.$$

This normalization of the generators gives the following non-zero traces,

$$
\begin{aligned}
\mathrm{Tr}\left[T^A T^B\right] &= 30\delta^{AB} \\
\mathrm{Tr}\left[T^a T^b\right] &= 30\delta^{ab} \\
\mathrm{Tr}\left[T^{iI}\bar{T}_{jJ}\right] &= 30\delta^i_j\delta^I_J \\
\mathrm{Tr}\left[T^A[\bar{T}_{iI},T^{jJ}]\right] &= 30\left(C^A\right)_I{}^J\,\delta^j_i \\
\mathrm{Tr}\left[T^a[\bar{T}_{iI},T^{jJ}]\right] &= 30\tfrac{1}{2}\left(\lambda^a\right)_i{}^j\,\delta^J_I \\
\mathrm{Tr}\left[T^{iI}\left[T^{jJ},T^{kK}\right]\right] &= 30i\epsilon^{ijk}\lambda^{IJK} \\
\mathrm{Tr}\left[\bar{T}_{iI}\left[\bar{T}_{jJ},\bar{T}_{kK}\right]\right] &= 30i\epsilon_{ijk}\bar{\lambda}_{IJK}
\end{aligned}
\qquad (2.40)
$$

$SO(1,9)$(Lorentz) generators

$SO(1,9)$ has 45 generators t^α , $\alpha = 1,\dots,45$ which can be labeled with two indices as

$$t^{MN}\ ,\ M,N = 1,\dots,10, \tag{2.41}$$

with the condition that $t^{NM} = -t^{MN}$. In the following, we will always consider $M > N$. They are normalized as[1]

$$\mathrm{tr}\left[t^{MN}t^{PQ}\right] = \delta^{MP}\delta^{NQ} \tag{2.42}$$

In an $SO(1,3) \times SO(6)$ basis, the 45 generators are labeled

$$t^{MN}\begin{cases} t^{mn} = -t^{nm} \\ t^{ab} = -t^{ba} \\ t^{ma} \end{cases} \quad \begin{array}{rcl} m,n &=& 1,...,4 \\ a,b &=& 1,...,6 \end{array} .$$

In this basis, the normalization becomes,

$$\begin{aligned} \mathrm{tr}\left[t^{mn}t^{pq}\right] &= \delta^{mp}\delta^{nq} \\ \mathrm{tr}\left[t^{ab}t^{cd}\right] &= \delta^{ac}\delta^{bd} \\ \mathrm{tr}\left[t^{ma}t^{nb}\right] &= \delta^{mn}\delta^{ab}. \end{aligned} \tag{2.43}$$

All the other traces are zero.

2.4.2 Spectrum of the gauge sector

The zero-modes of the gauge fields are in one to one correspondence with harmonic form on K [19, chapt.16]. The number of massless fields depends on their transformation properties under the holonomy group.

In ten dimensions, the gauge fields, $\hat{A}^\alpha, \alpha = 1,\dots,496$, belong to the adjoint representation of $E_8 \times E_8'$,

$$(\mathbf{248},\mathbf{1}) \oplus (\mathbf{1},\mathbf{248}).$$

Upon compactification à la Candelas et al.[11], the first E_8 factor is broken to the maximal subgroup $SU(3) \times E_6$. By embedding the spin connection in $E_8 \times E_8'$, $SU(3)$ becomes the holonomy group of the internal space. The four-dimensional gauge group is then composed of two factors, $E_6 \times E_8'$. Since E_6 is generally used to produce the low-energy gauge group of phenomenological models, it is be

[1] The usual normalization is $\mathrm{tr}\left[t^{MN}t^{PQ}\right] = 2\delta^{MP}\delta^{NQ}$ but is not well suited for our work since it is not compatible with the branching rules.

called the *visible* sector of the gauge group by opposition to E_8' which is called the *hidden* sector. The adjoint representation reduces to,

$$(\mathbf{248},\mathbf{1}) \oplus (\mathbf{1},\mathbf{248}) = (\mathbf{1},\mathbf{1},\mathbf{248}) \oplus (\mathbf{1},\mathbf{78},\mathbf{1}) \oplus (\mathbf{3},\mathbf{27},\mathbf{1}) \oplus (\bar{\mathbf{3}},\overline{\mathbf{27}},\mathbf{1}) \oplus (\mathbf{8},\mathbf{1},\mathbf{1}) \tag{2.44}$$

Using the convention (2.39) for the generators, the gauge fields can be written in a matrix form as 1-form over $M_4 \times K$,

$$\begin{aligned} \hat{A} &= i\hat{A}^{\alpha}T^{\alpha} \\ &= \underbrace{i\hat{A}'^{\beta}T'^{\beta}}_{\equiv \hat{A}'^{(1)}} + \underbrace{i\hat{A}^{A}T^{A}}_{\equiv \hat{A}^{(1)}} + \underbrace{i\hat{A}^{\text{I}i}\bar{T}_{\text{I}i}}_{\equiv \hat{A}^{(3)}} + \underbrace{i\bar{\hat{A}}_{\text{I}i}T^{\text{I}i}}_{\equiv \hat{A}^{(\bar{3})}} + \underbrace{i\hat{A}^{a}T^{a}}_{\equiv \hat{A}^{(8)}}. \end{aligned} \tag{2.45}$$

The lowercase greek indices $\alpha, \beta, \ldots = 1, \ldots, 248$ refer to the adjoint representation of E_8. The adjoint of E_6 will be labeled with latin uppercase letters of the begining of the alphabet $A, B, \ldots = 1, \ldots, 78$ while letter from the middle of the alphabet $I, J, \ldots = 1, \ldots, 27$ will represent matter multiplet as described below. The above expression (2.45) contains also two types of $SU(3)$ indices. They are both written with lowercase latin letters. Indices in the adjoint ($\mathbf{8}$) will use the beginning of the alphabet $a, b, \ldots = 1, \ldots, 8$ and $i, j, \ldots = 1, 2, 3$ will be reserved for the 3 or $\bar{\mathbf{3}}$ of $SU(3)$. Recall that, as described in the introduction of this section, for complex representations, upper and lower indices are conjugate to each others. The superscript on the matrix valued fields labels the $SU(3)$ representation. In the following, fields which represent massless modes in four dimensions will be written *without* a hat.

Fields which are invariant under $SU(3)$ each produce one vector zero-mode in 4 dimensions. They correspond to the $E_6 \times E_8'$ gauge fields and are written as 1-form over M_4

$$A^{(1)}(\hat{x}) = A^{(1)}_{\mu}(x)\mathrm{d}x^{\mu} \quad , \quad A'^{(1)}(\hat{x}) = A'^{(1)}_{\mu}(x)\mathrm{d}x^{\mu}. \tag{2.46}$$

The massless modes of the triplets and anti-triplets of $SU(3)$ are determined by the number of independent harmonic (2,1) and (1,1) forms over K. These numbers are called Hodge numbers and will be written h_{11} and h_{12}. We will use the following notation for the forms on K. The indices $u, v, w, ..$ go from 1 to 3, z and $\bar{z}$ are complex coordinates K. The covariantly constant $(3,0)$ form present in all Calabi-Yau manifolds is

$$\Omega_{(3,0)} = \frac{1}{3!}\Omega_{uvw}\mathrm{d}z^{u} \wedge \mathrm{d}z^{v} \wedge \mathrm{d}z^{w}. \tag{2.47}$$

The (1,1)-forms are,

$$\omega^{(j)}_{(1,1)}(z,\bar{z}) = \omega^{(j)}{}_{u}{}^{v}(z,\bar{z})\mathrm{d}z^{u} \wedge \mathrm{d}\bar{z}_{v}. \tag{2.48}$$

where $j = 1, \ldots, h_{11}$, while the (2,1) forms on K are written as

$$\omega^{(k)}_{(2,1)}(z,\bar{z}) = \frac{1}{2}\omega^{(k)}{}_{uv}{}^{w}(z,\bar{z})\mathrm{d}z^u \wedge \mathrm{d}z^v \wedge \mathrm{d}\bar{z}_w. \tag{2.49}$$

where $k = 1, \ldots, h_{21}$. For convenience, we also define $\pi^{(k)}$ forms by:

$$\pi^{(k)}{}_{iu} = \Omega_{ivw}\omega^{(k)}{}^{vw}{}_{u}.$$

We then introduce complex fields A^{Ii} and their complex conjugate $\bar{A}_{\mathrm{Ii}} = (A^{\mathrm{Ii}})^*$. The harmonic expansion is

$$\begin{aligned}
A^{(3)}(x,z) &= i\left[A^{Ii}{}_u\mathrm{d}z^u + A^{Ii\,u}\mathrm{d}\bar{z}_u\right]\bar{T}_{Ii} \\
&= \left[\sum_{j=1}^{h_{11}} C'^{(j)\,I}(x)\omega^{(j)}{}^{i}{}_{u}(z)\mathrm{d}z^u \right.\\
&\quad \left. + \sum_{j=1}^{h_{21}} C^{(j)\,I}(x)\Omega^{ikl}\omega^{(j)}{}_{kl}{}^{u}(z)\mathrm{d}\bar{z}_u\right]\bar{T}_{Ii}, \\
A^{(\bar{3})}(x,z) &= i\left[\bar{A}_{Ii\,u}\mathrm{d}z^u + \bar{A}_{Ii}{}^{u}\mathrm{d}\bar{z}_u\right]T^{Ii} \\
&= \left[\sum_{j=1}^{h_{11}} C'^{(j)}_{I}(x)\omega^{(j)}{}_{i}{}^{u}(z)\mathrm{d}\bar{z}_u \right.\\
&\quad \left. + \sum_{j=1}^{h_{21}} C^{(j)}_{I}(x)\Omega_{ikl}\omega^{(j)\,kl}{}_{u}(z)\mathrm{d}z^u\right]T^{Ii}.
\end{aligned} \tag{2.50}$$

$C^{(j)}$ and $C'^{(j)}$ are scalar fields in four dimensions which live in the **27** and $\overline{\mathbf{27}}$ of E_6. They will be referred as matter fields in the following since they will form, with their supersymmetric partner, chiral matter superfields in four dimensions.

Fields in the adjoint of $SU(3)$ do not carry any gauge charge as can be seen from (2.44). They do not have gauge interaction but can have a phenomenological importance [37, 6]. Here, they will appear as a background field,

$$A^{(8)} = A^a_\alpha(z,\bar{z})\mathrm{d}z^\alpha T^a + A^{a\,\alpha}(z,\bar{z})\mathrm{d}\bar{z}_\alpha T^a \tag{2.51}$$

2.4.3 Gravitational sector

Recall that in ten dimensions, the $N = 1$ supergravity multiplet contains the metric $\hat{g}_{\hat{\mu}\hat{\nu}}$, the dilaton $\hat{\varphi}$ and the antisymmetric tensor $\hat{B}_{\hat{\mu}\hat{\nu}}$. The zero-modes of the gravitational sector are determined by first writing the theory in the product space $M_4 \times K$. This correspond to breaking the ten-dimensional Lorentz group $SO(1,9)$ to $SO(1,3) \times SO(6)$. Secondly, the massless degrees of freedom left after Calabi-Yau compactification are determined by topological properties of K. The dilaton $\hat{\varphi}(\hat{x})$ is a scalar under $SO(1,9)$ in ten dimensions and will therefore be a scalar in the 4+6 dimensional theory. It has one zero-mode, a four-dimensional scalar field constant on K,

$$\hat{\varphi}(\hat{x}) \Rightarrow \varphi(x). \tag{2.52}$$

The anti-symmetric tensor $\hat{B}_{\hat{\mu}\hat{\nu}}$ leads to a 0-form ($\hat{B}_{\mu\nu}$), 1-form ($\hat{B}_{\mu\alpha}$) and 2-form ($\hat{B}_{\alpha\beta}$) on K. The massless modes of these fields being in correspondence with harmonic forms on K, their number is determined by the Hodge diamond of the Calabi-Yau manifold.

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & h_{11} & & 0 \\ & 1 & & h_{12} & & h_{21} & & 1 \\ & & 0 & & h_{11} & & 0 \\ & & & 1 & & & \end{array} \tag{2.53}$$

They will lead respectively to 1, 0 and h_{11} massless fields. This can be summarized by

$$B(x,z) = B_4(x) + B_6(x,z), \tag{2.54}$$

$$B_4(x) = \frac{1}{2}B_{\mu\nu}(x)\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu \qquad ; \mu,\nu = 1,..,4 \quad , \tag{2.55}$$

$$B_6(x,z) = B_a{}^b(x,z)\mathrm{d}z^a \wedge \mathrm{d}\bar{z}_b \qquad ; a,b = 1,2,3 \quad , \tag{2.56}$$

$$B_a{}^b(x,z) = \sum_{i=1}^{h_{11}} a^{(i)}(x)\omega^{(i)}{}_a{}^b(z). \tag{2.57}$$

The ten-dimensional metric $\hat{g}_{\hat{\mu}\hat{\nu}}(\hat{x})$ gives rise to three types of fields in 4+6 dimensions, $\hat{g}_{\mu\nu}(\hat{x})$, $\hat{g}_{\mu\alpha}(\hat{x})$, $\hat{g}_{\alpha\beta}(\hat{x})$. Moreover, one degree of freedom, $\hat{\sigma}$, invariant under $SO(1,3) \times SO(6)$, can be used to rescale the four-dimensional metric to normalize the lagrangian in four dimensions. The analysis of zero-modes of the metric gives the following results:

- $\hat{g}_{\mu\nu}(\hat{x})$ will produce one graviton in four dimensions.
- $\hat{g}_{\mu\alpha}$ has no zero-modes, this is due to the absence of continuous symmetries on Calabi-Yau manifolds.
- The zero-modes of $\hat{g}_{\alpha\beta}$ are obtained by determining the number of deformations of the internal metric which preserve the $SU(3)$ holonomy. They are deformations of the Kähler structure and deformations of the complex structure of K. These deformations are in one to one correspondence with $(2,1)$ forms and $(1,1)$ forms respectively.

Problems involving gravitational fields can be formulated either with the metric or with the vielbein as the fundamental field. As can be seen from equations (2.9) to (2.11), the Green-Schwarz counterterm involves the spin connection $\hat{\omega}$, a field which has a simple expression in terms of the vielbein $\hat{e}_{\hat{\mu}}^{\hat{m}}$ and we will therfore use this formalisme in the following. An Ansatz for the zehnbein compatible with Calabi-Yau compactification is given by[7],

$$\hat{e}_{\hat{\mu}}^{\hat{m}} = \begin{pmatrix} e^{-\frac{3}{2}\sigma(x)} e_{\mu}^{\ m}(x) & 0 \\ 0 & e^{+\frac{1}{2}\sigma(x)} \phi_{\alpha}^{\ a}(x,y) \end{pmatrix}. \tag{2.58}$$

$$m, \mu = 0, \ldots, 3 \quad ; \quad a, \alpha = 4, \ldots, 9$$

$\sigma(x)$ is a scalar field used to keep a canonical normalization of the vierbein $e_{\mu}^{\ m}$. $\phi_{\alpha}^{a}(x,y)$ is the internal sechsbein. The spin connection coefficients associated to (2.58) can be readily computed from the formulas exposed in section 2.3 in the particular case where,

$$\varphi^k(x) = e^{-\frac{3}{2}\sigma(x)} \quad ; \quad A_{\mu}^{\alpha}(x,y) = 0 \quad ; \quad \partial_{\alpha}\phi_{\beta}^{a} \neq 0$$

A direct computation leads to,

$$\begin{aligned}
\hat{\omega}_{mnp} &= e^{\frac{3}{2}\sigma(x)} \left[\omega_{mnp} - \tfrac{3}{2}\left(\eta_{mn}E_p^{\mu} - \eta_{mp}E_n^{\mu}\right)\partial_{\mu}\sigma(x)\right] \\
\hat{\omega}_{anp} &= 0 \\
\hat{\omega}_{mbc} &= \tfrac{1}{2} e^{\frac{3}{2}\sigma(x)} E_m^{\mu} \left[\Phi_b^{\alpha}\partial_{\mu}\phi_{\alpha c} - \Phi_c^{\alpha}\partial_{\mu}\phi_{\alpha b}\right] \\
\hat{\omega}_{abc} &= e^{-\frac{1}{2}\sigma(x)} \omega_{abc}(x,y) \\
\hat{\omega}_{mnb} &= 0 \\
\hat{\omega}_{anb} &= -\tfrac{1}{2} e^{\frac{3}{2}\sigma(x)} \partial_{\mu}\sigma(x) E_n^{\mu} \eta_{ab} \\
&\quad -\tfrac{1}{2} e^{\frac{3}{2}\sigma(x)} E_n^{\mu} \left[\Phi_a^{\alpha}\partial_{\mu}\phi_{\alpha b} + \Phi_b^{\alpha}\partial_{\mu}\phi_{\alpha a}\right]
\end{aligned} \tag{2.59}$$

The harmonic expansion of the sechsbein takes the following form (in complex notation),

$$\begin{aligned}
\hat{e}_{\alpha}^{a}(x,z) &= e^{\frac{1}{2}\sigma(x)} \left[e_{\alpha}^{0\ a}(z) + e^{0\bar{\beta}a}(z) \sum_{j=2}^{h_{11}} f^{(j)}(x) \omega_{\alpha\bar{\beta}}^{(j)}(z) \right], \\
\hat{e}_{\alpha}^{\bar{a}}(x,z) &= e^{0\beta\bar{a}}(z) \sum_{k=1}^{h_{21}} g^{(k)}(x) \pi_{\alpha\beta}^{(k)}(z), \\
\hat{e}_{\bar{\alpha}}^{\bar{a}}(x,z) &= (\hat{e}_{\alpha}^{a}(x,z))^{*} \quad , \quad \hat{e}_{\bar{\alpha}}^{a}(x,z) = (\hat{e}_{\alpha}^{\bar{a}}(x,z))^{*}.
\end{aligned} \tag{2.60}$$

$$\alpha, \beta, \bar{\alpha}, \bar{\beta} a, b, \bar{a}, \bar{b} = 1,2,3$$

where $f^{(j)}(x)$ $(g^{(k)}(x))$ are h_{11} (h_{21}) scalar fields in four dimensions which in a sense parametrize the allowed deformations.

The number of harmonic forms varies from one manifold to the other but the Kähler form is always present ($h_{11} \geq 1$). Locally J can be written as $J = i dz^a \wedge d\bar{z}_a$ leading to

$$-h_{\alpha,\beta} = e^{\sigma(x)}\delta_{\alpha\beta}.$$

In this case, the spin connection one-forms simplify considerably,

$$\begin{aligned}
\hat{\omega}^{(M)}(x) &= e^{\frac{3}{2}\sigma(x)}\omega^{(M)}(x) - \partial_\mu e^{\frac{3}{2}\sigma(x)}(\eta_{mn}E^\mu_p - \eta_{mp}E^\mu_n)dx^m T^{np}, \\
\hat{\omega}^{(K)}(x,y) &= 0, \\
\hat{\omega}^{(MK)}(x,y) &= -\tfrac{1}{3}\left(\partial_m e^{\frac{3}{2}\sigma(x)}\right)\eta_{ab}dy^a T^{mb}.
\end{aligned} \tag{2.61}$$

In the general case, a complete expansion on harmonic forms leads to complicated formulas which are not written here, the following form being sufficient,

$$\omega = \omega^{(M)} + \omega^{(K)} + \omega^{(MK)}$$

where

$$\begin{aligned}
\omega^{(M)}(x) &= \tfrac{1}{2}\omega_{mn\,p}T^{mn}dx^p \\
\omega^{(K)}(x,y) &= \omega_4^{(K)} + \omega_6^{(K)} \\
&\qquad \omega_4^{(K)} = \tfrac{1}{2}\omega_{ab\,m}T^{ab}dx^p \\
&\qquad \omega_6^{(K)} = \tfrac{1}{2}\omega_{ab\,c}T^{ab}dy^c \\
\omega^{(MK)}(x,y) &= \omega_{ma\,b}T^{ma}dy^b
\end{aligned} \tag{2.62}$$

The curvature two form is then obtained as

$$R = d\omega + \omega^2. \tag{2.63}$$

The physical dimension of the various fields in mass units are determined by their canonical kinetic terms. The dimension of the scalars C^I,C_I and of the vector fields $A^{(1)}$, A' is one. In the gravitational sector, the dimension of the spin connection counts the number of space-time derivatives,

$$\dim[\omega^{(M)}] = \dim[\omega_4^{(K)}] = \dim[\omega^{(MK)}] = 1 \quad , \quad \dim[\omega_6^{(K)}] = 0.$$

The Green-Schwarz counterterm will therefore introduce, in the effective lagrangian, corrections of order greater than four.

If supersymmetry is unbroken, all these fields have fermionic counterparts and arrange themselves into supermultiplets. In the gauge sector, $A^{(1)}$ and $A'^{(1)}$ form the vector part of an $E_6 \times E_8'$ gauge vector multiplet. $A^{(3)}$ and $A^{(3)}$ produce scalars which are part of $h_{(2,1)}$ and $h_{(1,1)}$ chiral matter supermultiplets in the **27** and $\bar{\mathbf{27}}$ of E_6. In the gravity sector, it is interesting to see how fields from different origins appear in exactly the right number to form supermultiplets. The dilaton φ combined with the unique zero-mode of the four-dimensional part of the antisymmetric tensor $B_{\mu\nu}$ produces the scalar part of a chiral multiplet or can be embedded in a linear multiplet. Other chiral superfields are produced by the Kähler deformation of the metric combined with the massless scalars from the internal part of $B_{\alpha\beta}$.

2.5 Preliminary calculations

At the beginning of this chapter, we constructed the ten-dimensional Green-Schwarz counterterm and in the following sections introduced the necessary tools to describe its compactification on a Calabi-Yau manifold K. Now the actual computation is straightforward but lengthy.

In the literature, some terms arising from ΔS have been analyzed [14, 25, 26] but a complete calculation has yet to be done. Even though in principle one can compute all of them, practically already a few of them are enough to extend our knowledge of quantum corrections in four dimensions. Moreover, many of these terms have more than two derivatives and their interpretation in four-dimensional supergravity is delicate. The idea is therefore to concentrate on the parts of ΔS in which we expect to find interesting contributions and then use our knowledge of string theory and supergravity to look for specific terms in the remaining parts. We will therefore compute in detail,

$$\Delta S_B = -C \int B \wedge I_8(\hat{F}, \hat{R}). \tag{2.64}$$

The computation is organised as follows. We will compute the reduction of $\mathrm{Tr}F^2$ and $\mathrm{tr}R^2$, two expressions appearing many times in ΔS. Next we will write the polynomials I_8, I_3, I_7 in a form convenient for the particular cases analysed in the next chapter.

2.5.1 Reduction of the gauge field strength

Let us first compute the traces over the gauge sector. The hidden sector (E_8') is in a sense trivial since, as described earlier, its zero-modes are 248 gauge fields in four dimensions. Fields in the visible sector (E_8) have $SU(3)$ quantum numbers and carry the rich structure of Calabi-Yau compactification.

Traces involving the gauge curvature are easy to compute since they require only the computation of $\mathrm{Tr}\hat{F}^2$. In the case of E_8, $\mathrm{Tr}\hat{F}^4$ is proportional to $\left(\mathrm{Tr}\hat{F}^2\right)^2$ as is shown in (2.81). The trace over the gauge curvature in the hidden sector F' is also easily computed since the zero-modes have no dependence on the internal coordinates. We simply have

$$\begin{aligned}
\mathrm{Tr}\left[F'^2\right] &= \mathrm{d}\Omega_3^{(G)}(A') \\
&= d_4\Omega^{(G)}(A') \\
&= \mathrm{Tr}\left[(\mathrm{d}_4 A')^2\right] + 2\mathrm{Tr}\left[A'^2 \mathrm{d}_4 A'\right] \\
&\equiv \mathrm{Tr}\left[F_4'^2(x)\right].
\end{aligned} \tag{2.65}$$

The exterior derivative operator $\mathrm{d}\bullet = \frac{\partial \bullet}{\partial x^\mu}\mathrm{d}\hat{x}^\mu$ will be written as $\mathrm{d} = \mathrm{d}_4 + \mathrm{d}_6$. The trace of F^2 is computed as follows,

$$\mathrm{Tr}F^2 = d\Omega_3^{(G)}(A) = \mathrm{Tr}\left[(dA)^2\right] + 2\mathrm{Tr}\left[A^2 dA\right]. \tag{2.66}$$

By embedding the $SU(3)$ holonomy group in the gauge group, E_8 is broken to $SU(3) \times E_6$. Using (2.44), the gauge fields decompose as

$$A = A^{(1)} + A^{(3)} + A^{(\bar{3})} + A^{(8)}.$$

Expressed in terms of gauge fields of the broken group, (2.66) contains a large number of terms. Group theoretical arguments allow us to determine which traces are not zero. Since we will use this argument later on, it will be briefly described here.

The A field belongs to the **248** of E_8. From the group theory point of view, the traces appear as Clebsh-Gordan coefficients of the singlet. The first term in (2.66) belongs to

$$(\mathbf{248} \times \mathbf{248})_{Symmetric} \tag{2.67}$$

of E_8. Its decomposition into irreducible representations is [36, p.113]

$$\mathbf{248} \times \mathbf{248} = \mathbf{1}_s + \mathbf{248}_a + \mathbf{3875}_s + \mathbf{27000}_s + \mathbf{30380}_a. \tag{2.68}$$

The subscripts s and a indicate whether the representation belongs to the symmetric (s) or antisymmetric (a) part of the tensor product. The presence of $\mathbf{1}_s$ in this decomposition allows the presence of $\mathrm{Tr}\,(\mathrm{d}A^2)$. The absence of a singlet in the antisymmetric part of the decomposition forbids terms like $\mathrm{Tr}A^2$ since A is a one-form. To express the trace in terms of the $A^{(i)}$, one has to introduce the branching rule

$$\mathbf{248} = (\mathbf{1}, \mathbf{78}) + (\mathbf{3}, \mathbf{27}) + (\bar{\mathbf{3}}, \overline{\mathbf{27}}) + (\mathbf{8}, \mathbf{1})$$

into (2.67) and into

$$(\mathbf{248} \times \mathbf{248})_{Anti-symmetric} \otimes \mathbf{248}$$

for the second term in (2.66) before extracting all the terms which contain a singlet. (2.66) then becomes,

$$\begin{aligned}
\mathrm{Tr}F^2 \;=\; & \mathrm{Tr}\left[\left(dA^{(1)}\right)^2\right] + 2\mathrm{Tr}\left[\left(A^{(1)}\right)^2 dA^{(1)}\right] \\
+ \; & \mathrm{Tr}\left[\left(dA^{(8)}\right)^2\right] + 2\mathrm{Tr}\left[\left(A^{(8)}\right)^2 dA^{(8)}\right] \\
+2 \; & \mathrm{Tr}\left[dA^{(3)} dA^{(\bar{3})}\right] + 2\mathrm{Tr}\left[\left(A^{(3)}\right)^2 dA^{(3)} + \left(A^{(\bar{3})}\right)^2 dA^{(\bar{3})}\right] \\
+2 \; & d\mathrm{Tr}\left[\left(A^{(1)} + A^{(8)}\right)\left\{A^{(3)}, A^{(\bar{3})}\right\}\right]
\end{aligned} \tag{2.69}$$

One can rewrite the first two lines as traces over curvatures. They are respectively defined as the E_6 field strength and background gauge curvature. These 4-forms are explicitely written as,

$$\mathrm{Tr}\left[\left(dA^{(1)}\right)^2\right] + 2\mathrm{Tr}\left[\left(A^{(1)}\right)^2 dA^{(1)}\right] \;=\; d_4\Omega_3^{(G)}(A^{(1)}) \;\equiv\; \mathrm{Tr}\,[F_4^2], \tag{2.70}$$

and

$$\mathrm{Tr}\left[\left(dA^{(8)}\right)^2\right] + 2\mathrm{Tr}\left[\left(A^{(8)}\right)^2 dA^{(8)}\right] \;=\; d_6\Omega_3^{(G)}(A^{(8)}) \;\equiv\; \mathrm{Tr}\left[F_6^2\right]. \qquad (2.71)$$

The remaining traces of (2.69) can be computed using (2.40). By introducing the two operators D_4 and D_6 which act on the the gauge field 1-form as,

$$\begin{aligned}
\mathrm{D}_4 A^{Ii} &= \mathrm{d}_4 A^{Ii} - i\left(C^A\right)^I{}_J A^A \wedge A^{Ji} \\
\mathrm{D}_4 \bar{A}_{Ii} &= \mathrm{d}_4 \bar{A}_{Ii} - i\left(C^A\right)_I{}^J A^A \wedge \bar{A}_{Ji} \\
\mathrm{D}_6 A^{Ii} &= \mathrm{d}_6 A^{Ii} - i\left(\tfrac{\lambda^a}{2}\right)^i{}_j A^a \wedge A^{Ij} \\
\mathrm{D}_6 \bar{A}_{Ii} &= \mathrm{d}_6 \bar{A}_{Ii} - i\left(\tfrac{\lambda^a}{2}\right)_i{}^j A^a \wedge \bar{A}_{Ij}
\end{aligned} \qquad (2.72)$$

and the superpotential 3-forms,

$$\begin{aligned}
\mathrm{W}(A) &= \epsilon_{ijk}\bar{\lambda}_{IJK} A^{Ii} \wedge A^{Jj} \wedge A^{Kk}, \\
\bar{\mathrm{W}}\left(\bar{A}\right) &= \epsilon^{ijk}\lambda^{IJK} \bar{A}_{Ii} \wedge \bar{A}_{Jj} \wedge \bar{A}_{Kk},
\end{aligned} \qquad (2.73)$$

(2.69) can be written as,

$$\mathrm{Tr}F^2 \doteq \mathrm{d}\Omega(A), \qquad (2.74)$$

where

$$\begin{aligned}
\Omega &= \Omega_3(A^{(1)}) + \Omega_3(A^{(8)}) \\
&- (30)\left(A^{Ii} \wedge \mathrm{D}_4\bar{A}_{Ii} + \bar{A}_{Ii} \wedge \mathrm{D}_4 A^{Ii}\right) \\
&- (30)\left(A^{Ii} \wedge \mathrm{D}_6\bar{A}_{Ii} + \bar{A}_{Ii} \wedge \mathrm{D}_6 A^{Ii}\right) \\
&+ \frac{(30)}{3}\left(\mathrm{W}(A^{Ii}) + \bar{\mathrm{W}}(\bar{A}_{Ii})\right)
\end{aligned}$$

In order to obtain an expression of $\mathrm{Tr}F^2$ in term of four-dimensional fields, one has to introduce the harmonic expansion of the gauge fields into (2.74). For clarity, I split the resulting expression in four parts according to the number of space-time and internal indices.

$$\Omega = \Omega|_{(3,0)} + \Omega|_{(2,1)} + \Omega|_{(1,2)} + \Omega|_{(0,3)},$$

where $\Omega|_{(p,q)}$ refers to a p-form on M_4 and a q-form on K. Its full expansion is a complicated expression in terms of the scalar fields C and forms on the Calabi-Yau manifold which will be written, when needed, for the special case we will study.

2.5.2 Reduction of the gravitational sector

We will first start with the computation of the trace over R^2, which will be computed using the same tools as before for the trace in the gauge sector. This is possible since we are working in a formalism which mimics the gauge formalism. The *gauge* fields are the spin connection $\hat{\omega}$ and the local group of symmetry is the Lorentz group in ten dimensions $SO(1,9)$.

The calculation will be done in two steps. First we rewrite $\text{tr}R^2$ in $4+6$ dimensions. Next we introduce the spin connection Ansatz corresponding to Calabi-Yau compactification.

Using the Lorentz Chern-Simons three-form, the trace can be expressed as

$$\text{tr}\hat{R}^2 = \mathrm{d}\Omega^{(L)}(\hat{\omega}) = \text{tr}\left[(\mathrm{d}\hat{\omega})^2\right] + 2\,\text{tr}\left[\hat{\omega}^2\mathrm{d}\hat{\omega}\right]. \tag{2.75}$$

where the trace is taken in the adjoint representation of $SO(1,9)$ (**45**). In going to a $4+6$ dimensional theory, $SO(1,9)$ breaks down to $SO(1,3)\times SO(6)$ and $SO(6)$ down to $SU(3)$, the holonomy group. The branching rule for the **45** is,

$$\mathbf{45} = (\mathbf{6},\mathbf{1})\oplus(\mathbf{1},\mathbf{15})\oplus(\mathbf{4},\mathbf{6}). \tag{2.76}$$

In this basis, the spin connections can be written as

$$\hat{\omega} = \hat{\omega}^{(M)} + \hat{\omega}^{(K)} + \hat{\omega}^{(MK)}, \tag{2.77}$$

where

$$\begin{array}{rclrcl} \hat{\omega}^{(M)}(\hat{x}) & = & \frac{1}{2}\hat{\omega}_{np,\hat{m}}(\hat{x})t^{np}\mathrm{d}\hat{x}^{\hat{m}} & \hat{m} & = & 1,\ldots,10 \\ \hat{\omega}^{(K)}(\hat{x}) & = & \frac{1}{2}\hat{\omega}_{ab,\hat{m}}(\hat{x})t^{ab}\mathrm{d}\hat{x}^{\hat{m}} & n,p & = & 1,\ldots,4 \\ \hat{\omega}^{(MK)}(\hat{x}) & = & \hat{\omega}_{an,\hat{m}}(\hat{x})t^{an}\mathrm{d}\hat{x}^{\hat{m}} & a,b & = & 1,\ldots,6 \end{array}$$

The normalization of the generators is given in (2.43). The determination of the non-zero Clebsh-Gordan coefficients of

$$(\mathbf{45}\otimes\mathbf{45})_{Symmetric} \longrightarrow \mathbf{1}$$

for $\text{tr}[\mathrm{d}\hat{\omega}^2]$ and

$$(\mathbf{45}\otimes\mathbf{45})_{Anti-symmetric}\otimes\mathbf{45} \longrightarrow \mathbf{1}$$

for $\text{tr}[\hat{\omega}^2\mathrm{d}\omega]$ allows us to eliminate the vanishing traces. (2.75) then becomes,

$$\begin{aligned} \text{tr}\,[\mathbf{R}^2] &= \mathrm{d}\hat{\Omega}^L_3(\hat{\omega}) \\ &= \text{tr}\left[(\mathrm{d}\hat{\omega})^2\right] + 2\,\text{tr}\left[\hat{\omega}^2\mathrm{d}\hat{\omega}\right] \\ &= \mathrm{d}\Omega_3^{(L)}(\hat{\omega}^{(M)}) + \mathrm{d}\Omega_3^{(L)}(\hat{\omega}^{(K)}) \\ &\quad + \text{tr}\left[(\mathrm{d}\hat{\omega}^{(MK)})^2\right] + \text{tr}\left[(\hat{\omega}^{(MK)})^2\left(\mathrm{d}\hat{\omega}^{(K)} + \mathrm{d}\hat{\omega}^{(M)}\right)\right] \\ &\quad + \text{tr}\left[\left\{\hat{\omega}^{(M)} + \hat{\omega}^{(K)}\right\}\mathrm{d}\hat{\omega}^{(MK)}\right] \end{aligned} \tag{2.78}$$

with the Chern-Simons form Ω given by (2.11). $\mathrm{tr}R^2$ after the compactification on a Calabi-Yau manifold is given by the contribution of the zero-modes of the spin connection to (2.78). Inserting (2.62) into (2.78) gives a general expression which we order according to the number of space-time index of its terms.

$$
\begin{aligned}
\mathrm{tr}\,[\mathbf{R}^2]|_{(4,0)} &= \mathrm{d}_4\Omega_3^L(\omega_4^{(M)}) + \mathrm{d}_4\Omega_3^L(\omega_4^{(K)}) \\
\mathrm{tr}\,[\mathbf{R}^2]|_{(3,1)} &= 2\,\mathrm{tr}\left[\left(\mathrm{d}_4\omega_4^{(K)} + \omega_4^{(K)2}\right)\left(\mathrm{d}_4\omega_6^{(K)} + \mathrm{d}_6\omega_4^{(K)}\right)\right] \\
&\quad +2\,\mathrm{tr}\left[\left\{\omega_4^{(K)}, \omega_6^{(K)}\right\}\mathrm{d}_4\omega_4^{(K)}\right] \\
\mathrm{tr}\,[\mathbf{R}^2]|_{(2,2)} &= \mathrm{tr}\left[\left(\mathrm{d}_4\omega_6^{(K)} + \mathrm{d}_6\omega_4^{(K)}\right)^2 + 2\mathrm{d}_4\omega_4^{(K)}\mathrm{d}_6\omega_6^{(K)}\right] \\
&\quad +2\,\mathrm{tr}\left[\left(\omega_4^{(K)}\right)^2\mathrm{d}_6\omega_6^{(K)} + \left(\omega_6^{(K)}\right)^2\mathrm{d}_4\omega_4^{(K)}\right] \\
&\quad +2\,\mathrm{tr}\left[\left\{\omega_4^{(K)}, \omega_6^{(K)}\right\}\left(\mathrm{d}_6\omega_4^{(K)} + \mathrm{d}_4\omega_6^{(K)}\right)\right] \\
&\quad +\mathrm{tr}\left[\left(\mathrm{d}_4\omega^{(MK)}\right)^2\right] \\
&\quad +2\,\mathrm{tr}\left[\left(\omega^{(MK)}\right)^2(\mathrm{d}_4\omega_4^{(K)} + \mathrm{d}_4\omega^{(M)})\right] \\
&\quad +2\,\mathrm{tr}\left[\left\{\omega^{(M)} + \omega_4^{(K)}, \omega^{(MK)}\right\}\mathrm{d}_4\omega^{(MK)}\right] \qquad (2.79) \\
\mathrm{tr}\,[\mathbf{R}^2]|_{(1,3)} &= 2\,\mathrm{tr}\left[\left(\mathrm{d}_6\omega_6^{(K)} + \omega_6^{(K)2}\right)\left(\mathrm{d}_4\omega_6^{(K)} + \mathrm{d}_6\omega_4^{(K)}\right)\right] \\
&\quad +2\,\mathrm{tr}\left[\left\{\omega_4^{(K)}, \omega_6^{(K)}\right\}\mathrm{d}_6\omega_6^{(K)}\right] \\
&\quad +2\,\mathrm{tr}\left[\mathrm{d}_4\omega^{(MK)}\mathrm{d}_6\omega^{(MK)} + \omega^{(MK)2}\left(\mathrm{d}_6\omega_4^{(K)} + \mathrm{d}_4\omega_6^{(K)}\right)\right] \\
&\quad +2\,\mathrm{tr}\left[\left\{\omega^{(M)} + \omega_4^{(K)}, \omega^{(MK)}\right\}\mathrm{d}_6\omega^{(MK)}\right] \\
&\quad +2\,\mathrm{tr}\left[\left\{\omega_6^{(K)}, \omega^{(MK)}\right\}\mathrm{d}_4\omega^{(MK)}\right] \\
\mathrm{tr}\,[\mathbf{R}^2]|_{(0,4)} &= \mathrm{d}_6\Omega_3^L(\omega_6^{(K)}) \\
&\quad \mathrm{tr}\left[+\left(\mathrm{d}_6\omega^{(MK)}\right)^2\right] \\
&\quad +2\,\mathrm{tr}\left[\omega^{(MK)2}\mathrm{d}_6\omega_6^{(K)}\right] \\
&\quad +2\,\mathrm{tr}\left[\left\{\omega_6^{(K)}, \omega^{(MK)}\right\}\mathrm{d}_6\omega^{(MK)}\right]
\end{aligned}
$$

The previous expression is particulary long and leads to a large number of terms when inserted in the expressions of the Green-Schwarz counterterm. Fortunately, it simplifies enormously in special cases. For example, the spin connection used for the simple truncation (2.61) reduces (2.79) to

$$\begin{aligned}
\mathrm{tr}\,[\mathbf{R}^2]|_{(4,0)} &= \mathrm{d}_4\Omega_3^L(\omega_4^{(M)}) \\
\mathrm{tr}\,[\mathbf{R}^2]|_{(2,2)} &= \mathrm{tr}\left[\left(\mathrm{d}_4\omega^{(MK)}\right)^2\right] + 2\,\mathrm{tr}\left[\left(\omega^{(MK)}\right)^2 \mathrm{d}_4\omega^{(M)}\right] \\
&\quad +2\,\mathrm{tr}\left[\left\{\omega^{(M)},\omega^{(MK)}\right\}\mathrm{d}_4\omega^{(MK)}\right] \\
\mathrm{tr}\,[\mathbf{R}^2]|_{(3,1)} &= \mathrm{tr}\,[\mathbf{R}^2]|_{(1,3)} = \mathrm{tr}\,[\mathbf{R}^2]|_{(0,4)} = 0
\end{aligned} \tag{2.80}$$

The full computation of ΔS requires an expression for $\mathrm{tr}R^4$. By looking at the expression of $\mathrm{tr}R^2$ in terms of ω's, it is clear that the full expression for $\mathrm{tr}R^4$ contains higher derivatives and will therefore not enter our analysis. Moreover the first application of this computation will be the *one generation model* in which the gravitational sector is described by a metric independent of the internal coordinates (see below). This state of affairs forces us to leave this term out of the analysis since it only contains terms of dimension eight; all non-zero derivatives being in M_4. In this case, the analysis of $B \wedge I_8$ can be done using the above traces only.

2.6 Reduction of ΔS_B

A complete calculation of ΔS would be interesting but is not a necessity now. As it will be described in the next chapter, the antisymmetric tensor in four dimensions, $B_{\mu\nu}(x)$, plays an important role in the study of quantum corrections to effective field theories of superstrings. It will be therfore much more profitable to study in details the reduction of the Green-Schwarz counterterm which involve the $\hat{B}$ field than to try to write down a complete reduction of ΔS. Before looking at the reduction of ΔS_B itself, let us write the explicit expression of ΔS in function of the ten-dimensional fields. All the tools have been presented in section (2.2) and will be applied here to the case of the heterotic string compactified on a Calabi-Yau manifold.

We start by writting down the polynomials needed to compute the counterterm in our case. We have the eight-form $I_8(\hat{F},\hat{R})$ (see 2.8), the seven-form $I_7(\hat{A},\hat{\omega})$ and the three-form $I_3(\hat{A},\hat{\omega})$ (see2.10) polynomials. In our case,the eight-form I_8(2.8) is,

$$\begin{aligned} I_8(\hat{F},\hat{R}) &= \frac{1}{24}\mathrm{Tr}_{E_8\times E_8'}\hat{F}^4 - \frac{1}{7200}(\mathrm{Tr}_{E_8\times E_8'}\hat{F}^2)^2 \\ &+ \frac{1}{8}\,\mathrm{tr}\hat{R}^4 + \frac{1}{32}(\,\mathrm{tr}\hat{R}^2)^2 \\ &- \frac{1}{240}\mathrm{Tr}_{E_8\times E_8'}\hat{F}^2\,\mathrm{tr}\hat{R}^2, \end{aligned}$$

where the traces are taken over the adjoin of $E_8 \times E_8$ for the gauge curvature and over the fundamental of $SO(1,9)$ for the gravitational curvature. Introducing,

$$\begin{aligned} \mathrm{Tr}_{E_8\times E_8'}\hat{F}^2 &= \mathrm{Tr}_{E_8}\hat{F}^2 + \mathrm{Tr}_{E_8'}\hat{F}'^2 \\ \mathrm{Tr}_{E_8\times E_8'}\hat{F}^4 &= \mathrm{Tr}_{E_8}\hat{F}^4 + \mathrm{Tr}_{E_8'}\hat{F}'^4 \end{aligned}$$

in the above formula allows us to drop the subscript E_8 and E_8' on the traces since it is clear from the context on which E_8 sector the trace applies. I_8 is only function of $\mathrm{Tr}\,[F^2]$ and $\mathrm{Tr}\,[F'^2]$ since for E_8 groups, traces in the adjoint representation verify the property,

$$\mathrm{Tr}\left[F^4\right] = \frac{1}{100}\left(\mathrm{Tr}\left[F^2\right]\right)^2. \tag{2.81}$$

With this simplification, we can now write,

$$
\begin{aligned}
I_8 &= I_8|_{gauge} + I_8|_{gravit} + I_8|_{mixed} \\
I_8|_{gauge} &= \tfrac{1}{3600}\left[\left(\mathrm{Tr}\hat{F}^2\right)^2 + \left(\mathrm{Tr}\hat{F}'^2\right)^2 - \left(\mathrm{Tr}\hat{F}^2\right)\left(\mathrm{Tr}\hat{F}'^2\right)\right] \\
I_8|_{gravit.} &= \tfrac{1}{8}\,\mathrm{tr}\hat{R}^4 + \tfrac{1}{32}(\,\mathrm{tr}\hat{R}^2)^2 \\
I_8|_{mixed} &= -\tfrac{1}{240}\left(\mathrm{Tr}\hat{F}^2 + \mathrm{Tr}\hat{F}'^2\right)\,\mathrm{tr}\hat{R}^2 \\
I_3 &= I_3|_{gauge} + I_3|_{gravit} + I_3|_{mixed} \\
I_3|_{gauge} &= -\tfrac{1}{30}\Omega_3^{(G)}(\hat{A}), \\
I_3|_{gravit.} &= \Omega_3^{(L)}(\hat{\omega}), \\
I_3|_{mixed} &= 0 \\
I_7 &= I_7|_{gauge} + I_7|_{gravit} + I_7|_{mixed} \\
I_7(\hat{A},\hat{\omega})\Big|_{gauge} &= \tfrac{1}{24}\Omega_7^{(L)}(\hat{A}) - \tfrac{1}{7200}\mathrm{Tr}\hat{F}^2\Omega_3^G(\hat{A}) \\
I_7(A,\omega)|_{gravit.} &= \tfrac{1}{8}\Omega_{(7)}^{(L)}(\hat{\omega}) + \tfrac{1}{32}\,\mathrm{tr}\hat{R}^2\Omega_3^L(\hat{\omega}) \\
I_7(A,\omega)|_{mixed} &= -\tfrac{1}{480}\left(\Omega_3^{(G)}(\hat{A})\,\mathrm{tr}\hat{R}^2 + \Omega_3^{(L)}(\hat{\omega})\mathrm{Tr}\hat{F}^2\right)
\end{aligned}
\tag{2.82}
$$

For reasons of clarity, we present ΔS as a sum of a pure gauge ΔS_1, pure gravitational ΔS_2 and mixed part,

$$\Delta S = \Delta S_1 + \Delta S_2 + \Delta S_3$$

where,

$$\begin{aligned}
\Delta S_1 &= \Delta S_{1\,B} + \Delta S_{1\,\alpha} \\
&= -\frac{C}{3600}\int_{M_4\times K} B\wedge\left(\left(\mathrm{Tr}F^2\right)^2+\left(\mathrm{Tr}F'^2\right)^2-\left(\mathrm{Tr}F^2\right)\wedge\left(\mathrm{Tr}F'^2\right)\right) \\
&- C\left(\frac{2}{3}+\alpha\right)\int_{M_4\times K}\frac{1}{24}\Omega_7^{(L)}(\hat{A})-\frac{1}{7200}\mathrm{Tr}\hat{F}^2\Omega_3^G(\hat{A})\wedge\frac{1}{30}\Omega_3^{(G)}(\hat{A})
\end{aligned}$$

$$\begin{aligned}
\Delta S_2 &= \Delta S_{2\,B} + \Delta S_{2\,\alpha} \\
&= -\frac{C}{8}\int_{M_4\times K} B\wedge\left(\left(\mathrm{tr}R^4\right)+\frac{1}{4}\left(\mathrm{tr}R^2\right)^2\right) \\
&+ \frac{C}{8}\left(\frac{2}{3}+\alpha\right)\int_{M_4\times K}\Omega_7^{(L)}(\hat{\omega})\wedge\Omega_3^{(L)}(\hat{\omega})
\end{aligned}$$

$$\begin{aligned}
\Delta S_3 &= \Delta S_{3\,B} + \Delta S_{3\,\alpha} \\
&= \frac{C}{240}\int_{M_4\times K} B\wedge\mathrm{Tr}F^2\wedge\mathrm{tr}R^2 \\
&+ \frac{C}{24}\left(\frac{2}{3}+\alpha\right)\int_{M_4\times K}\Omega_7^{(G)}(\hat{A})\wedge\Omega_3^{(L)}(\hat{\omega})-\frac{1}{10}\Omega_7^{(L)}(\hat{\omega})\wedge\Omega_3^{(G)}(\hat{A}) \\
&+ \frac{C}{240}\left(\frac{2}{3}+\alpha\right)\int_{M_4\times K}\left[\frac{1}{30}\mathrm{Tr}\hat{F}^2-\frac{1}{4}\mathrm{tr}\hat{R}^2\right]\wedge\Omega_3^{(G)}(\hat{A})\wedge\Omega_3^{(L)}(\hat{\omega})
\end{aligned} \tag{2.83}$$

The reduction of $\Delta S_{1\,B}$ is obtained by doing an expansion of the ten-dimensional fields over the harmonic forms of the internal manifold keeping only the massless modes in four dimensions. The reduction of the antisymmetric tensor $\hat{B}$ is given by (2.54-2.57) while the one of $\mathrm{Tr}F^2$ has been explicitely done in (2.74). Let us split the integral in three parts,

$$\Delta S_{1\,B} = \mathcal{I}_1 + \mathcal{I}_2 + \mathcal{I}_3.$$

The first part is

$$\mathcal{I}_1 = -\frac{C}{3600}\int_{M_4\times K} B\wedge\left(\mathrm{Tr}F^2\right)^2 = +\frac{C}{3600}\int \mathrm{d}B\wedge\left(\Omega\wedge\mathrm{d}\Omega\right), \tag{2.84}$$

where Ω is given by (2.74) in terms of harmonic $(1,1)$-forms and $(1,2)$-forms on K. The second part vanishes identically since it contains an 8-form on M_4,

$$\mathcal{I}_2 = -\frac{C}{3600}\int B\wedge\left(\mathrm{Tr}F'^{\,2}\right)^2 = 0. \tag{2.85}$$

For the same reason, the third part also simplifies ,

$$\begin{aligned}
\mathcal{I}_3 &= -\tfrac{C}{3600}\int_{M_4\times K} B\wedge \mathrm{Tr}F'^{\,2}\wedge \mathrm{Tr}F^2 \\
&= -\tfrac{C}{3600}\int_{M_4\times K}\left(\mathrm{d}_4B_6\wedge\Omega_3^{(G)}(A')\wedge \mathrm{d}_6\boldsymbol{\Omega}+\mathrm{d}_6B_6\wedge\Omega_3^{(G)}(A')\wedge \mathrm{d}_4\boldsymbol{\Omega}.\right).
\end{aligned} \tag{2.86}$$

Putting everything together leads to

$$\Delta S_{1\ B} = -\frac{C}{3600}\int_{M_4\times K}\mathrm{d}B\wedge\boldsymbol{\Omega}\wedge\mathrm{d}\boldsymbol{\Omega}+\mathrm{d}_4B_6\wedge\Omega_3^{(G)}(A')\wedge\mathrm{d}_6\boldsymbol{\Omega}+\mathrm{d}_6B_6\wedge\Omega_3^{(G)}(A')\wedge\mathrm{d}_4\boldsymbol{\Omega}. \tag{2.87}$$

The ΔS_2 in (2.83) describes the gravitational contribution to ΔS. Its interpretation is more delicate since it contains many higher derivative terms. One sees that, after integration by parts, ΔS_B is function of $\mathrm{d}B$ only. A non-derivative coupling of the B field would have been impossible to interpret in the linear multiplet formalism.

It is straightforward to derive the factorized expressions from (2.83). The complete results are long expressions which include operators of dimensions 6 or higher. If for example one concentrates only on the lowest dimensional operators, we can write,

$$\begin{aligned}
\Delta S_{1\ B}\big|_{dim.6} = \frac{C}{30} & \Bigg\{\sum_{j,k=1}^{h_{11}} -\mathcal{K}_1^{(jk)}\int_{M_4}\mathrm{d}_4B_4\wedge \mathcal{C}'^{(k)}{}_I\mathcal{D}_4\mathcal{C}'^{(j)\,I} \\
& +\sum_{j,k=1}^{h_{21}}\mathcal{K}_2^{(jk)}\int_{M_4}\mathrm{d}_4B_4\wedge\mathcal{C}^{(j)I}\mathcal{D}_4\mathcal{C}_I^{(k)} \\
& -\sum_{j=1}^{h_{11}}\sum_{k=1}^{h_{21}}\mathcal{K}_3^{(jk)}\int_{M_4}\mathrm{d}_4B_4\wedge\mathcal{C}^{(k)}{}_I\mathcal{D}_4\mathcal{C}'^{(j)\,I} \\
& -\sum_{j=1}^{h_{11}}\sum_{k=1}^{h_{21}}\left(\mathcal{K}_3^{(jk)}\right)^*\int_{M_4}\mathrm{d}_4B_4\wedge\mathcal{C}^{(k)I}\mathcal{D}_4\mathcal{C}'^{(j)}{}_I\Bigg\} \\
-\frac{C}{1800} & \sum_{j=1}^{h_{11}}\mathcal{K}_{(1,1)}^{(j)}\int_{M_4}a^{(j)}(x)\left(\mathrm{Tr}[F_4^2]-\frac{1}{2}\mathrm{Tr}[F'^2]\right)
\end{aligned} \tag{2.88}$$

where,

$$
\begin{aligned}
\mathcal{K} &= \int_K \mathrm{d}z^a \wedge \mathrm{d}\bar{z}_a \wedge \mathrm{Tr}F_6^{\;2} \\
\mathcal{K}^{(j)}_{(1,1)} &= \int_K \omega^{(j)}_{(1,1)} \wedge \mathrm{Tr}F_6^{\;2} \\
\mathcal{K}^{(jk)}_1 &= \int_K \eta^{(jk)}_1 \wedge \mathrm{Tr}F_6^{\;2} \\
\mathcal{K}^{(jk)}_2 &= \int_K \eta^{(jk)}_2 \wedge \mathrm{Tr}F_6^{\;2} \\
\mathcal{K}^{(jk)}_3 &= \int_K \eta^{(jk)}_3 \wedge \mathrm{Tr}F_6^{\;2}
\end{aligned}
\tag{2.89}
$$

The 2-forms $\eta^{(jk)}$ are the following combinations of the harmonic forms on K,

$$
\begin{aligned}
\eta^{(jk)}_1 &= \omega^{(j)}{}^{i}{}_{u}\,\omega^{(k)}{}_{i}{}^{v}\mathrm{d}z^u \wedge \mathrm{d}\bar{z}_v \\
\eta^{(jk)}_2 &= \pi^{(j)}{}^{iu}\pi^{(k)}{}_{iv}\mathrm{d}\bar{z}_u \wedge \mathrm{d}z^v \\
\eta^{(jk)}_3 &= \omega^{(j)}{}^{i}{}_{u}\,\pi^{(k)}{}_{iv}\mathrm{d}z^u \wedge \mathrm{d}z^v .
\end{aligned}
\tag{2.90}
$$

They depend only on the internal coordinates. If the expression (2.88) is in principle also computable, the general result would contain a large number of terms whose interpretation in terms of four-dimensional supergravity has to be done.

In order to compare with previous work, we will use in the following the Ansatz of simple truncation for the zehnbein. In this case, ΔS_2 and ΔS_3 simplify greatly. With the simple truncation, ΔS_2 has only contributions of dimension greater than 6.

$$\Delta S_{2\,B}|_{dim.6} = 0. \tag{2.91}$$

The counterterms for the mixed anomaly are given by

$$
\begin{aligned}
\Delta S_{3\,B}|_{dim.6} &= \frac{C}{240}\mathcal{K}^{(i)}\int_{M_4} a^{(i)}(x) \quad \mathrm{tr}[R_4^2] \\
&+ \frac{C}{240}\mathcal{K}\int_{M_4} B_4(x)\wedge \quad \Big(\mathrm{tr}\left[\left(\mathrm{d}_4\omega^{(MK)}\right)^2\right] \\
&\qquad +2\,\mathrm{tr}\left[\left(\omega^{(MK)}\right)^2 \mathrm{d}_4\omega^{(M)}\right] \\
&\qquad +2\,\mathrm{tr}\left[\left\{\omega^{(M)},\omega^{(MK)}\right\}\mathrm{d}_4\omega^{(MK)}\right]\Big)
\end{aligned}
\tag{2.92}
$$

with the components of $\omega^{(MK)}$ given by (2.61).

The correction introduced in a four-dimensional effective lagrangian by the compactification of the Green-Schwarz counterterm on a Calabi-Yau manifold were already studied in the second half of the eighties by various authors.

Derendinger et al. [14], Ibanèz and Nilles [25] and Itoyama and Léon [26] studied the modification to the coupling of the axion. As is shown below, our calculation includes their results if one keeps only the dimension six corrections and assume their Ansatz for the metric.

The correction to the axion coupling has its origin in

$$\Delta S_{B_6} = -C \int_M B_6 \wedge I_8. \tag{2.93}$$

Using (2.88,2.91,2.92) and keeping only the lowest dimensional operators, one obtains,

$$\begin{aligned}
\Delta S_{B_6}|_{dim.6} &= -\frac{C}{80}\sum_{i=1}^{h_{11}} \\
&\left\{ \int_K \omega^{(i)}{}^b{}_a \mathrm{d}z^a \wedge \mathrm{d}\bar{z}_b \wedge \left(\frac{4}{3\cdot 30}\mathrm{Tr}F_6^2 - \frac{1}{3}\,\mathrm{tr}R_6^2\right) \int_{M_4} a^{(i)}(x)\mathrm{Tr}F_4^2(x) \right.\\
&+ \int_K \omega^{(i)}{}^b{}_a \mathrm{d}z^a \wedge \mathrm{d}\bar{z}_b \wedge \left(\frac{-2}{3\cdot 30}\mathrm{Tr}F_6^2 - \frac{1}{3}\,\mathrm{tr}R_6^2\right) \int_{M_4} a^{(i)}(x)\mathrm{Tr}F'^2(x) \\
&\left. + \int_K \omega^{(i)}{}^b{}_a \mathrm{d}z^a \wedge \mathrm{d}\bar{z}_b \wedge \left(-\frac{1}{3}\mathrm{Tr}F_6^2 + 5\,\mathrm{tr}R_6^2\right) \int_{M_4} a^{(i)}(x)\,\mathrm{tr}R_4^2(x) \right\}
\end{aligned}$$

Giving a vacuum expectation value to the background fields and imposing the Calabi-Yau condition

$$\mathrm{Tr}F_6^2(z,\bar{z}) = 30\,\mathrm{tr}R_6^2(z,\bar{z}) \tag{2.94}$$

one gets

$$\begin{aligned}
\Delta S_{B_6}|_{dim.6} &= \frac{C}{80}\sum_{i=1}^{h_{11}} \int_K \omega^{(i)} \wedge \mathrm{tr}R_6^2 \\
&\int_{M_4} a^{(i)}(x)\left\{-\mathrm{Tr}F_4^2(x) + \mathrm{Tr}F'^2(x) + 5\,\mathrm{tr}R_4^2(x)\right\}.
\end{aligned} \tag{2.95}$$

I recall that (2.95) does not includes corrections of dimensions higher than 6. It is interesting to note that four-dimensional matter does not appear in these expressions.

Chapter 3

The Kähler mode.

[redacted]

This chapter presents the analysis of the reduction of the Green-Schwarz counterterm in the presence of a single h_{11} modulus. It contains an explicit derivation of the four-dimensional effective theory at tree-level. The explicit form of the reduced terms of ΔS_B are given. These will allow us to compute in the large radius limit the Kähler potential including the matter fields. It will be seen that these contributions take the form

$$c\left(T+\bar{T}-2\Sigma\bar{\Sigma}\right)$$

where c is a constant determined by the normalization of the ten-dimensional counterterm and by an integral over the internal space.

This chapter can be read independently of the first two. It is self contained and only refers to chapter two for explicit results on the reduction of ΔS.

3.1 Introduction.

The analysis we propose here is an attempt to analyse the role of the ten-dimensional Green-Schwarz counterterm ΔS in Calabi-Yau compactification. The moduli space of a Calabi-Yau manifold K has a product structure, $\mathcal{M}_1 \times \mathcal{M}_2$. These two spaces control the number h_{11} of harmonic $(1,1)$ forms and the number h_{21} of harmonic $(2,1)$ forms on K. The first one are associated with deformation of the Kähler structure and the second one with deformation of the complex structure of the manifold. An important distinction one can make between these two numbers is

$$h_{11} \geq 1 \quad ; \quad h_{21} \geq 0$$

This reflects at the level of Hodge numbers the fact that every Calabi-Yau manifold is a Kähler manifold and has a natural $(1,1)$ form, the Kähler form. Since the harmonic forms are assoicated with the zero-modes, this means that the particular mode associated with the Kähler form will be present in all Calabi-Yau compactification. This mode, sometimes called the "breathing mode", is related to the radius of compactification. It is then natural to study the model where

$$h_{11} = 1 \quad ; \quad h_{21} = 0$$

This model is called, *the one generation model* since in the case of Calabi-Yau compactification, the number of matter generation at the weak scale is related to the Euler constant χ by,

$$N_{gen.} = \frac{1}{2}\left|\chi\right| = \left|h_{11} - h_{21}\right|.$$

The first point to stress is that this analysis is done in the limit of large compactification radius. Ultimately, field theories describing physics at the electroweak scale, will have to be derived from a fundamental microscopic theory like string theory. The natural scale of a string theory, M_{String}, is related to the Planck scale M_P and contact with phenomenological physics is now done through its low-energy limit. In taking this limit, two different solutions are at hand. On one hand, if the compactification scale M_c is smaller than the string scale, one will first have to deal with an effective ten-dimensional supergravity theory and then describe compactification process in terms of field theory. On the other hand, if the compactification scale is higher than the string scale the compactification will take place at the string level.

In our analysis, the first case is assumed since we start with the Green-Schwarz counterterm which is defined in the ten-dimensional supergravity. This puts stringent limitations on the validity of our results since our analysis will be sensitive only to the large T limit effects. This handicap will be overshot by the fact that it brings simplicity and allows to write down explicit expressions which should give us some insight to where to look for quantum corrections at the string level.

The effective action will be described using the linear multiplet formulation and computation will be done using the superconformal approach of supergravity. The results will be checked against the known results on quantum corrections in effective four-dimensional effective theories from strings. In a first step, I will compute in components the four-dimensional lagrangian with corrections arising from a Green-Schwarz counterterm of the type described by Derendinger, Ferrara, Kounnas and Zwirner. We will concentrate on the terms containing up to two space-time derivatives, the higher derivatives contributions being less understood. This correction includes a linear multiplet L, the supersymmetric non-abelian Chern-Simons form and a Kähler function K. In our analysis we will need to compute the component expression of the effective lagrangian at tree-level $\mathcal{L}_0$ with its 1-loop corrections $\mathcal{L}_{GS}$. In computing the component expressions, one has to be careful to correctly fix the compensator and integrate out the auxiliary fields.

3.2 Spectrum of Calabi-Yau compactification.

The spectrum of massless fields of a Calabi-Yau compactification with the spin connection embedded in the gauge group is determined by its topological structure as is fully explained in [19]. In chapter two, the general case was considered and allows us to directly write the spectrum of our one generation model. The single h_{11} form has to be proportional to the Kähler form J and is chosen to be,

$$\omega = -iJ = \delta^b_a \mathrm{d}z^a \wedge \mathrm{d}\bar{z}_b. \tag{3.1}$$

In this section, we will always work in the complex notation for the internal space. Upper and lower indices are conjugate to each other by convention. In the gauge sector, the zero modes are gauge fields of the surviving $E_6 \times E_8'$ four-dimensional gauge group, these are not affected by the specific values of the Hodge numbers, scalars belonging to the **27** of E_6. In addition to these, there are background fields which are singlets of E_6. They are explicitly given, in a matrix notation, by (2.50),

$$A' = iA^{\alpha}_{\mu}(x)T^{\alpha}\mathrm{d}x^{\mu} \quad , \quad A^{(1)} = iA^{A}_{\mu}(x)T^{A}\mathrm{d}x^{\mu} \quad , \quad A^{(8)} = iA^{a}_{i}\mathrm{d}z^{i} + iA^{a\,i}\mathrm{d}\bar{z}_{i}$$
$$A^{(3)} = iC'^{I}(x)\mathrm{d}z^{i}\bar{T}_{Ii},$$
$$A^{(\bar{3})} = iC'_{I}(x)\mathrm{d}\bar{z}_{i}T^{Ii}. \tag{3.2}$$

The gauge indices are

$$\alpha = 1,\ldots,128\,,\; A = 1,\ldots,27\,,\; I = 1,\ldots,27\,,\; a = 1,\ldots,8 \quad ,$$

and the space time indices $\mu = 0,\ldots,4$ for M_4 and $i = 1,2,3$ for K. Note that only the uncharged background fields in the adjoint of $SU(3)$, $A^{(8)}$, still have a dependence on the internal coordinates.

The gravitational multiplet contains the metric, the antisymmetric tensor field and the dilaton. The metric is

$$\hat{g}_{\hat{\mu}\hat{\nu}} = \begin{pmatrix} \mathrm{e}^{-3\sigma(x)}g_{\mu\nu} & 0 \\ 0 & -\mathrm{e}^{\sigma(x)}\delta_{\alpha\beta} \end{pmatrix}. \tag{3.3}$$

It corresponds to the metric used in [14]. In complex coordinate, the internal metric becomes,

$$h_{a}{}^{b} = -h^{a}{}_{b} = \mathrm{e}^{\sigma(x)}\delta^{b}_{a} \quad , \quad h_{ab} = h^{ab} = 0. \tag{3.4}$$

The ten-dimensional dilaton $\hat{\varphi}$ has only one zero mode, a four-dimensional scalar field, $\varphi(x)$. The antisymmetric tensor $\hat{B}_{\hat{\mu}\hat{\nu}}$ produces a four-dimensional antisymmetric tensor $B_{\mu\nu}(x)$ and its internal part $\hat{B}_{\alpha\beta}$ has only one zero mode left, a pseudoscalar field $a(x)\epsilon_{\alpha\beta}$[1], or in terms of differential forms, $B_6 = ia(x)\mathrm{d}z^{j}\wedge\mathrm{d}\bar{z}_{j}$. This completes the bosonic sector of this model.

3.3 Construction of $\mathcal{L}_{4D}$.

The ten-dimensional effective lagrangian we are considering is the tree-level lagrangian of Chapline and Manton [13], augmented with the Green-Schwarz counterterm (2.9),

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{GS}. \tag{3.5}$$

[1] $\epsilon_{\alpha\beta}$ is an antisymmetric tensor invariant under $SU(3)$, $\epsilon_{45} = \epsilon_{67} = \epsilon_{89} = +1 = -\epsilon_{54} = -\epsilon_{76} = -\epsilon_{98}$, all other components are zero.

$\mathcal{L}_0$ is the field theory limit, or "zero slope limit", of the heterotic superstring in ten dimensions. The bosonic part of $\mathcal{L}_0$ is,

$$\hat{e}^{-1}\mathcal{L}_{10D,N=1} = -\frac{1}{2\hat{\kappa}^2}\hat{R}+\frac{3}{4}\hat{\varphi}^{-\frac{3}{2}}\hat{H}_{\hat{\mu}\hat{\nu}\hat{\rho}}\hat{H}^{\hat{\mu}\hat{\nu}\hat{\rho}}+\frac{9}{16\hat{\kappa}^2}\hat{\varphi}^{-2}\partial_{\hat{\mu}}\hat{\varphi}\partial^{\hat{\mu}}\hat{\varphi}-\frac{1}{4}\varphi^{-3/4}\mathrm{Tr}\left[\hat{F}_{\hat{\mu}\hat{\nu}}\hat{F}^{\hat{\mu}\hat{\nu}}\right], \tag{3.6}$$

where the ten-dimensional metric has the signature $(+,-,-,...)$ and all hatted quantities refer to ten-dimensional fields or indices. The trace of the field strengths $\hat{F}_{\hat{\mu}\hat{\nu}}$ is taken over the adjoint representation of the gauge group $E_8\times E_8'$. The antisymmetric tensor's field strength, $\hat{H}_{\hat{\mu}\hat{\nu}\hat{\rho}}$, is a dimension 5 operator. In addition to $\partial\hat{B}_{\hat{\mu}\hat{\nu}}$, it contains the gauge and Lorentz Chern-Simons terms which have a stringy origin [18],

$$\hat{H}_{\hat{\mu}\hat{\nu}\hat{\rho}} = \frac{1}{3}\left(\partial_{\hat{\mu}}\hat{B}_{\hat{\nu}\hat{\rho}} - \frac{1}{\sqrt{2}}\hat{\Omega}^{(G)}_{\hat{\mu}\hat{\nu}\hat{\rho}} + \frac{1}{\sqrt{2}}\hat{\Omega}^{(L)}_{\hat{\mu}\hat{\nu}\hat{\rho}} + \text{cyclic permutations}\right),$$

with

$$\begin{aligned}\hat{\Omega}^{(G)}_{\hat{\mu}\hat{\nu}\hat{\rho}} &= \hat{\kappa}\quad \mathrm{Tr}\left[\hat{A}_{\hat{\mu}}\hat{F}_{\hat{\nu}\hat{\rho}} - \tfrac{2}{3}\hat{g}\hat{A}_{\hat{\mu}}\hat{A}_{\hat{\nu}}\hat{A}_{\hat{\rho}}\right],\\ \hat{\Omega}^{(L)}_{\hat{\mu}\hat{\nu}\hat{\rho}} &= \frac{1}{\sqrt{\hat{\kappa}}}\quad \mathrm{tr}\left[\hat{\omega}_{\hat{\mu}}\hat{R}_{\hat{\nu}\hat{\rho}} - \tfrac{2}{3}\hat{\omega}_{\hat{\mu}}\hat{\omega}_{\hat{\nu}}\hat{\omega}_{\hat{\rho}}\right].\end{aligned} \tag{3.7}$$

Two dimensionful constants have been introduced, the ten-dimensional gravitational constant $\hat{\kappa}$ with a dimension $(\text{mass})^{-4}$ and a ten-dimensional gauge coupling constant with dimension $(\text{mass})^{-3}$.

The simplest way to reduce a higher dimensional field theory to four dimensions, is *dimensional reduction*. This means that all the fields are required to be invariant under translations in the extra dimensions. If one denotes by $\Phi(x^\mu, y^\alpha)$ a generic field, where y^α are the extra coordinates, this means that,

$$\partial_{y^\alpha}\Phi(x^\mu, y^\alpha) = 0.$$

Such reduction of (3.6) would produce a four-dimensional supergravity with an $N=4$ supersymmetry [12]. This is incompatible with a Calabi-Yau compactification which has explicitly being designed to produce an $N=1$ space-time supersymmetry. As realized by Witten [38], a more elaborate reduction can lead to an $N=1$ theory in four dimensions. One should not merely require the invariance of all fields under translation in the internal space but also the invariance under an $SU(3)$ subgroup of the rotation of the 6 internal coordinates. The four supersymmetries belong to the fundamental of $SU(4)$ which reduces under $SU(3)$ as $\mathbf{4}=\mathbf{1}+\mathbf{3}$ and leaves only one supersymmetry unbroken.

This truncation corresponds to the model we are analysing. It selects the zero-modes which are invariant under $SU(3)$. This forbids contributions from the harmonic (2,1)-forms since they cannot lead to $SU(3)$ singlets. The only

(1,1)-form leading to an invariant tensor is the complex structure introduced in (3.1). We are therefore left with a spectrum corresponding to $h_{11} = 1$, $h_{12} = 0$.

The surviving zero modes of this truncation are then directly obtained by a simple group theoretical analysis.

$$
\begin{aligned}
\hat{g}_{\hat{\mu}\hat{\nu}} &\in [\mathbf{10}\times\mathbf{10}]_{\mathrm{S}} = \underbrace{(\mathbf{4},\mathbf{1})^2_{\mathrm{S}}}_{\substack{\text{4Dmetric}\\ g_{\mu\nu}}} \oplus \underbrace{(\mathbf{1},\mathbf{6})^2_{\mathrm{S}}}_{\substack{\text{singlet}\\ e^{\sigma(x)}}} \oplus (\mathbf{4},\mathbf{6}) \\
\hat{B}_{\hat{\mu}\hat{\nu}} &\in [\mathbf{10}\times\mathbf{10}]_{\mathrm{A}} = \underbrace{(\mathbf{4},\mathbf{1})^2_{\mathrm{A}}}_{\substack{\text{4Danti-sym.}\\ B_{\mu\nu}}} \oplus \underbrace{(\mathbf{1},\mathbf{6})^2_{\mathrm{A}}}_{\substack{\text{pseudoscalar}\\ a(x)}} \oplus (\mathbf{4},\mathbf{6})
\end{aligned}
\qquad (3.8)
$$

The gauge fields have 2 types of indices, a space time index in the **10** of $SO(1,9)$ and a gauge index in $E_8 \otimes E_8'$. The simplest way to implement Witten's truncation is to include the $SU(3)_G$ subgroup in one of the E_8 factors. The gauge group in the reduced theory is then $E_6 \times E_8'$. This truncation will lead to gauge fields of E_8' in the hidden sector and to scalars and E_6 gauge fields in the visible sector.

$$
\begin{array}{ccccccccc}
\mathbf{248} & = & (\mathbf{78},\mathbf{1}) & \oplus & (\mathbf{27},\mathbf{3}) & \oplus & (\overline{\mathbf{27}},\bar{\mathbf{3}}) & \oplus & (\mathbf{1},\mathbf{8}) \\
 & & | & & | & & | & & \\
\mathbf{10} & = & (\mathbf{4},\mathbf{1}) & \oplus & (\mathbf{1},\bar{\mathbf{3}}) & \oplus & (\mathbf{1},\mathbf{3}) & & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 & & E_6 \text{ gauge fields} & & 27 \text{ 4D scalars} & & 27 \text{ 4D scalars} & & \\
 & & A^A_\mu & & A_a{}^{Ii} & \overset{\longleftrightarrow}{conjugate} & \bar{A}^a{}_{Ii} & &
\end{array}
\qquad (3.9)
$$

Notice that this truncation does not allow for contribution of uncharged scalars. They are in the adjoint of $SU(3)$ (**8**) and are generically present in any Calabi-Yau compactification. The reduction of (3.6) is then straightforward with the convention for the traces given in chapter 2. It leads to the following four-dimensional lagrangian,

$$\begin{aligned} e^{-1}\mathcal{L}_{4D,N=1} &= -\frac{1}{2\kappa^2}R_4 + \frac{3}{\kappa^2}\partial_\mu\sigma\partial^\mu\sigma + \frac{9}{16\kappa^2}\varphi^{-2}(x)\partial_\mu\varphi(x)\partial^\mu\varphi(x) \\ & -\frac{\kappa^2}{4}e^{6\sigma}\varphi^{-\frac{3}{2}}V_\mu V^\mu \\ & +\frac{3\kappa^2}{2}e^{-2\sigma}\varphi^{-\frac{3}{2}}\left(\partial_\mu a(x) - \frac{i}{\sqrt{2}}(30)\left(C'^*\overleftrightarrow{\mathcal{D}}_\mu C'\right)\right)^2 \\ & -\frac{1}{4}e^{3\sigma(x)}\varphi^{-\frac{3}{4}}\mathrm{Tr}\left[F'^2 + F^2\right] + 3e^{-\sigma}\varphi^{-\frac{3}{4}}(x)\,(30)\,\mathcal{D}_\mu C'^I\mathcal{D}^\mu C'_I \\ & -\frac{9}{2}e^{-5\sigma}\varphi^{-\frac{3}{4}}(30)\,(C',(C^a)\,C')^2 \\ & +2\kappa^2\varphi^{-\frac{3}{2}}\left|W\right|^2 - \frac{2}{3}e^{-5\sigma}\varphi^{-\frac{3}{4}}(30)\left|\frac{\partial W}{\partial C}\right|^2 \end{aligned} \tag{3.10}$$

where

$$\begin{aligned} \mathcal{D}_\mu C'^I &= \partial_\mu C'^I - i\left(C^A\right)^I{}_J A^A_\mu C'^J, \\ \mathcal{D}_\mu C'_I &= \partial_\mu C'_I - i\left(C^A\right)_I{}^J A^A_\mu C'_J, \\ C'^*\overleftrightarrow{\mathcal{D}}_\mu C' &= \sum_{I=1}^{27} C'_I\mathcal{D}_\mu C'^I - C'^I\mathcal{D}_\mu C'_I, \\ \left(C',\left(C^A\right)C'\right) &= \sum_{I,J=1}^{27} C'_I\left(C^A\right)^I{}_J C'^J, \end{aligned}$$

and V_μ is related to the antisymmetric tensor field strength,

$$\begin{aligned} V_\mu &= v_\mu - \tfrac{1}{2}e^{-1}\epsilon_{\mu\nu\rho\sigma}\Omega^{(G)\,\nu\rho\sigma} \\ &= \tfrac{1}{\sqrt{2}}e^{-1}\epsilon_{\mu\nu\rho\sigma}\left(\partial^\nu B^{\rho\sigma} - \tfrac{1}{\sqrt{2}}\Omega^{(G)\nu\rho\sigma}\right). \end{aligned}$$

Its square verifies,

$$V^\mu V_\mu = -3H^{\mu\nu\rho}H_{\mu\nu\rho}.$$

The superpotential has been defined as,

$$W = \bar{\lambda}_{IJK}C'^I C'^J C'^K. \tag{3.11}$$

The four-dimensional gravitational constant κ is related to the ten-dimensional one by,

$$\kappa = \frac{1}{\sqrt{\mathcal{V}}}\hat{\kappa}$$

where $\mathcal{V}$ is the volume of the internal space. g is the dimensionless gauge coupling constant, related to $\hat{g}$ by ,

$$g = \frac{1}{\sqrt{\mathcal{V}}}\hat{g}.$$

Notice that the fields have been rescaled in order to have the canonical dimensions in 4d. The lagrangian (3.10) can be put in a standard $N = 1$ supergravity form with a suitable redefinition of the fields. This rewriting will be done using the linear multiplet as is shown below.

In [38] Witten wrote (3.10) with the dilaton field embedded in a chiral multiplet S. This can also be done in the linear multiplet formalism introduced in chapter 1. In the linear multiplet formalism, the Green-Schwarz counterterms can be introduced as a superconformal D density. Explicitly, one can write the 1-loop corrected lagrangian density as,

$$\mathcal{L} = \left[S_0\bar{S}_0\Phi\right]_D + \left[S_0^3 W\right]_F + \text{h.c.} \tag{3.12}$$

$$\Phi\left(\frac{\hat{L}}{S_0\bar{S}_0}, \Sigma, \bar{\Sigma}\mathrm{e}^V\right) = \Phi_0 + \Phi_{GS}$$

The two pieces of Φ are, Φ_0 the string tree-level contribution,

$$\Phi_0 = -\frac{1}{\sqrt{2}}\left(\frac{S_0\bar{S}_0}{\hat{L}}\right)^{1/2}\mathrm{e}^{-K/2}, \tag{3.13}$$

and Φ_{GS}, the Green-Schwarz counterterm [15, 9, 10], for the one modulus case.

$$\Phi_{\mathrm{GS}} = \frac{A}{12}\frac{\hat{L}}{S_0\bar{S}_0}K \tag{3.14}$$

where A is a constant. K is the Kähler potential. In our analysis, we expect to see only the large radius limit of the Green-Schwarz corrections and therefore we have to use the appropriate limit of K in (3.14). At tree-level, it can be determined from the kinetic terms of the above lagrangian.

The components expression of the bosonic part of (3.12) have been computed in [16] for an arbitrary function Φ[2].

$$\mathcal{L} = \mathcal{L}_E + \mathcal{L}_{Kin} + \mathcal{L}_{Aux}.$$

$\mathcal{L}_E$ is the Einstein term, $\mathcal{L}_{Kin}$ contains the kinetic terms of the scalars and antisymmetric tensor while $\mathcal{L}_{Aux.}$ is the lagrangian density for the auxiliary fields. In order to fix the compensator we use the Einstein term $\mathcal{L}_E$,

$$\mathrm{e}^{-1}\mathcal{L}_E = -\frac{1}{2}R\left[-\frac{2}{3}(z_0\bar{z}_0)\left(\Phi - C\frac{\partial\Phi}{\partial C}\right)\right]. \tag{3.15}$$

[2] In [16], they also compute the terms bilinear in gauginos since they are specifically interested in the problem of gauginos condensation.

The compensator is determined, in the so called Einstein frame, by imposing that $\mathcal{L}_E$ gives the canonical Einstein lagrangian,

$$z_0\bar{z}_0\left[\Phi - C\frac{\partial\Phi}{\partial C}\right] = -\frac{3}{2}\frac{1}{\kappa^2}, \tag{3.16}$$

where C is the dilaton, the lowest component of the linear multiplet L. The fixing is not affected by the presence of the Green-Schwarz term since

$$\Phi_{\text{GS}} - C\frac{\partial\Phi_{\text{GS}}}{\partial C} = 0.$$

$\mathcal{L}_{Kin.}$ acquires two new terms with respect to the tree-level case,

$$e^{-1}\mathcal{L}_{Kin.} = e^{-1}\mathcal{L}_{0\,Kin.} - \frac{A}{6}C\partial_\mu z^i\partial^\mu\bar{z}^i - i\frac{A}{12}V_\mu\left(K_{z^i}\partial^\mu z_i - K_{\bar{z}_i}\partial^\mu\bar{z}_i\right).$$

The z^i represent all the scalar degrees of freedom in the theory. In our analysis, they will be described by the general breathing mode modulus t and the scalars z^I in the **27** of E_6. Since the counterterms do not modify the equations of motions of the auxiliary field A_μ of (3.12), the corrections to the kinetic terms arising after the integration of the auxiliary fields are the same as in the one at tree-level. Finally, the kinetic terms can be written as,

$$\begin{aligned} e^{-1}\mathcal{L}_{\text{Scal.}} &= -\left(\frac{1}{\kappa^2}+\frac{AC}{6}\right)K_{z^I\bar{z}_J}\partial_\mu z^I\partial^\mu\bar{z}_J - \frac{1}{4\kappa^2C^2}\partial_\mu C\partial^\mu C + \frac{1}{4\kappa^2C^2}V_\mu V^\mu \\ &\quad -\left(\frac{1}{\kappa^2}+\frac{AC}{6}\right)K_{t\bar{t}}\partial_\mu t\partial^\mu\bar{t} \\ &\quad -i\frac{A}{12}V_\mu\left(K_{z^I}\partial^\mu z^I - K_{\bar{z}_I}\partial^\mu\bar{z}_I\right) \\ &\quad -i\frac{A}{12}V_\mu\left(K_t\partial^\mu t - K_{\bar{t}}\partial^\mu\bar{t}\right). \end{aligned} \tag{3.17}$$

Setting $A = 0$ gives the tree-level lagrangian in components found in (1.10) and allows us to determine the field redefinition needed to write (3.10) in the standard supergravity form. The Kähler potential at string tree-level including the t moduli and the matter fields is [38, 14],

$$K_{\text{Tree}} = -3\log\left(t+\bar{t}-2\bar{z}z\right) \tag{3.18}$$

with

$$\bar{z}z = \sum_{I=1}^{27}\bar{z}_I z^I.$$

The field redefinition is easily deduced by comparing the kinetic terms of the scalars C'^I and axion a fields,

$$
\begin{aligned}
C(x) &= \tfrac{1}{\kappa^2} e^{-3\sigma(x)} \varphi^{\frac{3}{4}}(x), \\
t(x) &= e^{\sigma(x)} \varphi^{\frac{3}{4}}(x) - i\sqrt{2}\kappa^2 a(x) + (30)\kappa^2 C'_I(x) C'^I(x), \\
z^I &= (30)^{\frac{1}{2}} \kappa C'^I(x).
\end{aligned}
\tag{3.19}
$$

One then check that by substituting (3.19) into (1.10) that we get back (3.10). Note that the vector field V_μ is not redefined.

3.4 The Green-Schwarz counterterm.

The computation of ΔS is straightforward. The counterterm is naturally split in two. One part, ΔS_B contains the counterterm with the antisymmetric tensor field while the second part, ΔS_I is determined by the gauge fields and spin connections only. Moreover, each of these parts can be split in three, gauge, gravitational and mixed parts. For these, the notation of Green and Schwarz [18] will be used. ΔS_B is given by (2.83),

$$\Delta S_B = \Delta S_{1\,B} + \Delta S_{2\,B} + \Delta S_{3\,B}$$

$$
\begin{aligned}
\Delta S_{1\,B} &= -\frac{C}{3600} \int B \wedge \left(\mathrm{Tr}\left[F'^2\right]^2 + \mathrm{Tr}\left[F^2\right]^2 - \mathrm{Tr}\left[F^2\right] \mathrm{Tr}\left[F'^2\right] \right) \\
\Delta S_{2\,B} &= -\frac{C}{8} \int B \wedge \left(\mathrm{tr}\left[R^4\right] + \frac{1}{4} \mathrm{tr}\left[R^2\right]^2 \right) \\
\Delta S_{3\,B} &= \frac{C}{240} \int B \wedge \mathrm{Tr}\left[F^2\right] \mathrm{tr}\left[R^2\right]
\end{aligned}
\tag{3.20}
$$

One can discard from the above all the terms with more than two space-time derivatives since we will analyse them in a framework which describes only terms up to two derivatives. This has drastic consequences on ΔS since all the terms involving R^2 have to be left out. For the terms coupling to ΔS_B,

$$\Delta S_{2\,B} = \Delta S_{3\,B} = 0.$$

Let's do explicitly the derivation of $\Delta S_{1\,B}$.

One starts with,

$$
\begin{aligned}
\Delta S_{1\,B} &= -\tfrac{C}{3600} \int B \wedge \left[(\mathrm{Tr} F^2)^2 + (\mathrm{Tr} F'^2)^2 - \mathrm{Tr} F'^2 \wedge \mathrm{Tr} F^2 \right] \\
&= \mathcal{I}_1 + \mathcal{I}_2 + \mathcal{I}_3
\end{aligned}
$$

The first term $\mathcal{I}_1$ can be written as,

$$I_1 = -\frac{C}{3600} \int \mathrm{d}B \wedge \mathcal{F} \wedge \mathrm{d}\mathcal{F} \tag{3.21}$$

where $\mathcal{F}$ is a the 3-form defined in (2.74). In our case, it reduces to,

$$\begin{aligned}\mathcal{F}_{\rm 1gen.} &= \Omega_3(A^{(1)})(x) + \Omega_3(A^{(8)})(z,\bar{z}) \\ &-(30){\rm d}z^i \wedge {\rm d}\bar{z}_i \wedge C'^I \overleftrightarrow{\mathcal{D}}_\mu C'_I {\rm d}x^\mu \\ &-(30)2\left(\tfrac{\lambda^a}{2}\right)^i_j A^a \wedge {\rm d}z^i \wedge {\rm d}\bar{z}_j \, |C'|^2 \\ &+\tfrac{(30)}{3}\left(\epsilon_{ijk}\bar{\lambda}_{IJK} C'^I C'^J C'^K {\rm d}z^i \wedge {\rm d}z^j \wedge {\rm d}z^k + {\rm c.c.}\right)\end{aligned} \tag{3.22}$$

The exterior derivative with respect to the internal space simplifies to,

$$\mathrm{d}_6\mathcal{F} = \mathrm{Tr}F_6^2 + 2(30)i\left(\frac{\lambda^a}{2}\right)^i_j |C'|^2\, \mathrm{d}_6 A^a \wedge \mathrm{d}\bar{z}_i \wedge \mathrm{d}z^j. \tag{3.23}$$

The second term $\mathcal{I}_2$ in the integrand vanishes since $(\mathrm{Tr}F'^2)^2$ is an 8-form with indices on M_4. The third term is given by ,

$$I_3 = -\frac{C}{3600}\int \mathrm{d}_4 B_6 \wedge \Omega_3(A') \wedge \mathrm{d}_6\mathcal{F} + \mathrm{d}_6 B_6 \wedge \Omega_3(A') \wedge \mathrm{d}_4\mathcal{F} \tag{3.24}$$

Finally we have,

$$\begin{aligned}\Delta S_{1B} &= \tfrac{iC_1}{120} \int_{M_4} \mathrm{d}_4 B_4 \wedge C'^I \overleftrightarrow{\underset{\mu}{\mathcal{D}}} C'_I \mathrm{d}x^\mu \\ &+ \tfrac{C_1}{3600} \int_{M_4} \mathrm{d}_4 a \wedge \left(\Omega_3^{(G)}(A^{(1)}) + \Omega_3^{(G)}(A')\right) \\ &- \tfrac{C_2}{120} \int_{M_4} \mathrm{d}_4\left(B_4 \wedge \mathrm{d}_4 |C'|^2\right) \\ &- \tfrac{C_3}{4} \int_{M_4} \mathrm{d}_4 B_4 \wedge |C'|^2 \mathrm{d}_4 |C'|^2 \\ &+ \tfrac{C_4}{4} \int_{M_4} \mathrm{d}_4 a \wedge \left(C'^I \overleftrightarrow{\underset{\mu}{\mathcal{D}}} C'_I \mathrm{d}x^\mu\right) \wedge \mathrm{d}_4\left(C'^I \overleftrightarrow{\underset{\nu}{\mathcal{D}}} C'_I \mathrm{d}x^\nu\right) \\ &+ \tfrac{iC_5}{4} \int_{M_4} \mathrm{d}_4 B_4 \wedge \left(C'^I \overleftrightarrow{\underset{\mu}{\mathcal{D}}} C'_I \mathrm{d}x^\mu\right) |C'|^2 \\ &+ \tfrac{C_5}{120} \int_{M_4} \mathrm{d}_4 a \wedge |C'|^2 \left(\Omega_3^{(G)}(A^{(1)}) + \Omega_3^{(G)}(A')\right)\end{aligned} \tag{3.25}$$

where the real constants C_i $i = 1, \ldots, 5$ are defined below in terms of integrals over the internal space.

$$
\begin{aligned}
C_1 &= -C\mathcal{K} \\
C_2 &= -C\int_K 2i\left(\frac{\lambda^a}{2}\right)^i_{\ j} \mathrm{d}\bar{z}_i \wedge \mathrm{d}z^j \wedge \Omega_3^{(G)} \wedge A^a \left(A^{(8)}\right) \\
C_3 &= -C\int_K 4\left(\frac{\lambda^a}{2}\right)^i_{\ j}\left(\frac{\lambda^b}{2}\right)^k_{\ l} \mathrm{d}\bar{z}_i \wedge \mathrm{d}\bar{z}_k \wedge \mathrm{d}z^j \wedge \mathrm{d}z^l \wedge A^a \wedge A^b \\
C_4 &= -C\int_K i\mathrm{d}\bar{z}_i \wedge \mathrm{d}\bar{z}_j \wedge \mathrm{d}\bar{z}_k \wedge \mathrm{d}z^i \wedge \mathrm{d}z^j \wedge \mathrm{d}z^k \\
C_5 &= -C\int_K 2\left(\frac{\lambda^a}{2}\right)^i_{\ j} \mathrm{d}\bar{z}_i \wedge \mathrm{d}\bar{z}_k \wedge \mathrm{d}z^j \wedge \mathrm{d}z^k \wedge \mathrm{d}_6 A^a
\end{aligned}
\tag{3.26}
$$

In order to directly compare with the component expressions in the linear multiplet, it will be usefull to rewrite (3.25) in terms of C, t and z^I.

$$
\begin{aligned}
\Delta S_{1B} &= \tfrac{-iC_1}{3600\sqrt{2}}\tfrac{1}{\kappa^2} \int_{M_4} \mathrm{d}V_4 \ \ v^\mu z^I \overleftrightarrow{\mathcal{D}}_\mu \bar{z}_I \\
&+ \tfrac{C_1}{3600}\tfrac{-1}{3!30\sqrt{2}\kappa^2} \int_{M_4} \mathrm{d}V_4 \ \ \epsilon^{\mu\nu\rho\sigma}\partial_\mu Imt\left(\Omega_{\nu\rho\sigma}(A^{(1)}) + \Omega_{\nu\rho\sigma}(A')\right) \\
&+ \tfrac{C_2}{120}\tfrac{1}{30\sqrt{2}\kappa^2} \int_{M_4} \mathrm{d}V_4 \ \ v_\mu \partial^\mu \left(\bar{z}_I z^I\right) \\
&+ \tfrac{C_3}{4}\tfrac{1}{30^2\sqrt{2}\kappa^4} \int_{M_4} \mathrm{d}V_4 \ \ v_\mu \partial^\mu \left(\bar{z}_I z^I\right)\left(\bar{z}_J z^J\right) \\
&+ \tfrac{iC_5}{4}\tfrac{1}{30^2\sqrt{2}\kappa^4} \int_{M_4} \mathrm{d}V_4 \ \ v_\mu \left(z^I \overleftrightarrow{\mathcal{D}}_\mu \bar{z}_I\right)\left(z^J \bar{z}_J\right) \\
&- \tfrac{C_5}{120}\tfrac{1}{3!30\sqrt{2}\kappa^4} \int_{M_4} \mathrm{d}V_4 \ \ \epsilon^{\mu\nu\rho\sigma}\partial_\mu Imt \ \ z^I \bar{z}_I \left(\Omega^{(G)}_{\nu\rho\sigma}(A^{(1)}) + \Omega^{(G)}_{\nu\rho\sigma}(A')\right) \\
&- \tfrac{C_4}{4}\tfrac{1}{30^2\sqrt{2}\kappa^6} \int_{M_4} \mathrm{d}V_4 \ \ \epsilon^{\mu\nu\rho\sigma}\partial_\mu Imt \left(z^I \overleftrightarrow{\mathcal{D}}_\nu \bar{z}_I\right)\partial_\rho\left(z^J \overleftrightarrow{\mathcal{D}}_\nu \bar{z}_J\right)
\end{aligned}
\tag{3.27}
$$

The expressions have been ordered in power of $\frac{1}{\kappa^2}$.

3.5 The Kähler potential.

The Kähler potential appears in the four-dimensional Green-Schwarz counterterm through the combination (3.14),

$$\frac{A}{12}\left[\hat{L}K\right]_D$$

its component expression can easily be read from (3.17). The terms involving zero modes of the antisymmetric tensor are,

$$\begin{aligned} e^{-1}\mathcal{L}_{\mathrm{GS(B)}} &= -\frac{AC}{6}K_{t\bar{t}}\,\partial_\mu Im[t]\partial^\mu Im[t] \\ &\quad -i\frac{A}{12}v_\mu\left(K_{z^I}\partial^\mu z^I - K_{\bar{z}_I}\partial^\mu \bar{z}_I\right) \\ &\quad -i\frac{A}{12}v_\mu\left(K_t\partial^\mu t - K_{\bar{t}}\partial^\mu \bar{t}\right). \end{aligned} \tag{3.28}$$

where $A = \dfrac{3C(E_8)}{8\pi^2}$ (see [15]). The problem now is to determine K. As described in the introductory parts of this thesis, the reduction of ΔS should determine the four-dimensional Green-Schwarz counterterm in its large t limit.

The first line involves the real dilaton field C coupling to the zero-mode coming from the internal part of the antisymmetric tensor through a coupling ,

$$\frac{Ae}{6}K_{t\bar{t}}C\left(\partial_\mu\,\mathrm{Im}\,t\right)\left(\partial^\mu\,\mathrm{Im}\,\bar{t}\right).$$

Since this coupling is absent from ΔS, we must have,

$$K_{t\bar{t}} = 0. \tag{3.29}$$

The second and the third lines can be directly compared with (3.25). Keeping only the terms with lowest order in $\frac{1}{\kappa^2}$ one finds that K is,

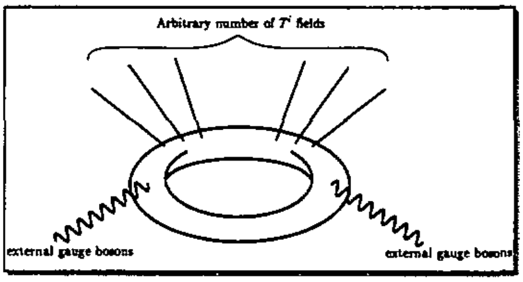
$$K(t,\bar{t},z^I,\bar{z}_I) = \frac{C_1}{600\sqrt{2}A}\frac{1}{\kappa^2}\left[t+\bar{t}-2z^I\bar{z}_I\right] \tag{3.30}$$

It is interesting to note that both counterterms come exactly with the same form as the one would expect from the naïve Witten's truncation. This result is valid for the lowest order in $\frac{1}{\kappa^2}$.

Chapter 4

Conclusions

As a conclusion we will show the link between the work done in this thesis and the string computation of thresholds corrections. In 1991, Dixon, Kaplunovsky and Louis computed the moduli dependent threshold corrections to gauge couplings [17]. Their computation was based on the one-loop string diagram with two external gauge bosons and an arbitrary number of moduli T^i.



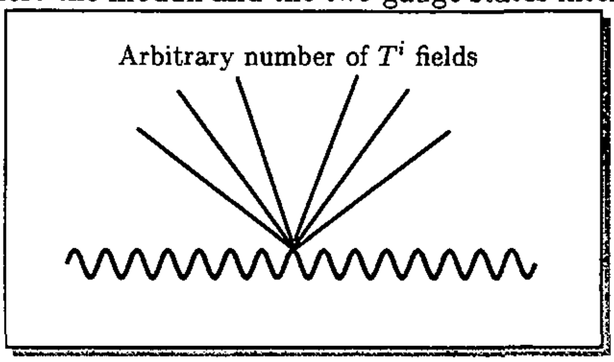
If we consider only the overall modulus T, these threshold corrections Δ, for a unique gauge group factor, are

$$b \log \left[\left(T + \bar{T} \right) |\eta(T)|^4 \right] F_{\mu\nu} F^{\mu\nu} \tag{4.1}$$

where b is a constant and $\eta(T)$ is the Dedekind function,

$$\eta(T) = \mathrm{e}^{i\pi T/12} \prod_{n=1}^{\infty} \left(1 - \mathrm{e}^{2\pi i n T} \right) .$$

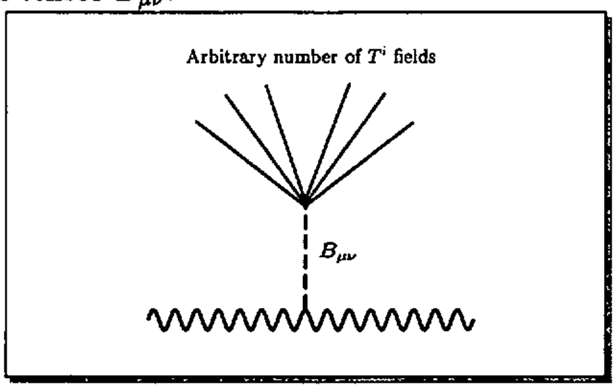
There are two important points to notice. First, these corrections are modular invariant as they should and, second, they contain a *non-harmonic* part. The presence of the $\log \left(T + \bar{T} \right)$ requires a careful analysis of the various contributions giving rise to (4.1) in the four-dimensional effective action. The modular invariant result (4.1) is generated by three modular *covariant* contributions. First there is a contact term where the moduli and the two gauge states interact in one vertex.



This contribution incorporates the effects of massive string states through an interaction term involving the Dedekind η functions. Its supersymmetrization leads to a term which can be expressed as an F density,

$$\sim [\log \eta(T) WW]_F . \tag{4.2}$$

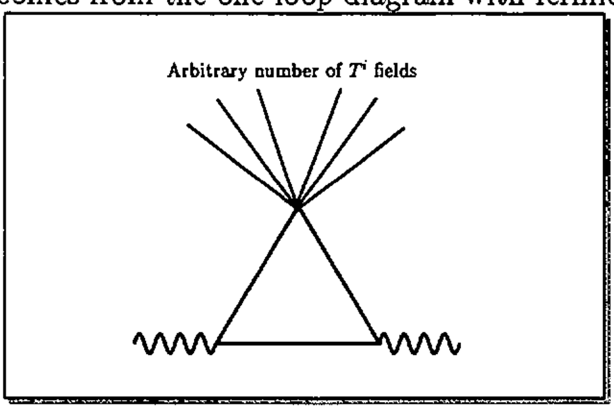
with a holomorphic T-dependence. In the above expression, W is a chiral multiplet representing the gauge field strength. The second diagram is the T mixing, in which the interaction between the moduli and the gauge fields is done through the antisymmetric tensor $B_{\mu\nu}$.



This diagram will contribute to the $\log T + \bar{T}$ diagram through a term involving the antisymmetric tensor field $B_{\mu\nu}$. Its supersymmetrization leads to,

$$\sim \left[\log\left(T + \bar{T}\right) \hat{L}\right]_D . \tag{4.3}$$

These two diagrams are local anomalies which arise at string one-loop order. The last contribution comes from the one-loop diagram with fermion internal lines



and arises from a non local term,

$$\sim \left[W^a W^a \mathcal{P}_C \log(T + \bar{T})\right]_F . \tag{4.4}$$

$\mathcal{P}_C$ is the non-local projector of a vector multiplet into a chiral one. In the Wilsonian approach, this last contribution is a one-loop diagram constructed from the string tree-level effective action $\mathcal{L}^{(0)}$. The first two contributions appears in $\mathcal{L}^{(1)}$. Explicitly they can be written as,

$$\begin{aligned}\mathcal{L}^{(1)} &= A\left[\log(T+\bar{T})\hat{L}\right]_D + B\left[\log|\eta|^4 W^a W^a\right]_F + \text{h.c} \\ &= \left[\left(A\log(T+\bar{T}) + B\log|\eta(T)|^4\right)\ \hat{L}\right]_D\end{aligned} \tag{4.5}$$

A and B are two constants appearing each in a specific diagram. B is equal to b since it is the only term involving the Dedekind function. The local contribution of the triangle anomaly (4.4) to the gauge kinetic terms $F_{\mu\nu}F^{\mu\nu}$ are equivalent to the one produced by (4.3). This is due to the chiral projector $\mathcal{P}_C$. As shown in [15], its action on a vector multiplet leads to a chiral multiplet whose lowest component is proportional to the lowest component of the vector multiplet plus a non-local piece which does not enter in our analysis. This anomalous contribution is known in terms of the other two since the full result is modular invariant. These three constants are related to b by,

$$A + c = B = b. \tag{4.6}$$

$\mathcal{L}^{(1)}$ can therefore be written as,

$$\mathcal{L}^{(1)} = b\left[\log(T+\bar{T})\,|\eta|^4\,\hat{L}\right]_D - C\left[\log(T+\bar{T})\hat{L}\right]_D \tag{4.7}$$

where C, here, is a constant appearing in the contribution of (4.4).

In order to relate this analysis to the work done in this thesis, we have to take the large radius of compactification limit of (4.7) before interpreting the information obtained from ΔS. The large radius of compactification limit corresponds to taking the large $T+\bar{T}$ limit since the real part of the T modulus is related to the compactification radius, R, by

$$t = R^2 + i\varphi,$$

with R measured in units of the Planck length.

The modular group $SL(2,Z)$ acts on t as,

$$t \longrightarrow \frac{at - ib}{ict + d}\quad ,\quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,Z). \tag{4.8}$$

In the large t limit, this symmetry is broken down to a shift of the imaginary part of t,

$$t \longrightarrow t + i\varphi. \tag{4.9}$$

The large t limit of the moduli dependent threshold corrections is

$$\log\left[(t+\bar{t})\,|\eta|^4\right] \longrightarrow -\frac{\pi}{3}(t+\bar{t})$$

It is important to note that now

$$t+\bar{t}$$

is an invariant under (4.9).

The large T limit of $\mathcal{L}^{(1)}$ can be taken on (4.7). In this setting the limit is very easily obtained. It contains two term,

$$\begin{array}{ccc} & \text{large T limit} & \\ b\left[\log(T+\bar{T})\,|\eta|^4\,\hat{L}\right]_D & \longrightarrow & b\left[\left(T+\bar{T}\right)\hat{L}\right]_D \\ -C\left[\log(T+\bar{T})\hat{L}\right]_D & \longrightarrow & -C\left[\log(T+\bar{T})\hat{L}\right]_D\Big|_{\text{LargeT}} \end{array} \tag{4.10}$$

The exact form of the second term is not needed since we know that it exactly cancels the contribution of the triangle graph.

In this thesis it is the analysis of the first term,

$$b\left[\left(T+\bar{T}\right)\hat{L}\right]_D \tag{4.11}$$

which was developed. First it was derived directly from the compactification of the Green-Schwarz counterterm in ten dimensions. This shows that the mechanism of cancellation of anomalies arising from the hexagon diagram in ten dimensions is, after reduction, at the origin of the cancellation of triangle graph in four dimensions. The present analysis has been done only for the case of one h_{11} modulus but all the tools have been developed to carry this analysis first to the case of one h_{12} modulus and to more general cases. Our results also allow an explicit determination of the coefficient b of the anomaly in terms of integrals over the Calabi-Yau manifold.

As is shown at the end of chapter 3, our construction also allows us to include matter fields in our expressions. It was shown that the matter contributions were of the type,

$$\sim\left(t+\bar{t}-2z\bar{z}\right), \tag{4.12}$$

suggesting a supersymmetrization

$$\sim\left[T+\bar{T}-2\Sigma\bar{\Sigma}\right]_D. \tag{4.13}$$

The combination of $T,\bar{T},\Sigma$ and $\bar{\Sigma}$ appears exactly as in the tree-level Kähler potential with the same relative coefficient.

In conclusion I can say that the relation between the processes of anomaly cancelation in four and ten dimensions has been shown for the case of one modulus, the universal breathing mode. This shows the way to further investigations in this directions. For example, in the future I will complete this analysis by including an arbitrary number of moduli and matter fields. It will also be interesting to see how these corrections obtained in the large T limit will give more general informations.

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Acknowledgements

I would like to thanks first my thesis advisor, Jean-Pierre Derendinger for having supporting me during the years I spend in Neuchâtel. When I arrived here, the theory group of high energy physics was just born. He managed to create in a very short time a good working place with many opportunities to make contact with other physicists. He initiated me in a very practical way to string theory and deepened my knowledge of supersymmetry. On a more personal level, his enthusiastic and critical view of Physics gave me some courage when I was lost in the jungle of my computations.

Secondly I want to thanks Fernando Quevedo for all the discussions we had on physics or the world in general. I was particularly impressed by his encyclopedic knowledge of string theory and all the subjects related to it. His willingness to share his science (and his books) make me doubt about the existence of the *selfish gene*.

A work over such a long period of time would not have been possible without the friendly atmosphere of the institute. I particularly thanks Peter Bamert who had to live with my little habits daily.