

# A numerical minimization of perceptual error for the linear chromatic adaptation transform

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## Abstract

In this paper, new sensors for the chromatic adaptation transform (CAT) are found. They have been obtained by the independent numerical minimization of 6 perceptual error metrics over 16 corresponding color pair (CCP) data sets, including Lam's set, using as starting point for the minimization 4 known and commonly used sensors. An analysis of their performances has shown that the best performance is always achieved by the sensors resulting from the minimization process over Lam's data set. Some of these performances are statistically equivalent to - and even better than - those obtained using the CMCCAT2000 and the nonlinear Bradford transforms. This result reinforces both the use of Lam's set as a good representation for the other CCP data sets, and the efforts mentioned in the literature towards the removal of the nonlinearity of the CAT of the CIECAM97 model.

## Keywords

Chromatic Adaptation Transform, Color Constancy, Color Appearance, Chromatic Adaptation Sensors.

## Introduction

When we look at an object separately under two different illumination conditions, for example under a tungsten lamp and a daylight illumination, we maintain almost the same appearance of the colors of the object, even though the visual stimuli are quite different in the two situations. This is due to a natural mechanism of adaptation to the change in illumination conditions, inner to the human visual system, and called *chromatic adaptation*. The persistence of the same color appearance in presence of illumination variations is called *color constancy*.

The phenomenon of color constancy was known since the nineteenth century. Helmholtz, Ives, Hering, and Helson [1], first conducted tests to find a model for such adaptation of the human visual system. As a result of their efforts, the von Kries adaptation model was created [2, 3]. This model mainly relies on two hypotheses: 1) The adaptation is done by a separated scaling of the three photoreceptor responses of the human visual system without changing their sensitivity shape; 2) The scaling coefficients are adjusted to keep the adapted appearance of a white reference surface constant.

The first hypothesis means that, while adapting for an illuminant change, the cone responses do not interfere with each other, but are just scaled by a multiplicative factor. The second hypothesis suggests that the scaling factors are regulated by some conscious perception of the scene illuminant, and it is commonly referred to as the *Illumination Estimation Hypothesis*.

This model has been extensively tested up to now. It has been found that it is coherent with many results obtained during subjective visual tests conducted in laboratory [5, 6, 7], and that it can well explain some phenomena such as the good adaptation to yellow-blue shifts of the color components of an illuminant, and the bad adaptation to red-green shifts [1]. Anyway, it is still considered as an incomplete model because of its extreme simplicity. The chromatic adaptation should in fact also take into consideration other variables, such as the correlation between spatially local chromatic signals across illuminants, and the desensitization caused by the eye movement across spatial variations [4]. Moreover, the illumination estimation hypothesis has been recently reviewed [8], demonstrating that it is not valid if formulated in terms of conscious perception.

Despite these observed limitations, the von Kries model is still a valid model for the human chromatic adaptation, and its simplicity constitutes an advantage for the computational aspect of color constancy, i.e., to achieve color constancy for an artificial vision system. This is necessary, since systems like scanners, cameras, video displays and so on, use different kinds of light to acquire or reproduce an image, and they do not have the ability to adapt to illumination changes. Then, it is necessary to implement an operation that transforms a color obtained from one media under certain light conditions to the corresponding color used in another media, such that the perceived color appearance in the two cases is the same. This operation corresponds to a *chromatic adaptation transform*.

In many applications, the transformation is linear and can be represented by a  $3 \times 3$  matrix whose role is to obtain the post-adaptation cone response of the visual system to the color stimuli. It is in fact in this space that the adaptation takes place. The aim of the present paper is to propose several candidates for this matrix, and to compare the performance obtained by using them to those obtained by the Bradford [9], the CMCCAT2000 [15], and the Sharp CAT [11]. These known CATs differ from each

other, since they have been obtained by using a minimization technique over different corresponding color pair data sets and error metrics, as it will be recalled in the following sections.

### The Chromatic Adaptation Transform

Let  $(X_1, Y_1, Z_1)$  indicate the CIE XYZ color coordinate of a sample under a certain reference illuminant  $I_1$ . The chromatic adaptation transform is used to estimate the color coordinates  $(X_2, Y_2, Z_2)$  which would produce the same color appearance of the sample under  $I_1$  for an observer adapted to an illuminant  $I_2$ , denoted as test illuminant. In other words, the chromatic adaptation transform is used to obtain the *corresponding color* under illuminant  $I_2$ . This transform is obtained as follows:

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = T^{-1} \cdot \begin{bmatrix} \frac{R'_w}{R_w} & 0 & 0 \\ 0 & \frac{G'_w}{G_w} & 0 \\ 0 & 0 & \frac{B'_w}{B_w} \end{bmatrix} \cdot T \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} \quad (1)$$

where  $R_w, G_w, B_w$  and  $R'_w, G'_w, B'_w$  are the RGB color components of the reference and test illuminations, respectively. As it can be noticed, Eq. 1 represents the formulation of the von Kries model as mentioned in the introduction. In fact, according to this model, the adaptation is done separately for each color channel, and this is represented by the use of a diagonal matrix. Moreover, this adaptation does not take place in the XYZ space, but in a transformed cone space obtained by the use of the matrix  $T$ .

To find a suitable matrix  $T$ , Lam [9] created at Bradford University a database of corresponding color pairs by subjective inspection of 58 dyed wool samples considering two different illumination conditions. Using this database he found a transform that maps corresponding color data pairs, minimizing a certain perceptual error metric. The transform, known as Bradford transform, is nonlinear in the blue component of the tristimulus values. In most applications, however, the linear form is used instead, thus discarding the nonlinearity in the blue component. The matrix  $T$  corresponding to this transform, denoted by  $T_{BFD}$  in this paper, is given in Table 1.

Finlayson *et al.* [11, 13] proposed a method to obtain different matrices, based on numerical minimization of the euclidean distance in the CIE XYZ space between actual and predicted corresponding colors, while preserving white, i.e., no errors in the achromatic transform. This transform resulted in sharper, more decorrelated, sensors with respect to the Bradford transform. The corresponding  $T$  matrix is denoted by  $T_{Sharp}$ , cf. Table 1.

Another proposal, corresponding to the matrix  $T_{vonKries}$  in Table 1, was formulated by Hunt, Pointer, and Estevez [12], and consisted in a matrix which linearly transforms the tristimulus values XYZ to relative cone responses (LMS sensors).

The nonlinear Bradford transform was embedded in the CIECAM97 color appearance model [10], but new efforts concentrated on removing its nonlinear correction in the blue channel. These efforts gave origin to a completely linear CAT, called CMCCAT2000 [15] and here denoted by  $T_{2000}$ . This transform represents a candidate

for the substitution of the nonlinear Bradford CAT in the CIECAM97. The scientific community is currently discussing if this transform represents the only valid candidate or if there are some other equivalent or even better transforms. Some comparative studies between the Sharp and the other transforms [13, 16, 14] put in evidence that the standardization of CMCCAT2000 is still premature, since many other matrices ensuring equivalent performances have been found, thus shifting the problem to: 1) The choice of the criteria to establish which of the new CATs is the best; 2) The validity of this choice, considering the important experimental errors that are still present in the available CCP data sets.

### The Minimization Process

As illustrated in the previous section, the matrices  $T$  corresponding to the different CATs proposed in the literature are the result of an error minimization process applied over certain CCP data sets. They thus differ either due to the choice of the perceptual error metric, or due to the choice of the CCP used for the minimization process. The idea of the present paper is to separately minimize different error metrics using different CCP data sets, in order to compare the best results found.

We have chosen to use six different error metrics and sixteen different CCP data sets. Table 1 summarizes our choices. The first three error metrics, denoted by  $\{PEM = 1, 2, 3\}$ , correspond respectively to the mean of three known color distance formulas. The other three metrics, denoted by  $\{PEM = 4, 5, 6\}$  correspond respectively to the Root Mean Squared (RMS) error of these color distances. These error metrics are computed between the CCP data, obtained by perceptual tests, and their prediction obtained from Eq. 1. The aim of the minimization process is to find a matrix  $T$  that minimizes a certain error metric over a given CCP data set. We have considered the CCP data sets at disposal from [17] which correspond to those used by Süssstrunk [14, 16] to compare CATs.

We have chosen a modified descent minimization method to find the optimal coefficients of the  $3 \times 3$   $T$  matrix. Since this minimization method converges to local minima, we have chosen four different starting points, corresponding to the matrices shown in Table 1, which lead us to  $16 \times 6 \times 4 = 384$  resulting matrices. For each coefficient of the  $T$  matrix, a displacement  $\delta t$  was applied, i.e. we formed a new matrix with coefficients:

$$t_{new}^{ij} = t^{ij} + \delta t^{ij} \quad (2)$$

where  $t^{ij}$  with  $\{i, j = 1, 2, 3\}$  is the  $(i, j)$  coefficient of the  $T$  matrix. If the error metric obtained over the CCP data set using this new matrix has a smaller value than that obtained for the preceding matrix, then this displacement is kept, otherwise we changed the direction, i.e.:

$$t_{new}^{ij} = t^{ij} - \delta t^{ij}. \quad (3)$$

This is done individually for each of the nine coefficient. We iterate the process until a stable value of the error metric is reached.

This minimization process was repeated 384 times, one

for each combination of the starting point (SP), error metric (PEM), and CCP data set, resulting in as many matrices obtained at the end of the process, denoted by  $M_{(SP,PEM,CCP)}$  according to the corresponding parameters. For example the matrix  $M_{(4,2,13)}$  was achieved using the  $T_{vonKries}$  matrix (SP = 4) as starting point for the minimization of  $\Delta\overline{E}_{CIE94}$  (PEM = 2) over the ‘‘Breneman 1’’ data set (CCP = 13).

## Tests and Results

The performances of the 384 matrices found in the minimization process were compared to those obtained using the Sharp, the linear and nonlinear Bradford, and the CMCCAT2000 transforms over the sixteen available CCP data sets. We have chosen to use the same comparison test as in [13], i.e., a t-Student test with 95% confidence interval. The null hypothesis was that the mean difference between the prediction errors obtained using respectively the reference and test transforms, is equal to zero, where the prediction errors are expressed in the  $\Delta E_{Lab}$  or  $\Delta E_{CIE94}$  perceptual metric. In case the null hypothesis was rejected, we determined which from the reference and test transforms provided the best results.

To have a global indication of the performance of a particular transform, we calculated an index which is the difference between the number of times the test transform was evaluated better, and the number of times it was evaluated worse than the reference transform. This was done over the sixteen available CCP data sets. This index, denoted by  $I_n$  with  $n = 1, 2, \dots, 384$ , spans from  $I_n = -16$  to  $I_n = 16$ , corresponding to the cases in which the test transform performs respectively worse or better than the reference transform for all the 16 CCP data sets considered. In case  $I_n = 0$ , the test and reference transforms are considered equivalent. Moreover, we define the factor  $A_k$  indicating the number of transforms whose index  $I_n$  is greater than  $k$ , i.e.:

$$A_k = \#\{n : I_n > k\} \quad (4)$$

where the symbol  $\#$  specifies the cardinality of the set considered. In Table 2 we show the number of transforms which are statistically equivalent to or better than the reference transforms according to  $A_k$ .

In Table 3 we list the transforms that provided the best performance, including the corresponding parameters that led to these transforms after application of the minimization process. The index  $I_{max}$  corresponds to the maximum value of  $I_n$  obtained among the 384 transforms with respect to the reference transform considered at each time. As an example, the transform based on the matrix obtained starting from  $T_{2000}$  (SP = 3) by minimizing  $\Delta\overline{E}_{CIE94}$  (PEM = 2) over Lam’s set (CCP = 1) has an index  $I_{max} = 5$  comparing the performance to the linear Bradford transform, and this represents the best result found among the proposed transforms.

Figures 1– 4 illustrate the shapes of the sensors that are statistically equivalent to - or better than - a reference transform, while Figure 5 compares the reference sensors to those corresponding to  $M_{(4,2,1)}$ , which represents one of the seven sensors that ensure performances equivalent

to the nonlinear Bradford transform, as indicated in Table 3.

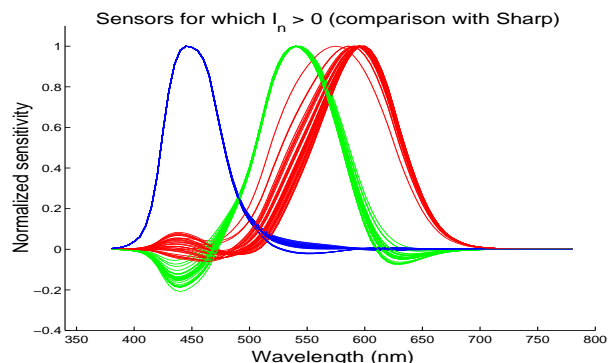


Figure 1: Sensitivity curves for the sensors statistically better than the reference transform ( $I_n > 0$ ). Comparison with Sharp transform (for  $\Delta E_{CIE94}$ ).

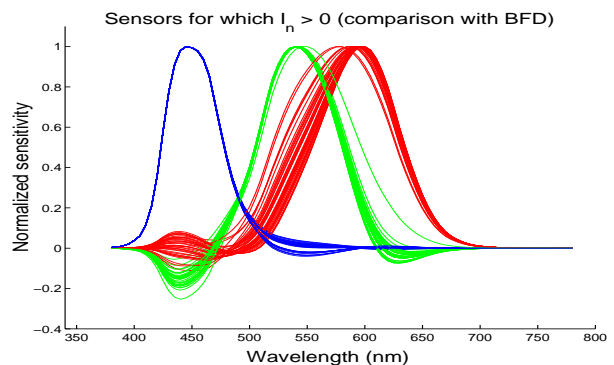


Figure 2: Sensitivity curves for the sensors statistically better than the reference transform ( $I_n > 0$ ). Comparison with linear Bradford transform (for  $\Delta E_{CIE94}$ ).

Following Table 3, we also report three among the best matrices obtained, according to the t-Student test for  $\Delta\overline{E}_{CIE94}$ :

$$M_{(3,2,1)} = \begin{bmatrix} 1.1059 & 0.0642 & -0.1702 \\ -0.8630 & 1.8195 & 0.0435 \\ -0.0012 & 0.0053 & 0.9959 \end{bmatrix}$$

$$M_{(1,5,1)} = \begin{bmatrix} 1.0922 & 0.0890 & -0.1811 \\ -0.8793 & 1.8121 & 0.0211 \\ -0.0023 & 0.0156 & 0.9868 \end{bmatrix}$$

$$M_{(4,2,1)} = \begin{bmatrix} 0.9732 & 0.2000 & -0.1732 \\ -0.8509 & 1.8244 & 0.0265 \\ 0.0009 & -0.003 & 0.9994 \end{bmatrix} \quad (5)$$

## Discussion

A certain number of observations can be made from the tests we have performed.

Firstly, it is clear from Table 2 that a certain number of equivalent or better CATs exists with respect to the

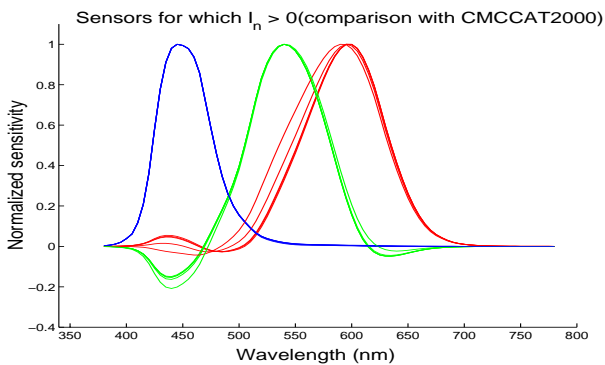


Figure 3: Sensitivity curves for the sensors statistically better than the reference transform ( $I_n > 0$ ). Comparison with CMCCAT2000 transform (for  $\Delta E_{CIE94}$ ).

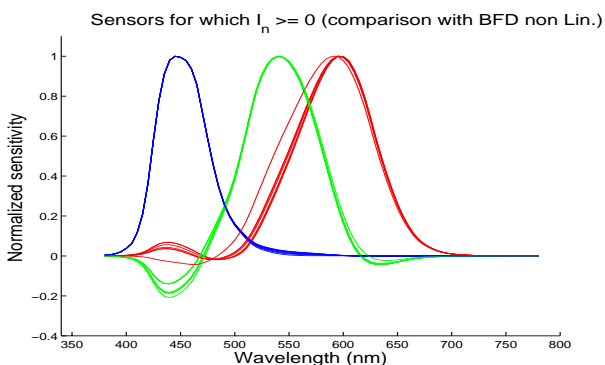


Figure 4: Sensitivity curves for the sensors statistically equal to - or better than - the reference transform ( $I_n \geq 0$ ). Comparison with nonlinear Bradford transform (for  $\Delta E_{CIE94}$ ).

reference ones, at least from the statistical analysis performed by the t-Student test. This shifts the problem in establishing some criteria to choose a good CAT matrix. Fairchild [18] recommends that the best choice should be a CAT which ensures performances similar to those obtained by the current CIECAM97 model. However, two main problems are still not considered: 1) The CCP data sets used for comparisons are still affected by a considerable noise; 2) The Von Kries model is still far from being complete, since it is either too simple to describe what actually happens in the visual system, or the visual system applies a von Kries transformation to a combination of the cone signals, rather than scaling individual cone signals [18].

Secondly, we have shown that Lam's database reveals to represent quite efficiently the other data sets used in our tests. In fact, it can be noticed in Table 3 that the best performances have always been obtained by the minimization of Lam's CCP data set. In other words, the best transform  $T$  that minimizes a certain error metric between the CCPs of Lam's database and their predictions obtained from Eq. 1, is the best transform for the prediction of the entire collection of 16 CCP data sets. This is an important result, since it tends to show that it is possible to limit the data to Lam's data only, an observa-

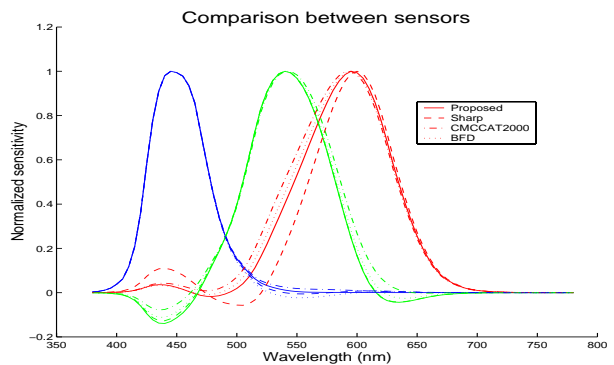


Figure 5: Comparison between reference sensors and  $M_{(4,2,1)}$ , best sensor with respect to nonlinear Bradford transform.

tion that corroborates choices that were made in several papers [13, 14].

Thirdly, another interesting result is obtained by the comparison with the CMCCAT2000 matrix. This matrix was found by Li *et al.* [15] by the minimization of  $\Delta E_{Lab}$  over certain data sets, including Lam's data set. In Table 3 we show that better results can be obtained by minimizing the RMS of  $\Delta E_{CIE94}$  or  $\Delta E_{CMC1:1}$ , which are clearly different from the perceptual error metrics used by Li. We also show that in the minimization of the RMS of  $\Delta E_{CIE94}$ , equivalent results are obtained by selecting any of the four starting points considered. Thus the minimization does not depend, in this case, on the starting point (Table 3, right column, comparison with CMCCAT2000). It is interesting to notice that the best results, apart from the one corresponding to the CMCCAT2000, are obtained by the minimization of  $\overline{\Delta E}_{CIE94}$  (boldface values indicated in Table 3), which can thus be considered as a good error metric to take into consideration when minimizing the color difference in a data set.

Finally, Table 3 shows us that similar or even better performances are found with respect to the nonlinear Bradford transform. A better performance is obtained considering the mean  $\Delta E_{Lab}$  error metric, while an equivalent performance is obtained in the sense of  $\Delta E_{CIE94}$ . This reinforces the interest in orienting the research towards removing the nonlinearity present in the CAT of the CIECAM97 model.

## Conclusions

In this paper we show that new sensors for the chromatic adaptation transform can be found with respect to those already proposed. The performances obtained by some of the new sensors are equivalent or even better for a certain database of corresponding color pairs. We show that Lam's set can significantly represent the other considered data sets, since the performance of the CAT obtained by the minimization process performed on Lam's set permits to obtain the best performances also for the other CCP data sets considered. Moreover, some sensors have proven to ensure a performance equivalent to the one obtained by the nonlinear Bradford transform. This suggests that this transform, embedded in the CIECAM97 model, could be

substituted by a more economic linear one, as it is currently wished.

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### Biography

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*Fausto Pellandini* obtained his Ph.D. from the ETH Zurich in 1967. In 1972 he was appointed as professor of electronics at UniNe where he was in charge of creating IMT UniNe, which he headed for 10 years. Since 1975 he heads the ESPLAB at IMT, where he leads research activities mostly concerned with recognition, compression and coding of information (speech and images), algorithms and architectures for ultra low power signal processing. From 1989 he is part-time professor at the EPFL, where he teaches signals and systems theory.

CCP Data Sets	PEM (Perceptual Err. Metric)	SP (Starting Point)
1 → Lam	1 → $\overline{\Delta E}_{Lab}$	1 → $T_{Sharp} = \begin{bmatrix} 1.2694 & -0.0988 & -0.1706 \\ -0.8364 & 1.8006 & 0.0357 \\ 0.0297 & -0.0315 & 1.0018 \end{bmatrix}$
2 → Helson	2 → $\overline{\Delta E}_{CIE94}$	2 → $T_{BFD} = \begin{bmatrix} 0.8951 & 0.2664 & -0.1614 \\ 0.0367 & 1.7135 & 0.0367 \\ 0.0389 & -0.0685 & 1.0296 \end{bmatrix}$
3 → CSAJ		
4 → Lutchi		
5 → Lutchi D50		
6 → Lutchi WF	3 → $\overline{\Delta E}_{CMC1:1}$	3 → $T_{2000} = \begin{bmatrix} 0.7982 & 0.3389 & -0.1371 \\ -0.5918 & 1.5512 & 0.0406 \\ 0.0008 & 0.0239 & 0.9753 \end{bmatrix}$
7 → Kuo&Luo		
8 → Kuo&Luo TL84		
9 → Braun&Fairchild 1	4 → RMS of $\Delta E_{Lab}$	4 → $T_{vonKries} = \begin{bmatrix} 0.3897 & 0.6890 & -0.0787 \\ -0.2298 & 1.1834 & 0.0464 \\ 0 & 0 & 1 \end{bmatrix}$
10 → Braun&Fairchild 2		
11 → Braun&Fairchild 3	5 → RMS of $\Delta E_{CIE94}$	
12 → Braun&Fairchild 4		
13 → Breneman 1		
14 → Breneman 8		
15 → Breneman 4	6 → RMS of $\Delta E_{CMC1:1}$	
16 → Breneman 6		

Table 1: List of data sets, perceptual error metrics, and starting points used in the article [11, 16, 17, 19].

Perceptual Err. Metric	Reference transforms			
	Sharp ( $A_0$ )	CMCCAT ( $A_0$ )	BFD ( $A_0$ )	Nonlinear BFD ( $A_{-1}$ )
$\Delta E_{Lab}$	62	55	87	16
$\Delta E_{CIE94}$	46	15	63	7

Table 2: Number of test transforms with  $I_n > 0$  ( i.e.  $A_0$ ) or  $I_n \geq 0$  ( i.e.  $A_{-1}$ ) with respect to the reference transforms.

Reference Transforms		$\Delta E_{Lab}$	$\Delta E_{CIE94}$
<b>Sharp</b>	$I_{max}$	7	6
	SP	<b>2</b>	<b>3</b>
	PEM	<b>2</b>	<b>2</b>
	CCP Data Set	<b>1</b>	<b>1</b>
<b>CMCCAT</b>	$I_{max}$	5	1
	SP	2,4	1,2,3,4,4
	PEM	6,6	5,5,5,5,6
	CCP Data Set	1,1	1,1,1,1,1
<b>BFD</b>	$I_{max}$	5	5
	SP	<b>1,1,2,2,2,3,3,4,4,4</b>	<b>1,1,2,2,3,3,4</b>
	PEM	<b>2,3,1,2,3,2,3,2,3,6</b>	<b>1,2,1,2,1,2,3</b>
	CCP Data Set	<b>1,1,1,1,1,1,1,1,1,1</b>	<b>1,1,1,1,1,1,1</b>
<b>Nonlinear BFD</b>	$I_{max}$	3	0
	SP	4,4	1,2,2,3,3,4,4
	PEM	<b>2,6</b>	6,3,6,3,6,2,6
	CCP Data Set	1,1	1,1,1,1,1,1,1

Table 3: Summary of the conditions (choice of starting point, minimizing function, and data set) that led to the transforms ensuring the best performances in comparison with the reference transforms. The SP, PEM, and CCP values have to be read in a vertical sense, as a triplet which actually represents the matrix  $M_{(SP,PEM,CCP)}$ .