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OPTICAL FIBER SENSORS USING BIREFRINGENT RESONATORS

THESE

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IMPRIMATUR POUR LA THÈSE

Capteurs à fibre optique utilisant des résonateurs biréfringents

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OPTICAL FIBER TRANSDUCERS USING FABRY-PEROT RESONATORS

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Abstract

The basic properties and the different types of opto-electronic detection schemes for high sensitivity polarimetric Fabry-Perot transducers are discussed. Two examples of optical fiber transducers, one a magneto-optic current monitor and the other a static force sensor, are presented to illustrate the subject.

Introduction

Optical fiber transducers using single-mode fibers and interferometric measurement techniques can provide convenient solutions to a number of particular sensing problems. To meet the various requirements dictated by each particular situation, it is necessary to adapt the properties of the fiber to the application and also to find an appropriate opto-electronic detection scheme. For static measurands, the problem of temparature drift of the interferometer can be solved by polarimetric techniques. These techniques use the two eigenpolarizations supported by the single-mode fiber and can be seen as differential interferometry using a single piece of fiber. The effect of the measurand on the fiber material provokes a difference for the propagation constants of the two eigenpolarizations, which results in an optical phase difference at the output end of the fiber. Changes of the optical length of the fiber are common to both eigenpolarizations and therefore they do not perturb the measurement. To adjust the sensitivity of the phase response to the effect of the measurand, the fiber is usually arranged so that it passes many times in the field of the measurand, like in fiber coils for magnetooptic current monitors. Different approaches using Fabry-Perot

resonators have been suggested [1] [2] [3]. In that case, the sensitivity of the phase response is increased by the multiple passes of the light in the same piece of fiber. The detection schemes for polarimetric Fabry-Perot transducers are particular, because they have to cope with the narrow resonances of Fabry-Perot resonator (Fig. 1). According to the dynamic range of a particular transducer, different detection schemes must be applied to measure the phase difference between the two eigenpolarizations of the resonator with high accuracy. A common feature to all these systems is the fact, that the optical frequency v has to be tuned to the resonance peak of the Fabry-Perot. This can be conveniently realized with a single-mode laser diode as light source and an opto-electronic feedback acting on the diode current.

Birefringent Fabry-Perot resonator

This section presents the relevant properties of birefringent Fabry-Perot resonators. Optical fiber Fabry-Perot resonators with low loss and high finesse can be produced from single-mode fibers [4]. Conveniently, the length of the fiber can be adjusted to obtain the desired free spectral range. A birefringent Fabry-Perot resonator can be considered as the combination of two polarization encoded resonators implemented in parallel on a common length of fiber. The two eigenpolarizations are determined by the roundtrip polarization properties of the resonator cavity and the position of each set of resonances (Fig. 1) is determined by the effective index n_i (i = 1,2) of each eigenpolarization [2]. Figure 1 shows the intensity and the phase of the light transmitted through a birefringent Fabry-Perot resonator vs frequency v. The free spectral range for the eigenpolarizations is

$$FSR_i = c/2n_id \ . \qquad (1)$$

It depends on the optical length of the resonator n_id and the speed of light in vacuum c. Note that the effective index difference $\Delta n = n_1 - n_2$ never exceeds about 10^{-3} in optical fibers and therefore the free spectral range is nearly the same for the two eigenpolarizations. It follows that the separation between the corresponding resonance

frequencies of the two eigenpolarizations is linearly related to the induced birefringence by

$$\Delta\nu_B = (\nu/n)\,\Delta n\ . \qquad (2)$$

The effective reflectivity R_1 and R_2 of the mirrors with $R = (R_1R_2)^{1/2}$, determine the finesse F of the resonator

$$F = \frac{\pi\sqrt{R}}{1-R}\ , \qquad (3)$$

and the spectral width the resonances

$$\Delta\nu_R = FSR/F\ . \qquad (4)$$

Fabry-Perot resonators can be used in the transmission or in the reflection mode with similar detection schemes. The phase function in transmission (Fig. 1) depends only on the finesse of the resonator [2] and the phase difference between the two eigenpolarizations at the output of the resonator reads

$$\Delta\psi_T = 2\,\tan^{-1}(\rho\,\tan(\Delta\phi/2)) \qquad (5)$$

where $\Delta\phi = (2\pi/c)\nu d\Delta n$ is the retardation for a single pass through the cavity and $\rho^2 = 1+4F^2/\pi^2$ is the contrast factor of the resonator. In reflection, the phase function depends not only on the contrast factor but also on the respective reflectivities of the input and output mirrors. It will not be presented here for the sake of simplicity.

According to the dynamic range $\Delta\nu_B$ of a particular transducer, three different detection schemes must be applied to measure the phase difference between the two eigenpolarizations of the resonator with high accuracy.

Detection schemes

Transducers with a dynamic range on the order of the resonance width, i.e. $\Delta\nu_B \cong \Delta\nu_R$, require only one laser source to excite both resonances and to measure the phase difference between the two eigenpolarizations at the output of the resonator. The optical frequency of the laser is locked to the maximum total tranmission of the resonator. At this point the intensities of the two eigenpolarizations are equal (Fig. 1). The feedback loop uses a small optical frequency modulation

$\Delta\nu_L \cong \Delta\nu_R$ of the laser and phase sensitive detection of the total transmitted light to extract sign and magnitude of the frequency offset. For $|\Delta\nu_B| \cong \Delta\nu_R$ the intensity changes around the resonances are large and the phase difference must be measured with a technique which is independent of the intensities of the two eigenpolarizations. Heterodyne phase detection is adequate for that purpose. In this case, the dynamic range for the measurand is about $\Delta\phi \cong \pm 0.4(\pi/F)$ and corresponds to one resonance width, i.e. $\Delta\psi_T = \pm 45°$ [2].

Near the resonance, the intensity transmission can be approximated by a quadratic function and the phase function is approximated by

$$\Delta\psi_T = \rho\ \Delta\phi \quad . \tag{6}$$

If the dynamic range is restricted to $\Delta\nu_B < \Delta\nu_R$ (Fig. 1), the intensity variations due to the laser modulation $\Delta\nu_L$ are small and therefore direct polarimetric phase mesurement is possible. It can be shown that, for differential detection of two orthogonal polarizations [5], the averaged error in the phase is less than 0.2 % for $|\Delta\nu_B| \leq 0.1\ \Delta\nu_R$. Therefore the dynamic range for the retadation is about $\Delta\phi \cong \pm 0.1(\pi/F)$ and corresponds to $\Delta\psi_T = \pm 12°$.

For larger dynamic range, $|\Delta\nu_B| > \Delta\nu_R$, the two resonances do not overlap anymore. Two different laser diodes, each locked to one resonance peak of each eigenpolarizations have to be used. In that case, the induced birefringence is first converted into a frequency difference $\Delta\nu_B$ between the two lasers. Then, a single separate unbalanced interferometer with an optical path difference L performs the frequency to phase conversion [3] with the same scale factor for both lasers. At the output of this interferometer a precise determination of the phase variations $\Delta\phi_1$ and $\Delta\phi_2$ for both optical frequencies is accomplished by counting the periods and interpolating between them. The difference

$$\Delta\Phi 1 - \Delta\Phi 2 = (2\pi/c)L\Delta\nu_B, \tag{7}$$

modulo 2π, corresponds exactly to the retardation produced by the birefringence inside the resonator cavity. In that case, the dynamic range is only limited by the continuous tuning range of the lasers.

Laser source requirements

The opto-electronic detection schemes which have been presented require laser sources which are both continuously tunable over many free spectral ranges of the resonator to maintain the optical frequency at resonance, and sufficiently coherent to take advantage of the spectral properties of the resonator. The effect of the laser linewidth $\delta\nu$ (FWHM) on the response curve of the resonator is to average the intensity and the phase transfer functions and therefore to reduce the performance of the different detection schemes. For the direct polarimetric system and the heterodyne system which use the phase response of the resonator, it can be shown that for $\delta\nu = \Delta\nu_R$, the sensitivity drops to 75% of its value for a monochromatic source [1]. For the system with large dynamic range using the two lasers, broad linewidths change the accuracy of the frequency locking. For the same condition $\delta\nu = \Delta\nu_R$, the sensitivity drops to 25% of its value for a monochomatic source [5] and results in a lower resolution for the measurement of the output phase difference in Eq. (7). Since the linewidths of commercially available free running single-mode laser diodes are about 20 MHz at 5 mW output power, the finesse for a Fabry-Perot resonator with a free spectral range 1 GHz (10 cm of fiber) should not exceed 50 ($\Delta\nu_R$ = 20 MHz). Short resonators with larger free spectral range relax the requirements on laser linewidth but increase the requirements on tuning range. The typical continuous tuning range for Fabry-Perot type single-mode laser diodes is 10 GHz by changing the current and 100 GHz by changing the case temperature. Values which are one order of magnitude larger have recently been reported for multielectrode DFB lasers [6], but these devices are not yet commercially available.

Faraday isolators have to be used to isolate the laser from optical feedback, since the reflections from fiber optical Fabry-Perot resonators cannot be suppressed by oblique injection.

Magneto-optic electrical current sensor

This section presents an optical fiber current sensor which uses a fiber-optic Fabry-Perot resonator shaped into a single-turn helix (Fig. 2) instead of a fiber coil [7]. The shape of the single-turn fiber helix is chosen so that the eigenpolarizations are originally degenerate and become circular by the Faraday effect. The increase in sensitivity is proportional to the finesse of the resonator. To illustrate the properties which can be expected for current monitors using a resonator, the maximum current corresponding to the dynamic range for the two polarimetric detection schemes discussed above are evaluated for the fixed value of F = 100 for the finesse. The sensitivity of the detection is then equivalent to that of a single pass trough a fiber coil with 66 turns (i.e. $\rho = 66$).

1) Direct polarimetric detection allows phase excusions up to $\Delta\psi_T = \pm$ 12 °. Consequently the induced circular retardation should not exceed $\Delta\phi = \pm 0.1(\pi/F)$ which gives $\Delta\phi = \pm 0.18°$ for F = 100. At λ=780 nm the magneto-optic induced circular retardation is $\Delta\phi = 3.4\times10^{-4}$ °/A·turn and therefore the maximum allowed current is ± 500 A.

2) With heterodyne phase detection larger currents can be measured since phase excursions up to $\Delta\psi_T = \pm 45°$ can be accepted. The dynamic range for this system is $\Delta\phi \cong \pm 0.4(\pi/F)$ which gives $\Delta\phi = \pm 0.72°$ for F = 100 and corresponds to a maximum allowed current of ± 2000 A.

Note that in a practical case the finesse is limited by the laser linewidth and the free spectral range of the resonator. To demonstrate the principle of the sensor, the helix shown in Fig. 2 was constructed. The length of the resonator is about 16 cm and gives a free spectral range of FSR = 600 MHz. The finesse is 6 and gives a resonance width of $\Delta\nu_R$ = 100 MHz, much larger than the laser linewidth. The semiconductor laser source was locked to the maximum transmission of the Fabry-Perot resonator and heterodyne phase detection was used to measure the phase shift between the two eigenpolarizations [7]. Experimental results are shown in Fig. 3. The experimental points follow closely the theoretical curve for a Fabry-Perot resonator of

finesse F = 6.2 and the slope is $\Delta\psi/I = 1.4 \times 10^{-3}$ °/A, which is 4 times larger than the sensitivity of a single-pass sensor. This is in good agreement with the measured finesse F = 6 for this resonator.

High resolution static force sensor

Because of their very good material properties and small transverse dimensions, single-mode silica fibers are well adapted to force sensors using the elasto-optic effect. For 125 μm diam. silica fibers, the induced retardation is 350° for one kilogram of applied weight on the bare fiber, limiting the practical precision of direct polarimetric measurements to about 1 g [9]. Fabry-Perot resonators can be used to increase the precision of the measurement [3]. To demonstrate the principle, a Fabry-Perot resonator with a free spectral range of 850 MHz and a finesse of 10 was constructed from a 125 μm diam. fiber with low intrinsic birefringence (Fig. 4). The optical frequencies of two laser diodes were locked to one resonance peak of each of the two linear eigenpolarizations of the resonator. The optical frequencies were compared in an unbalanced Mach-Zehnder reference interferometer with polarization maintaining fibers (York HB600) to avoid cross-talk between the two polarizations. The optical path difference was L = 3 m, which is shorter than the coherence length of the lasers and gives a sensitivity for the frequency-to-phase conversion of $2\pi/100$ MHz. Two fringe counters performed fringe counting and interpolation to $2\pi/1000$ and the induced birefringence was evaluated through Eq. (7).

Figure 4 shows experimental results obtained with this setup. Open circles represent measured values of the phase difference at the output of the Mach-Zehnder as a function of the weight applied to the fiber. The straight line corresponds to the theoretical value for a silica fiber with a diameter of 125 μm and a Mach-Zehnder with an optical path difference of 3 m. The dashed line corresponds to the retardation induced inside the cavity. It shows that the frequency-to-phase conversion system increases the sensitivity by a factor 15.

Conclusion

Polarimetric sensors using high finesse Fabry-Perot resonators have been presented. Compared to single-pass polarimetric sensors, the sensitivity is increased in proportion with the finesse. According to the expected dynamic range of the birefringence induced in the resonator cavity, different detection schemes must be applied to perform the measurement. The case of two particular sensors, a magneto-optic current monitor and a static force sensor have been presented together with experimental results. Although the requirements on the laser source are severe, commercially available compact-disc type lasers diodes can be used. If multielectrode compound-cavity laser diodes reach mass production, fiber-optical transducers using resonators will offer an excellent solution to many difficult sensing problems.

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Figure captions

Fig. 1. Intensity and phase of the two eigenpolarizations transmitted through a birefringent Fabry-Perot resonator vs optical frequency ν. The phases show a steep slope within the resonance. $\Delta\nu_R$ is the width of the resonance and $\Delta\nu_B$ the birefringence induced shift.

Fig. 2. Fiber optical Fabry-Perot current sensor. The single-turn fiber helix is chosen so that the eigenpolarizations are originally degenerate and become circular by the Faraday effect.

Fig. 3. Faraday induced optical phase shift in the fiber optical sensor. 1) response for a single pass through one loop of fiber, 2) response for a Fabry-Perot with a finesse of F = 6.2, compared with the experimental results (open circles).

Fig. 4. Fiber optical Fabry-Perot force sensor and experimental results. The dashed line corresponds to the induced retardation inside the cavity for a 125 μm silica fiber. The solid line is the theoretical response of the system with 3 m optical path difference of the reference interferometer. Open circles are measured values.

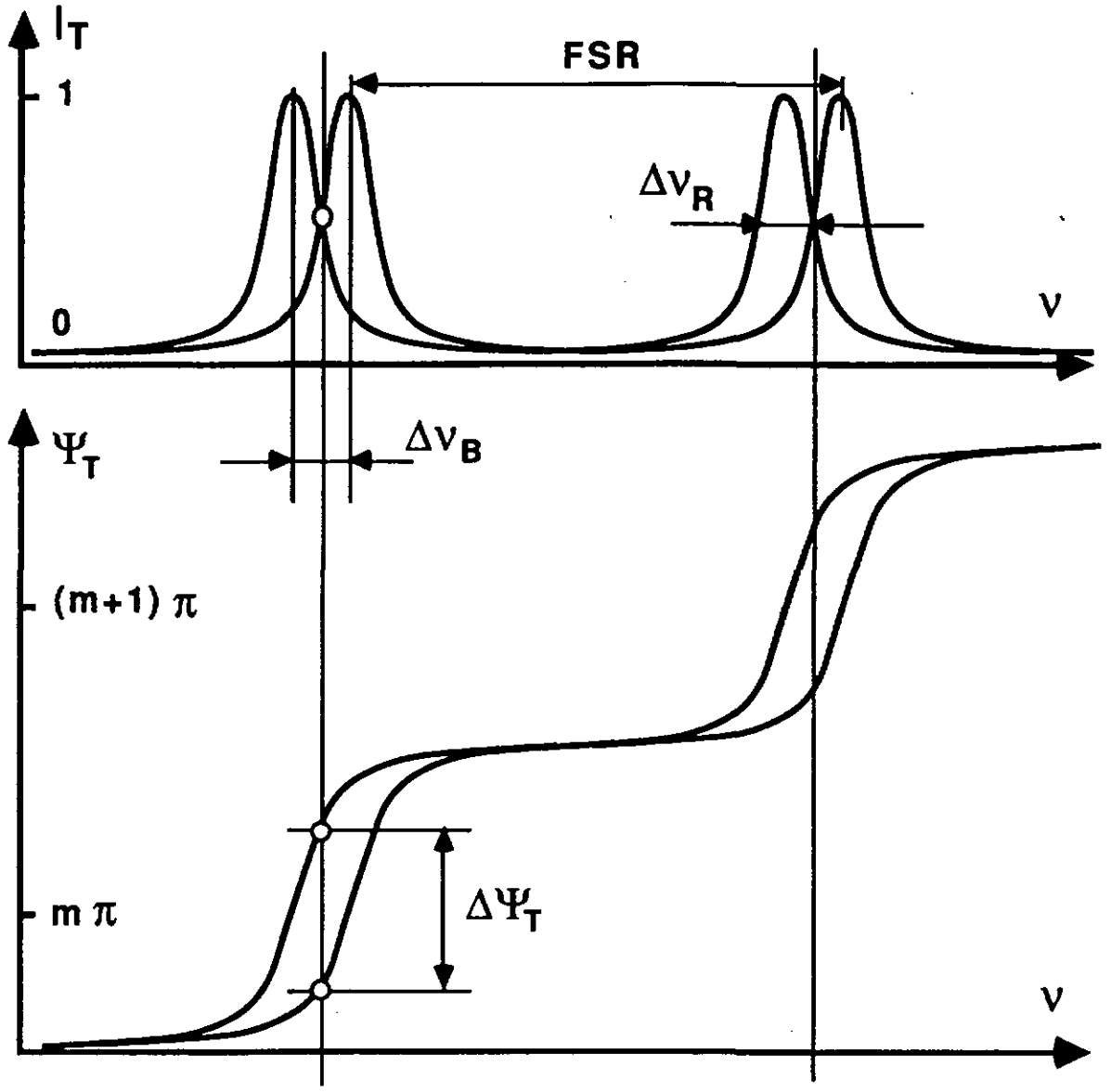


Fig. 1

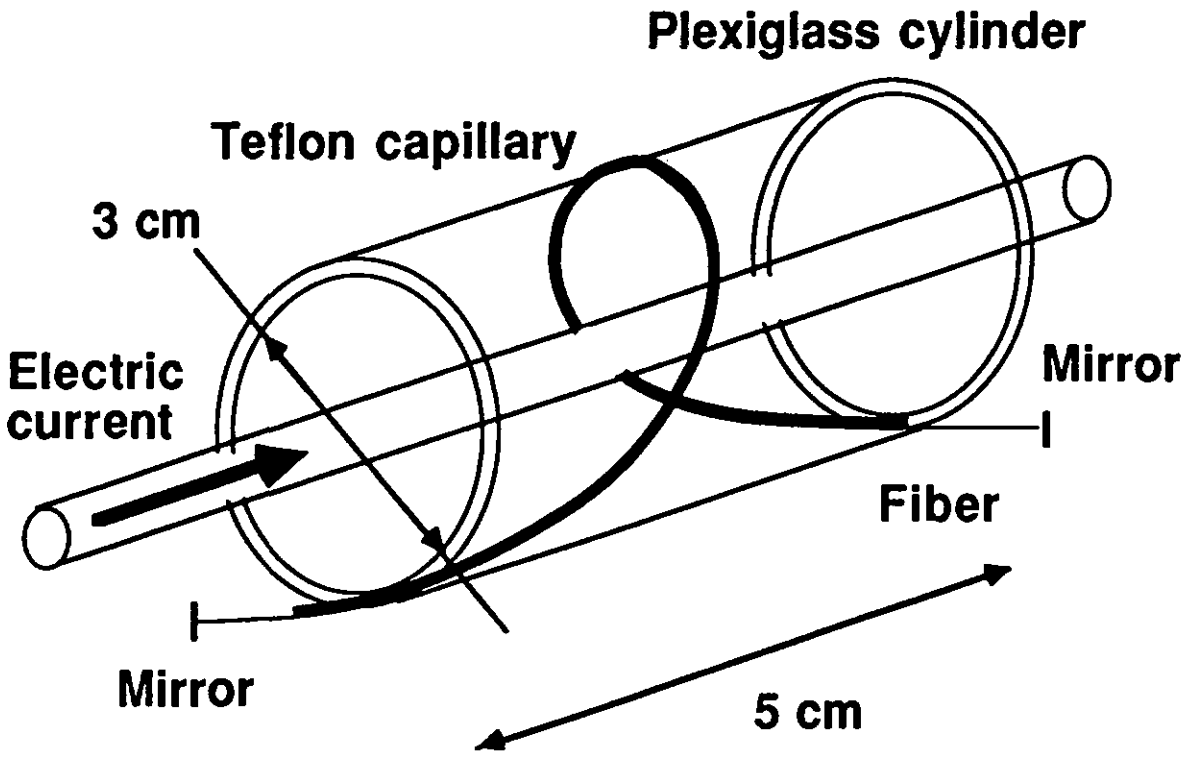

Plexiglass cylinder
Teflon capillary
3 cm
Electric
current
Mirror
Fiber
Mirror
5 cm

Fig. 2

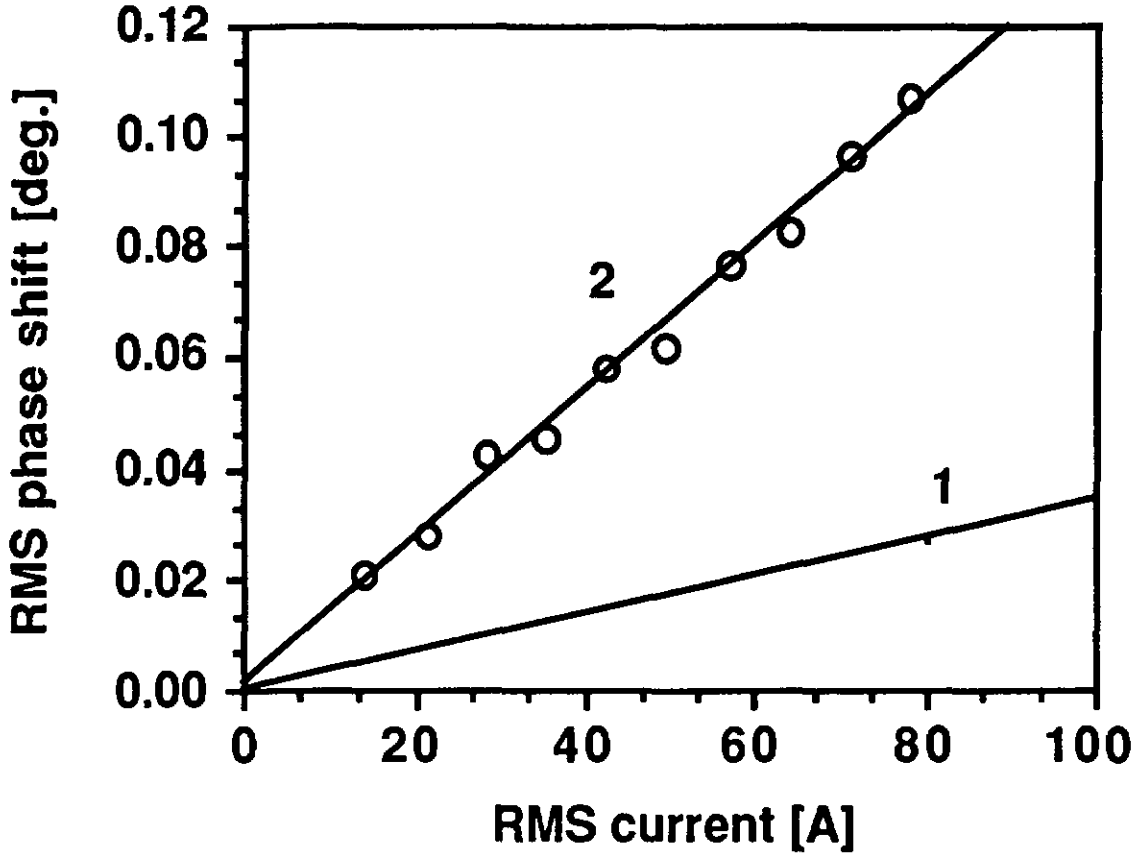
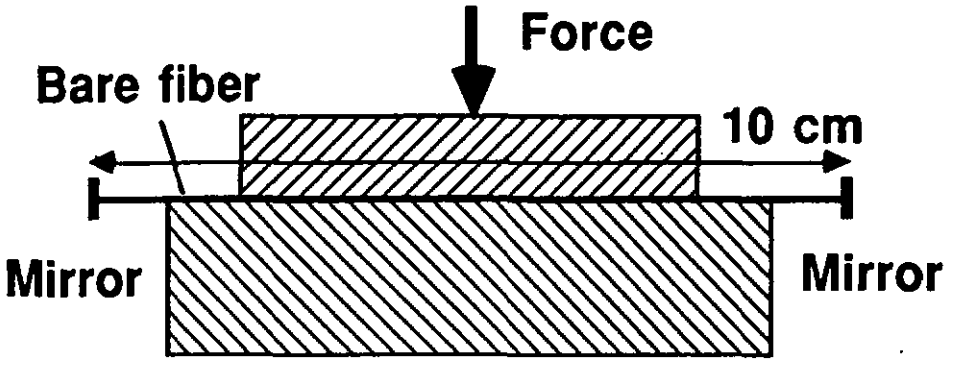

RMS phase shift [deg.]
0.12
0.10
0.08
0.06
0.04
0.02
0.00
0
20
40
60
80
100
RMS current [A]
2
1

Fig. 3


Force
Bare fiber
10 cm
Mirror
Mirror

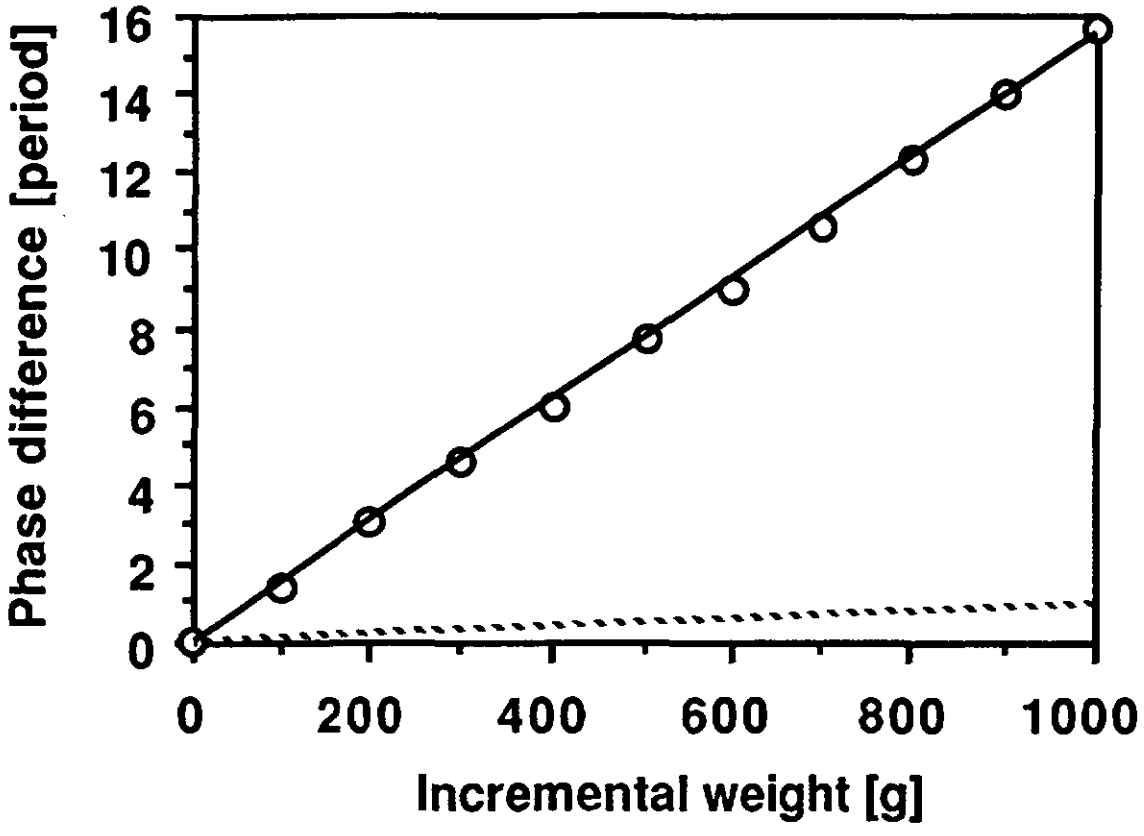

Phase difference [period]
16
14
12
10
8
6
4
2
0
0
200
400
600
800
1000
Incremental weight [g]

Fig. 4

HIGH SENSITIVITY FABRY-PEROT OPTICAL FIBER SENSOR FOR THE MEASUREMENT OF MECHANICAL FORCE

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A high sensitivity polarimetric fiber sensor has been realized, using a high finesse fiber optical Fabry-Perot resonator. The frequency of the diode laser source is locked to the resonance peak of the Fabry-Perot, where the sensitivity is highest. Heterodyne polarization interferometry is then applied to measure the phase difference between the two eigenpolarizations, independently of their amplitudes. The sensor is insensitive to temperature changes of the Fabry-Perot cavity and to other perturbations that affect the length of the cavity. The sensitivity may be as much as 100 times larger than for a single-pass sensor. The fiber optical Fabry-Perot as a passive, lightweight sensor element, which can be made very compact. The application to the measurement of static force by induced birefringence is reported in detail.

High finesse fiber Fabry-Perot resonators can be produced from a piece of single-mode fiber by carefully polishing the fiber ends and gluing properly oriented chip size dielectric mirrors on both ends [1]. When the fiber, which constitutes the resonator cavity, is pressed between two parallel plates, the stress distribution in the core region is anisotropic and brings linear birefringence in the cavity [2]. The light that propagates in such a cavity can be separated into two linear eigenpolarizations, oriented along the two orthogonal axes of birefringence, which see sightly different indices of refraction, or optical length. One gets a dual Fabry-Perot cavity, implemented in parallel on a common length of fiber. In this paper, it will be shown how a high sensitivity static force sensor can be made from a fiber Fabry-Perot resonator by measuring the phase difference between the two eigenpolarizations at the output of the resonator. External perturbations affecting the length of the fiber or the wavelength of the source are common-mode perturbations and do not change the observed value of induced birefringence. The gain in sensitivity compared to a single-pass sensor is proportional to the finesse and can become more than 100.

Figure 1 shows the intensity and the phase transfer functions for the transmission of a birefringent Fabry-Perot resonator of finesse F = 10 as a function of the mean accumulated single pass phase $\phi_0 = (2\pi/c)\nu d n_0$, where c is the speed of light in vacuum, ν is the light frequency, d is the length of the resonator and n_0 is the mean refractive index of the two eigenmodes. The transmitted intensity consists of two identical Airy combs, separated by the

phase difference $\Delta\phi = (2\pi/c)\nu d\Delta n$, where $\Delta n = n_1 - n_2$ is the difference of the refractive indices of the two eigenpolarizations. For a transverse force K applied to the uncoated fiber, the induced birefringence yields $d\Delta n = 4CK/\pi R$, where C is the stress-optic coefficient ($C = -3.7\times10^{-12}$ m^2/N for silica) and R is the fiber diameter [2]. The phase difference between the two transmitted eigenpolarizations is given by $\Delta\psi = \psi_1 - \psi_2 = 2\tan^{-1}[\rho\tan(\Delta\phi/2)]$, with $\rho \cong (1 + 2F/\pi)$ for a resonator finesse of $F > 1$ [3].

The fiber Fabry-Perot force sensor is based on the measurement of this phase difference $\Delta\psi$. Around the resonance, $\Delta\phi = m\pi$, $\Delta\psi$ depends linearly on $\Delta\phi$ and the slope, or sensitivity, is readily seen to be $\Delta\psi/\Delta\phi = \rho$ (for example $\rho = 7$ for $F = 10$), instead of $\Delta\psi/\Delta\phi = 1$ in the case of a single-pass sensor. However, it is important to note that the dynamic range of the described Fabry-Perot sensor is limited to the width of the resonance and that, because of the large amplitude variations near the resonance, it is imperative to measure the phase independently of the amplitude. The detection scheme which has been chosen can be divided into two parts. The first part consists of a feedback loop, which locks the laser frequency to the Fabry-Perot resonance peaks, where the sensitivity is maximum, and the second part measures the phase difference $\Delta\psi$ accurately using heterodyne detection.

The electronics for the stabilization of the laser frequency is shown in the upper part of Fig. 2. The temperature stabilized single-mode laser diode is frequency modulated at 1 kHz via the injection current over a range corresponding to a fraction of the resonance width. The error signal is proportional to the corresponding frequency component in the output intensity of the Fabry-Perot [4]. This error signal is extracted from the mean intensity of the two eigenmodes, detected by the photodiode D2 behind a polarizer P2 oriented at 45°. The frequency locking works properly as long as the two resonance peaks (Fig. 1) are separated by less than their width, which is about $\Delta\phi = \pm1.2/F$. This limits the dynamic range. At this point, the sensitivity $\Delta\psi/\Delta\phi$ is still about 85% of its maximum value for $\Delta\phi = m\pi$.

The heterodyne polarization interferometer, which allows to measure the phase difference $\Delta\psi$ of the two eigenpolarizations independently of their amplitudes, is shown in the lower part of Fig. 2. The light from the laser is split by the polarizing beam splitter PBS into two beams with orthogonal polarizations. Two bragg cells, M1 and M2, shift the optical frequency in the two arms by 40.0 MHz and 40.1 MHz, respectively, so that the beat frequency is 100 kHz, which is compatible with the electronic phasemeter used in the experiment. The two polarizations are recombined through the non-polarizing beam splitter BS. The photodiode D1, placed behind a polarizer P1 oriented at 45°, supplies the heterodyne reference signal. The complementary beam is launched into the fiber Fabry-Perot resonator FFP. Care is taken to ensure that the two orthogonal polarizations are aligned parallel and perpendicular to the principal axis of birefrin-

gence of the pressed fiber. At the output, the photodiode D2, placed behind a polarizer P2 oriented at 45°, detects the phase shifted heterodyne signal. The two signals are fed through a band pass filter to the electronic phasemeter, where the phase difference $\Delta\psi$ is determined by zero-crossing detection.

Measurements were made with a Fabry-Perot consisting of a 10 cm long single-mode fiber of low intrinsic birefringence with two gold coated mirrors of 76% reflectivity glued to both ends. The free spectral range of the resonator was 1 GHz and the finesse has been measured to be F = 10, which is in good agreement with the theoretical value for 76% mirrors. The fiber resonator is placed in a mechanical mounting that permits the application of a static transverse force on a short length of the fiber, where the coating has been removed. A bias load was applied in order to have the two resonances of the eigenpolarizations exactly superimposed (i.e. $\Delta\phi = \pi$). Around this working point, the measurements were made by adding and removing calibrated weights. The laser diode is optically isolated from the Fabry-Perot resonator and emits at 780 nm a single line of 20 MHz width (FWHM), which is about 5 times narrower than the Fabry-Perot resonance peaks. At the output of the optical isolator (OI), the light passes through a single-mode fiber (SMF), which acts as a spatial filter, before entering the heterodyne polarimeter.

The experimental results are shown in Fig. 3. The resolution for $\Delta\psi$ was limited to ±3° by instabilities of the single-mode fiber (SMF), acting as the spatial filter. Further improvements should bring the resolution closer to that of the phasemeter, which is 0.01°. The reproducibility was of the same order as the resolution. The experimental points follow closely the theoretical curve for a Fabry-Perot resonator of finesse F = 10 and a silica fiber of 125 µm outer diameter. The maximum slope is 2.1°/g, which is 7 times larger than the sensitivity of a single-pass sensor.

A novel high resolution polarimetric Fabry-Perot sensor concept, with a diode laser source and heterodyne phase detection of the transmitted light, has been successfully applied to measure static force by induced birefringence with increased sensitivity. In some cases, it might be more convenient to use the Fabry-Perot in reflection rather than in transmission, since this requires access to only one fiber end. However, in order to have enough reflected intensity at the resonances, the Fabry-Perot has to be asymmetric with 100% reflectivity at the far end and a limited finesse F, namely F = 40 for $I_{res} = 0.2\ I_{max}$.

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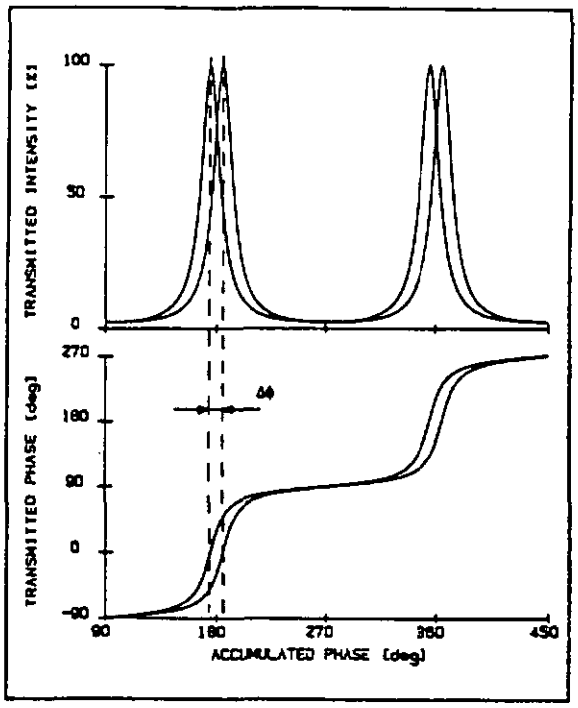
Figure captions

Fig. 1 Intensity and phase of the transmitted eigenpolarizations in a birefringent Fabry-Perot resonator as a function of the mean accumulated single pass phase.

Fig. 2 Experimental setup for the measurement of the accumulated birefringence in a Fabry-Perot resonator using heterodyne polarimetry. LD: laser diode, L1, L2, L3, L4: objective lenses, OI: optical isolator, SMF: single-mode fiber (spatial filter), PC: polarization controller, FFP: fiber Fabry-Perot.

Fig. 3 Measured phase difference $\Delta\psi$ of the transmitted eigenmodes versus applied transverse load. Solid line: theoretical curve for a Fabry-Perot of finesse F = 10 and a silica fiber of 125 μm outer diameter. Dashed line: single-pass sensor for comparison.



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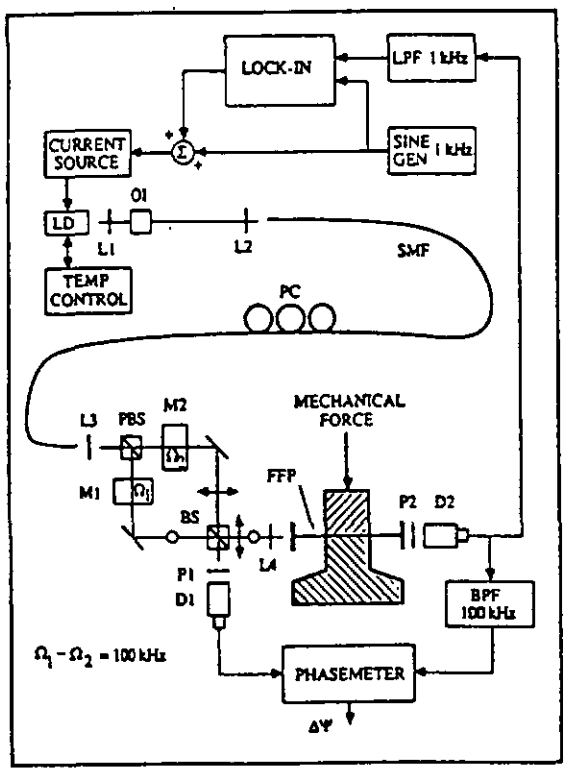

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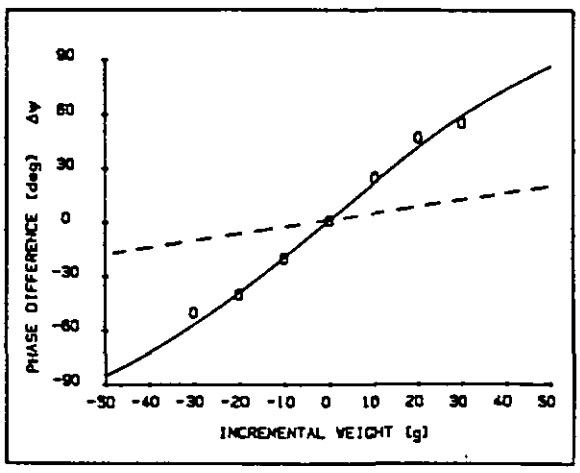

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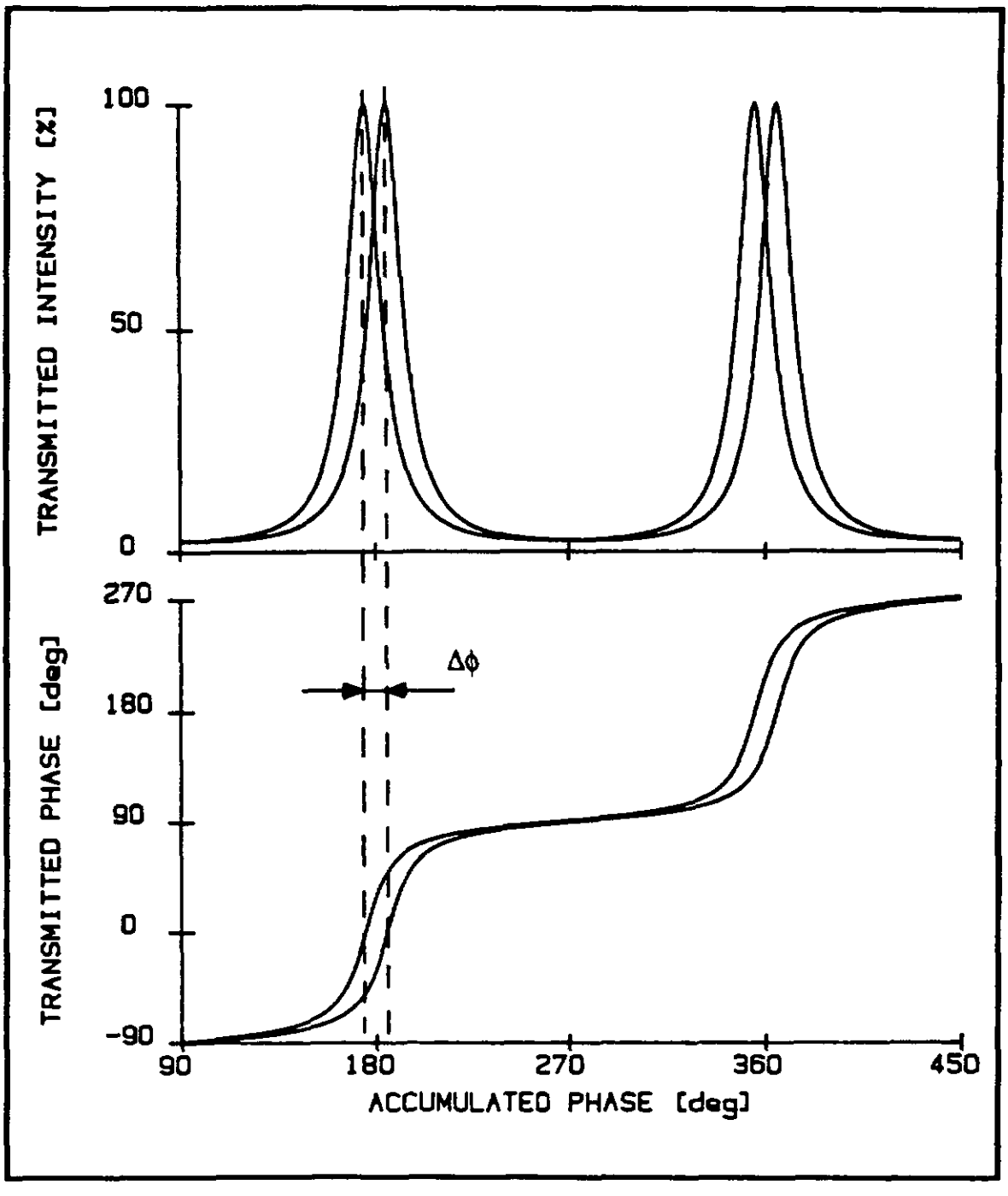
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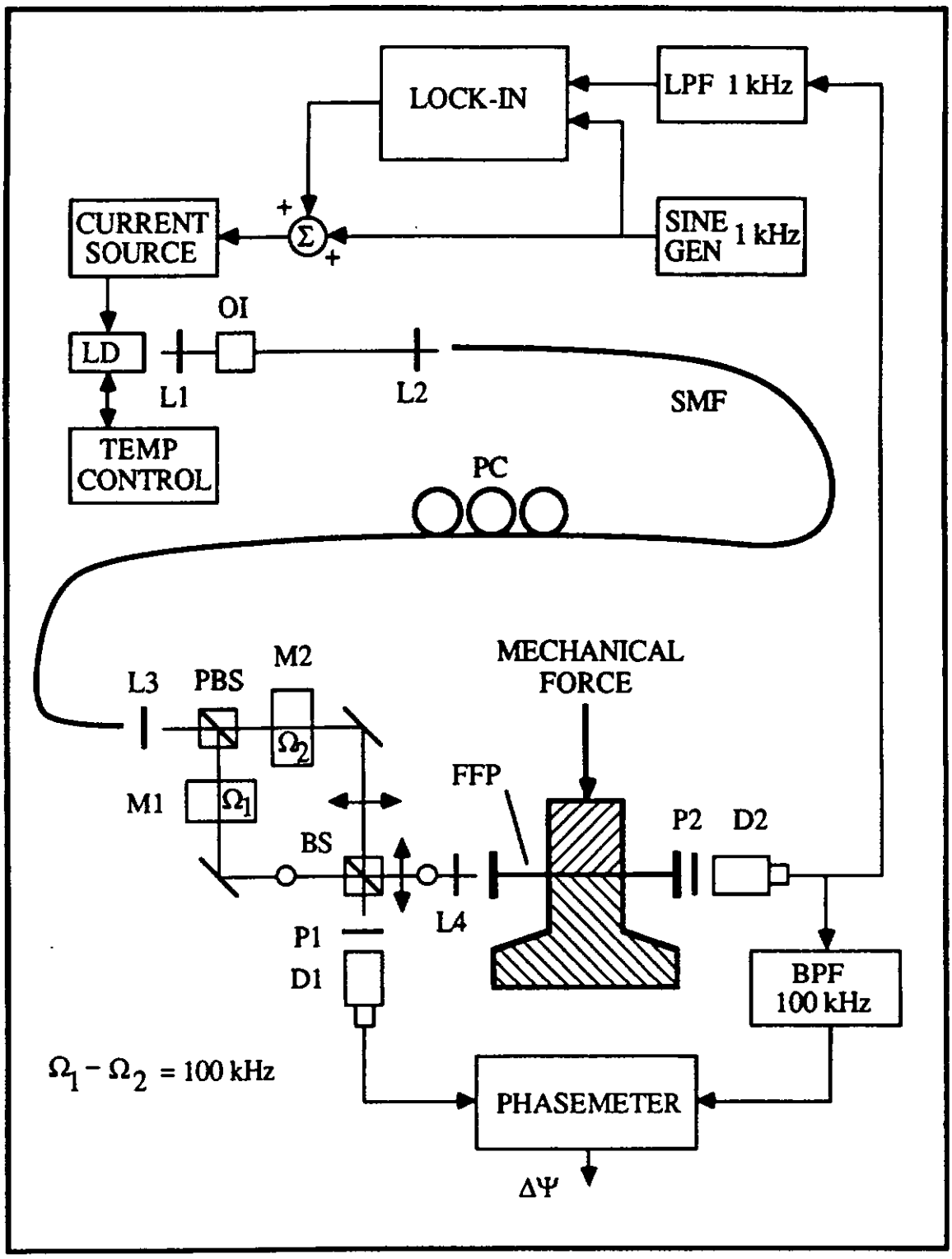
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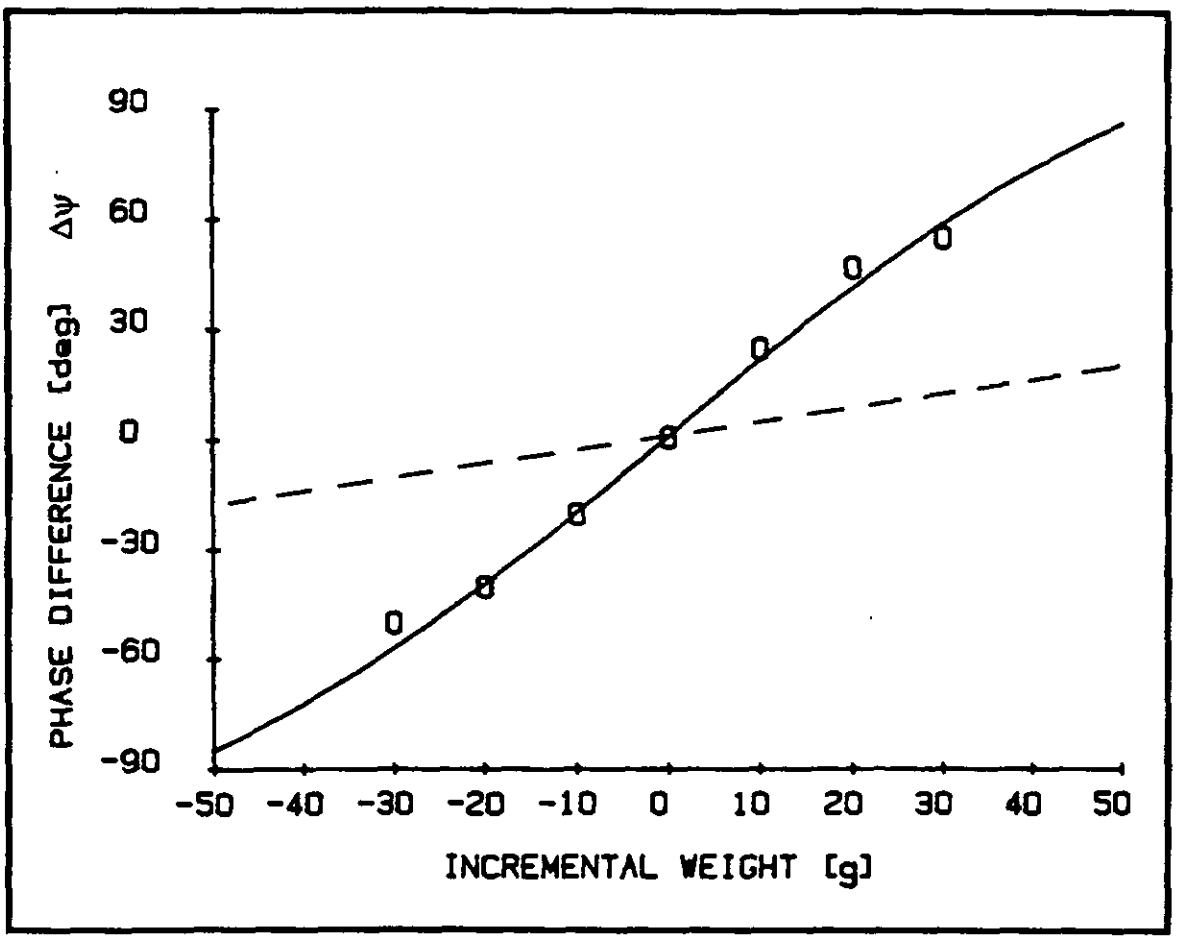
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Polarimetric fiber optical sensor with high sensitivity using a Fabry-Perot structure

François Maystre and Rene Dandliker

A novel polarimetric Fabry-Perot sensor concept, based on the phase detection of the transmitted light, is presented in detail. This concept has been successfully applied to measure static force by stress induced birefringence in an optical fiber with high sensitivity. The detection scheme consists of locking the optical frequency of a laser diode to a resonance peak, where the sensitivity is highest, and using heterodyne detection to measure the phase difference between the eigenpolarizations.

I. Introduction

Fiber optic polarimetric sensors are well known.[1,2] They have the basic property that the sensing element is a single piece of single-mode fiber and that all the perturbations which affect the length of the fiber are common to both eigenpolarizations, and therefore they do not perturb the measurement. As a major drawback, the sensitivity of the induced retardation to an external measurand is usually low (lateral force on a 125-μm diam fiber: 0.35 deg/g,[3] Faraday rotation: 1.7×10^{-4} deg/A·turn at 780 nm).[4] However, a number of early works on electrooptic modulators have demonstrated that the phase modulation can be greatly enhanced by inserting the electrooptic crystal in the cavity of an optical resonator.[5,6] Based on the same concept, it has recently been shown that the net phase shift between the two eigenpolarizations which pass through a birefringent medium can be enhanced by designing it as a Fabry-Perot resonator and maintaining the optical frequency close to a resonance peak.[7] In this paper, the relevant properties of birefringent fiber optic Fabry-Perot resonators are reviewed and a sensor concept using a resonance tracking loop and heterodyne polarimetry is presented. Experimental results for a static force sensor using stress induced birefringence will be presented and the performance and limitations of the detection scheme are discussed. Experimental data of a measurement system for static force using Fabry-Perot resonators of finesses 8 and 17 are presented.

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II. Theory

In this section, the basic theoretical properties of birefringent fiber optic Fabry-Perot resonators are presented. To begin with, the phase of the light reflected by and transmitted through a Fabry-Perot is discussed. Then the properties of a lossless birefringent cavity are discussed and the particular case of a fiber with low intrinsic birefringence pressed between two parallel plates is presented.

A. Phase Response of Fabry-Perot Resonators

Using the standard approach,[8,9] it can be shown that for the two incident waves which correspond to the two orthogonal eigenpolarizations of a fiber optic Fabry-Perot resonator, the phase added by reflection on the resonator is

$$\psi_R = \tan^{-1}[\rho \tan(\phi_i)] - \tan^{-1}[\mu \tan(\phi_i)], \quad (1)$$

and depends on the accumulated phase for a single pass through the resonator

$$\phi_i = (2\pi/c)\nu n_i d, \quad (2)$$

where c is the speed of light in vacuum, ν is the light frequency, d is the length of the resonator, and n_i ($i = 1,2$) is the refractive index of the medium for each of the two polarizations. The shape of the phase curve is given by the contrast factor

$$\rho^2 = 1 + 4F^2/\pi^2, \quad (3)$$

where F is the finesse of the resonator, and by

$$\mu = \frac{\sqrt{R_1}/(1 - A_1) + \sqrt{R_2}}{\sqrt{R_1}/(1 - A_1) - \sqrt{R_2}}, \quad (4)$$

where R_1 and R_2 are the reflectivities of the input and

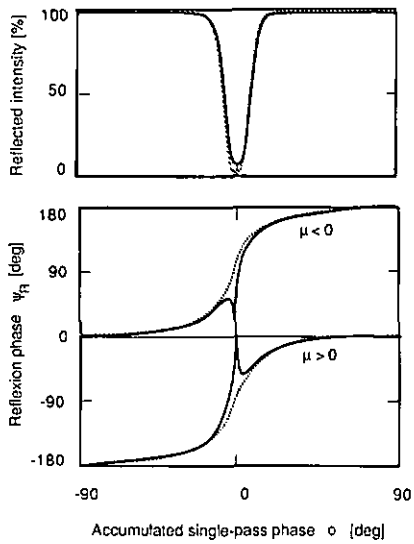

Fig. 1. Intensity and phase of a monochromatic wave reflected by a Fabry-Perot resonator of finesse $F = 10$ as a function of the accumulated single-pass phase ϕ. The cases of an input mirror with higher reflectivity than the output mirror $\mu > 0$ (e.g., $R_1 = 76\%$, $R_2 = 71\%$) and the opposite case $\mu < 0$ (e.g., $R_1 = 71\%$, $R_2 = 76\%$) are shown. The dashed line represents the limiting case of a symmetrical resonator ($\mu = \infty$).

output mirrors, respectively, and A_1 is the loss coefficient of the input mirror. Figure 1 shows the intensity and the phase of the light reflected by a Fabry-Perot resonator of finesse $F = 10$ for various configurations. Note that for the case of input mirrors of higher reflectivity than the output mirror ($\mu > 0$), the phase curve is fundamentally different from the opposite case ($\mu < 0$), where the input mirror is of lower reflectivity than the output mirror. Using Eq. (1) and assuming that $(\phi_1 + \phi_2)/2 = \phi_0 = m\pi$, it is readily verified that after reflection the phase difference between the two linearly polarized waves which correspond to the eigenpolarizations of the resonator reads

$$\Delta\psi_R = 2\tan^{-1}[\rho\tan(\Delta\phi/2)] - 2\tan^{-1}[\mu\tan(\Delta\phi/2)], \quad (5)$$

where $\Delta\phi = (2\pi/c)\nu(n_1 - n_2)d$ is the difference of the accumulated phases of the two waves.

In a similar way it can be shown that, at the output of the resonator, the total phase of each eigenpolarization transmitted through the resonator (Fig. 2) is given by

$$\psi_T = \tan^{-1}[\rho\tan(\phi_i)]. \quad (6)$$

Note that ψ_T depends, through ρ, only on the finesse F of the resonator, and not on μ, which accounts for the distribution of the reflectivities and the losses among the two mirrors. It follows that for $\phi_0 = m\pi$, the phase difference between the two eigenpolarizations at the output of the resonator is

$$\Delta\psi_T = 2\tan^{-1}[\rho\tan(\Delta\phi/2)]. \quad (7)$$

B. Polarimetric Fabry-Perot Sensor

When an optical fiber is pressed between two parallel plates, the stress distribution in the fiber material is anisotropic and adds linear birefringence to the core of the fiber.[3] Because of this anisotropy, the light that

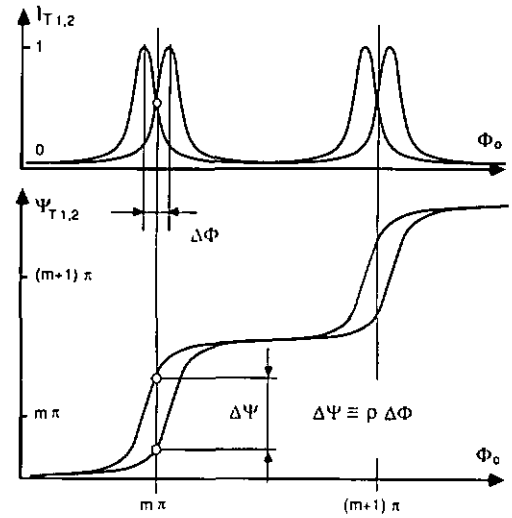

Fig. 2. Intensity and phase of the transmitted eigenpolarizations in a birefringent Fabry-Perot resonator of finesse $F = 10$ as a function of the mean accumulated single-pass phase.

propagates in such a fiber can be separated into two linear eigenpolarizations, oriented parallel and perpendicular to the direction of the force, with different phase velocities. For a force per unit length p on the fiber, the induced birefringence is given by[3]

$$\beta = (8C/\lambda r)p, \quad (8)$$

where C is the stress-optic coefficient ($C = -3.7 \times 10^{-12}$ m^2/N for silica), r is the fiber radius, and λ is the wavelength of the light. If the fiber has been mechanically twisted before being pressed, a circular birefringence α is also present in the core and amounts to[10]

$$\alpha = g\tau, \quad (9)$$

where g is the stress-optic rotation coefficient ($g = 0.13$ for silica), and τ is the mechanical rotation rate. The combination of linear and circular birefringence on the polarization properties of optical fibers has been described in detail[10] using coupled wave equations. Further, it can be shown that the resulting polarization properties of a piece of fiber of length z with constant linear and circular birefringence can be represented by a Jones matrix,[11] which reads

$$\mathbf{B} = \begin{pmatrix} \cos\gamma z - i(\beta/2\gamma)\sin\gamma z & (\alpha/\gamma)\sin\gamma z \\ -(\alpha/\gamma)\sin\gamma z & \cos\gamma z + i(\beta/2\gamma)\sin\gamma z \end{pmatrix} \quad (10)$$

in a linear polarization basis which is aligned with the axis of the linear birefringence. For this matrix, the eigenpolarizations are elliptical and do not depend on z, and the parameter $\gamma = [\alpha^2 + (\beta/2)^2]^{1/2}$ gives the polarization beat length $L_b = \pi/\gamma$.

In general, the eigenpolarizations of a Fabry-Perot resonator with a birefringent cavity are given by the eigenvectors of the matrix for one total round trip, and their accumulated round trip phase is obtained from the eigenvalues of that matrix, which can be written as

$$\lambda_{1,2} = \exp(\pm i\Delta\phi). \quad (11)$$

Using the first equivalence theorem for lossless birefringent elements,[12] it can be demonstrated (Appendix

A) that the eigenvectors of a matrix for one total round trip in a medium which contains any distribution of linear birefringence and reciprocal optical activity always correspond to two linear orthogonal polarizations. These linear polarizations are those which remain linear after a single pass through the medium. Their azimuth can be computed from the coefficients of the single pass matrix using the equivalence formulas given by Hurwiz and Jones.[12] Therefore, the eigenpolarizations of a birefringent Fabry-Perot resonator are always linear. The azimuth of these linear eigenpolarizations might be different at the input and at the output of the cavity. However, as soon as there is some nonreciprocal optical activity (i.e., Faraday rotation) in the cavity, the eigenmodes become elliptical.

In the particular case where the twist induced circular birefringence is much smaller than the linear birefringence, i.e., $\beta/2 \gg \tau$, the eigenpolarizations of the round trip matrix are nearly parallel to the axis of the linear birefringence, and the eigenvalues become

$$\lambda_{1,2} = \exp(\pm i\Delta\phi) = \exp(\pm i2\gamma z), \tag{12}$$

with $\gamma \cong \beta/2$. Thus, the phase difference $\Delta\phi$ for one round trip inside the resonator is linearly related to the total force $P = zp$ applied to the fiber, namely,

$$\Delta\phi = (8C/\lambda r)P. \tag{13}$$

For a silica fiber with an outer diameter of $2r = 125\ \mu$m, the sensitivity is $\Delta\phi/P = 35°/\text{N}$ at $\lambda = 780$ nm.[3] Figure 2 shows the intensity and the output phase of the eigenpolarizations transmitted through a birefringent Fabry-Perot resonator of finesse $F = 10$ as a function of the mean accumulated single-pass phase ϕ_0. The transmitted intensity consists of two identical Airy combs, separated by the phase difference $\Delta\phi$. Around the resonances, $\Delta\phi = m\pi$, $\Delta\psi$ depends linearly on $\Delta\phi$ and the slope, or sensitivity, is readily seen to be $\Delta\psi_T/\Delta\phi = \rho$ (for example, $\rho = 7$ for $F = 10$), instead of $\Delta\psi_T/\Delta\phi = 1$ in the case of a single-pass sensor. The sensor concept described here takes advantage of the steep sections of the phase functions and is based on the measurement of $\Delta\psi$ close to resonance. The detection scheme for this Fabry-Perot sensor concept consists therefore of two parts. The first part is a feedback loop, which locks the laser frequency to a resonance peak, where the sensitivity is maximum. The second part measures the phase difference $\Delta\psi$ accurately. Because of the large amplitude variations near the resonances, it is necessary to measure the phase between the two eigenpolarizations independently of their amplitudes, e.g., by heterodyne phase detection. Finally, the external perturbations affecting the length of the fiber are common mode and do not change the observed value of induced linear birefringence. Thus, the measurement is insensitive to temperature changes of the Fabry-Perot cavity.

III. Limitations

One important aspect of this sensor scheme is the fact that the sensitivity drops severely when the resonance conditions are not met. For example, only 50%

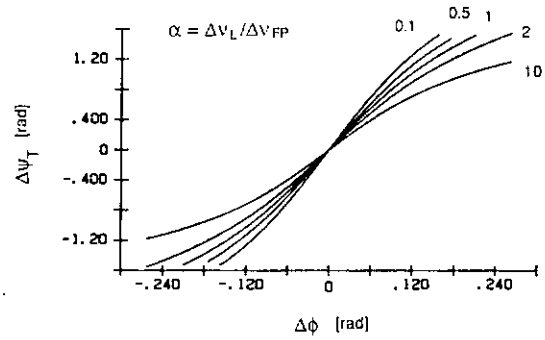

Fig. 3. Numerical calculation of the sensor response curve for a nonmonochromatic source. $\Delta\nu_L$ is the laser linewidth and $\Delta\nu_{F.P.}$ is the Fabry-Perot resonance width. The calculation is made for a resonator of finesse $F = 20$. The response for $\alpha = 0$ is the same as for $\alpha = 0.1$.

of the maximum sensitivity remains at $\Delta\phi = m\pi \pm \pi/F$. Therefore, although its response is periodic, the dynamic range of a Fabry-Perot sensor is limited to the resonance width, which is about $\Delta\psi = \pm 0.8$ rad ($\pm 45°$). The dynamic range for the measurand $\Delta\phi$ then becomes $\Delta\phi \cong \pm 1.2/F$, where F is the finesse of the Fabry-Perot. At the end of this dynamic range, the sensitivity $\Delta\psi/\Delta\phi$ is still ~85% of its maximum value at resonance and therefore the measurement requires only a small correction with respect to the linear relationship. Note that this condition assures at the same time that the amplitudes of the eigenpolarizations at the output are not too much attenuated by the transmission of the resonator. Finally, the relative resolution is determined by the accuracy $\delta\psi$ of the phase detection, which depends on the phase measurement technique and the SNR of the detector signals, compared to the dynamic range of $\Delta\psi$ given above.

Long and short Fabry-Perot resonators of high finesse can be produced from optical fibers. However, when the spectral width of the Fabry-Perot resonances $\Delta\nu_{F.P.}$ becomes comparable with the spectral width of the laser $\Delta\nu_L$ which is used as the light source, the ideal response of the measurement system is modified. Figure 3 shows numerical calculations of the response curve for a Fabry-Perot of finesse $F = 20$ and a laser with a Lorentzian spectrum for different values of the ratio $\alpha = \Delta\nu_L/\Delta\nu_{F.P.}$. The details of these calculations are reported in Appendix B. It can be seen that if α approaches 1, the response curve is flattened by averaging and the slope decreases. For example, at $\alpha = 1$ the maximum sensitivity corresponds to 75% of its value at $\alpha = 0$. In a practical measurement system, this effect limits the maximum finesse which can be used, or the length of the resonator (i.e., its free spectral range). Since free running single-mode laser diodes have spectral widths in the 20–40-MHz range, a 5-cm long resonator of finesse 20, which gives a resonance width of $\Delta\nu_{F.P.} = 100$ MHz, corresponds to a value of α between 0.2 and 0.4.

IV. Experimental Setup

The electronics for the control of the laser frequency is shown in the upper part of Fig. 4. The temperature

stabilized single-mode laser diode is frequency modulated at 1 kHz via the injection current over a range corresponding to a fraction of the resonance width. The error signal is extracted from the mean intensity of the two eigenmodes, detected by photodiode $D2$. This signal is proportional to the fundamental frequency component in the output intensity of the Fabry-Perot.[13] It is important to note that this frequency locking technique works properly as long as the superposition of the two resonance peaks in Fig. 2 gives a single maximum in transmission. It can be verified that the total transmitted intensity for the two eigenmodes will represent a single maximum only up to a phase difference of $\Delta\phi = \pm\pi/\sqrt{6}F \cong \pm 1.3/F$ rad.

The heterodyne polarization interferometer, which allows us to measure the phase difference $\Delta\psi$ of the two eigenpolarizations independent of their amplitudes, is shown in the lower part of Fig. 4. The light from the laser is split by the polarizing beam splitter (*PBS*) into two beams with orthogonal polarizations. Two Bragg cells, $M1$ and $M2$, shift the optical frequency in the two arms by 40.0 and 40.1 MHz, respectively, so that the beat frequency is 100 kHz, which is compatible with the electronic phasemeter used in the experiment. The two polarizations are recombined through the nonpolarizing beam splitter *BS*. The photodiode $D1$, placed behind a polarizer $P1$ oriented at 45°, supplies the heterodyne reference signal. The complementary beam is launched into the fiber Fabry-Perot resonator (*FFP*). Care is taken to ensure that the two orthogonal polarizations are aligned parallel and perpendicular to the principal axis of birefringence of the pressed fiber. At the output, the photodiode $D2$, placed behind a polarizer $P2$ oriented at 45°, detects the phase shifted heterodyne signal. The two signals are fed through a bandpass filter to the electronic phasemeter, where the phase difference $\Delta\psi$ is determined by zero crossing detection.

V. Experimental Results

High finesse fiber Fabry-Perot can be produced from a piece of single-mode fiber by carefully polishing the fiber ends and gluing properly oriented chip-size mirrors on both ends.[14] In this work, the measurements are made with two different Fabry-Perot cavities consisting of a 10-cm long single-mode fiber of low intrinsic birefringence (York LB 600), which gives 1-GHz free spectral range. One of them has two gold coated mirrors of 76% reflectivity and the other has one 76% mirror and one dielectric mirror of 97% reflectivity. The finesses of the resonators were measured to be $F = 8$ and $F = 17$, which is in good agreement with the theoretical value for the reflectivity of the mirrors. To perform the measurement, the fiber resonator is placed in a mechanical mounting that permits the application of a static transverse force on a short length of the fiber where the coating has been removed. The fiber is fixed without any mechanical torsion and a bias load is applied to have the two resonances of the eigenpolarizations exactly superimposed (i.e., $\Delta\phi = \pi$). Around this constant value, the measurements are

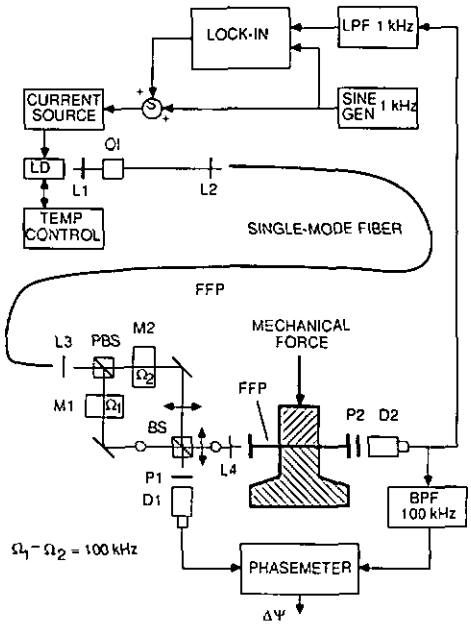

Fig. 4. Experimental setup for the measurement of the accumulated birefringence in a Fabry-Perot resonator (*FFP*) using heterodyne polarimetry: *LD*, laser diode; *L*, lenses; *OI*, optical isolator; *M*, acoustooptic modulators; *PBS*, polarizing beam splitter; *BS*, beam splitter; *P*, polarizers; *D*, detectors.

made by adding and removing calibrated weights. The laser diode is optically isolated from the Fabry-Perot resonator and emits at 780 nm a single line of 20-MHz width (FWHM), which gives $\alpha = 0.16$ for the resonator of finesse 8 and $\alpha = 0.34$ for the resonator of finesse 17. Efficient optical isolation of the laser diode is necessary to assure free running conditions and a sufficiently wide, continuous tuning range. As shown in Fig. 4, the light passes through a high birefringence single-mode fiber, which acts as a spatial filter and delivers a fixed linear polarization before entering the heterodyne polarimeter.

The experimental results are shown in Fig. 5. The experimental points follow closely the theoretical curves computed for Fabry-Perot resonators of finesses $F = 8$ and $F = 17$, made of a silica fiber of 125-μm o.d. The maximum slopes are 1.8 and 3.8°/g, which are 5 and 11 times larger, respectively, than the sensitivity of a single-pass sensor. The resolution for $\Delta\psi$ is limited to ±1° by instabilities of the heterodyne interferometer. Figure 6 shows the time response of the measurement system for the resonator of finesse 17, recorded with a 20-Hz bandwidth, when a weight of 10 g is applied and removed. Note that the residual noise at the output of the resonator is essentially the same as for the heterodyne interferometer alone, even for large phase shifts. As shown in this figure, the reproducibility is better than the resolution and, in standard laboratory conditions, the laser remained locked for hours.

VI. Conclusion

A novel polarimetric Fabry-Perot sensor concept, based on the phase detection of the transmitted light, has been successfully applied to measure static force

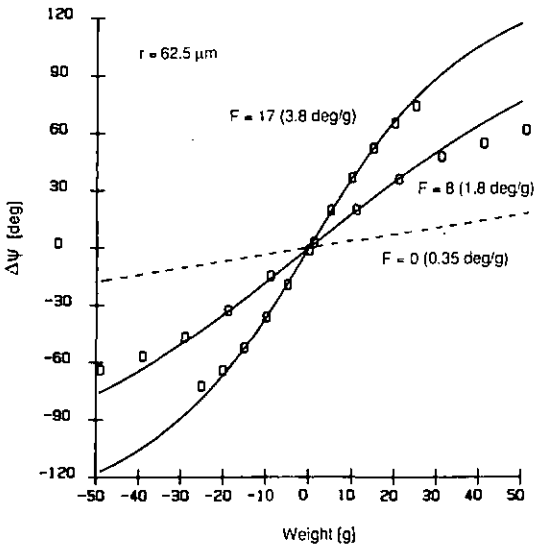

Fig. 5. Measured phase difference $\Delta\psi$ of the transmitted eigenmodes vs applied transverse load. Solid line: theoretical curves computed from Eq. (6) for a silica fiber of 125 μm outer diameter and resonators with finesses 17 and 8, respectively. Dashed line: single pass sensor for comparison. The circles are experimental data points.

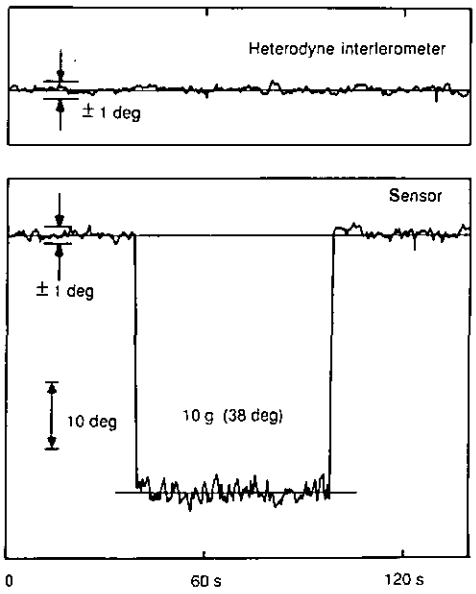

Fig. 6. Time response of the measurement system when a load of 10 g is applied and removed. The upper trace shows the noise of the heterodyne interferometer alone.

by stress induced birefringence in an optical fiber with high sensitivity. The detection scheme consists of locking the optical frequency of a laser diode to a resonance peak, where the sensitivity is highest, and using heterodyne detection to measure the phase difference between the eigenpolarizations. In some cases, it might be more convenient to use the Fabry-Perot in reflection rather than in transmission, since this requires access to only one fiber end. However, to have enough reflected intensity at the resonances, the Fabry-Perot should be asymmetrical and the reflectivities of both mirrors should be chosen so that they produce a suitable phase response curve in reflection. The requirements on the spectral properties of the source are severe, but numerical calculations have shown that if the spectral width of the light source is equal to the resonance width of the Fabry-Perot, the maximum sensitivity is still 75% of its value for a monochromatic source. Experimental results, using a free running single-mode laser diode and optical fiber Fabry-Perot resonators of finesses 8 and 17, have shown that the sensitivity to stress induced birefringence is increased by factors of 5 and 11, respectively, as predicted by the theory.

Parts of this paper were presented at the Conference on Optical Fiber Sensors, New Orleans, January 1988.

Appendix A

The first equivalence theorem[12] states that "for a light of a given wavelength, an optical system containing any number of retardation plates and rotators is optically equivalent to a system containing only two elements—one a retardation plate, and the other a rotator." It follows that the Jones matrix of any piece of optical fiber can be put in the form of

$$\mathbf{M} = \mathbf{A}\,\mathbf{R}\,\mathbf{B}\,\mathbf{R}^{-1}, \tag{A1}$$

where

$$\mathbf{A}(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}, \tag{A2}$$

$$\mathbf{R}(\theta)\mathbf{B}(\delta)\mathbf{R}^{-1}(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \exp(-i\delta) & 0 \\ 0 & \exp(i\delta) \end{pmatrix} \times \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{A3}$$

correspond to an equivalent optical activity α, and an equivalent birefringent retardation 2δ with its axis oriented at the angle θ with respect to the reference frame. The matrix for one total round trip in the Fabry-Perot must take into account one reflection at each mirror and therefore reads

$$\mathbf{U} = \mathbf{S}\,\mathbf{R}^{-1}\,\mathbf{B}\,\mathbf{R}\,\mathbf{A}\,\mathbf{S}\,\mathbf{A}\,\mathbf{R}\,\mathbf{B}\,\mathbf{R}^{-1}, \tag{A4}$$

where

$$\mathbf{S} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \tag{A5}$$

is the matrix of an ideal mirror, which takes into account the fact that the reflection changes the handedness of circular polarizations. Note that, because the optical activity is assumed to be reciprocal, the corresponding matrix is identical for both directions, forth and back. The product on the right-hand side of Eq. (A4) is readily carried out and gives

$$\mathbf{U} = \begin{pmatrix} \cos^2\theta\, e^{-i2\delta} + \sin^2\theta\, e^{i2\delta} & \sin\theta\cos\theta(e^{-i2\delta} - e^{i2\delta}) \\ \sin\theta\cos\theta(e^{-i2\delta} - e^{i2\delta}) & \cos^2\theta\, e^{i2\delta} + \sin^2\theta\, e^{-i2\delta} \end{pmatrix}, \tag{A6}$$

which can also be written as

$$\mathbf{U} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \exp(-i2\delta) & 0 \\ 0 & \exp(i2\delta) \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \tag{7}$$

and corresponds to a linear retardation of value 4δ with

its axis oriented at angle θ. The eigenvectors of this matrix correspond therefore to the two orthogonal linear polarizations with azimuth at θ and at $\theta + \pi/2$, and the corresponding eigenvalues are $\lambda_{1,2} = \exp(\pm i2\delta)$.

Appendix B

The eigenmodes of the resonator correspond to two waves V_1 and V_2 with linear orthogonal polarizations. Let us describe the corresponding optical fields at the input of the resonator by the Fourier representation of their analytical signals.

$$V_1(z = 0,t) = \int d\nu U_1(\nu) \exp(i2\pi\nu t), \tag{B1}$$

$$V_2(z = 0,t) = \int d\nu U_2(\nu) \exp(i2\pi\nu t), \tag{B2}$$

where the integration extends from $-\infty$ to ∞, and the $U_i(\nu)$ are the spectra of the complex amplitudes. The source is locked to a resonance of the Fabry-Perot and its spectrum is

$$U_L(\nu - \nu_0) = u_L(\nu - \nu_0) \exp[\varphi(\nu - \nu_0)], \tag{B3}$$

where ν_0 is given by

$$\nu_0 n_0 d2\pi/c = \phi_0 = m\pi, \tag{B4}$$

with $n_0 = (n_1 + n_2)/2$. Acoustooptic frequency shifters preserve the phase relations and therefore the spectra of the complex amplitude of the two fields are

$$U_1(\nu) = u_L(\nu - \nu_0 + \delta\nu/2) \exp[i\varphi(\nu - \nu_0 + \delta\nu/2)], \tag{B5}$$

$$U_2(\nu) = u_L(\nu - \nu_0 - \delta\nu/2) \exp[i\varphi(\nu - \nu_0 - \delta\nu/2)], \tag{B6}$$

where $\delta\nu/2$ accounts for the frequency shift produced by the acoustooptic modulators. At the output of the resonator, the fields become

$$V_i(z = d,t) = \int d\nu H_i(\nu) U_i(\nu) \exp(i2\pi\nu t), \quad i = 1,2. \tag{B7}$$

$H_i(\nu)$ is the complex transmittance of the Fabry-Perot

$$H_i(\nu) = a_i \exp(i\psi_i), \tag{B8}$$

where the amplitude and the phase are given by the expressions[8]

$$a_i(\nu) = \frac{T}{\sqrt{(1 - R)^2 + 4R \sin^2\phi_i(\nu)}}, \tag{B9}$$

$$\psi_i(\nu) = \tan^{-1}[\rho \tan\phi_i(\nu)], \tag{B10}$$

with $\phi_i = \nu n_i d2\pi/c$ and $\rho^2 = 1 + 4F^2/\pi$. The instantaneous intensity passing through a polarizer placed at 45° with respect to the orientation of the eigenpolarizations is

$$2I(t) = |\int d\nu H_1 U_1 \exp(i2\pi\nu t) + \int d\nu H_2 U_2 \exp(i2\pi\nu t)|^2. \tag{B11}$$

The corresponding electrical signal, detected with a limited bandwidth $f = 1/T$, can be written as

$$i(t) = 1/T \int_{-T/2}^{T/2} dt I(t) = \langle I(t) \rangle_\varphi, \tag{B12}$$

assuming that the process is ergodic and that $\delta\nu < 1/T < \Delta\nu_L$. Evaluation of this expression with $\delta\nu \ll \delta\nu_{\text{F.P.}}$, yields the following result:

$$\begin{aligned} i(t) \propto & \int d\nu a_1^2 u_L^2 + \int d\nu a_2^2 u_L^2 \\ & + \exp(i2\pi\delta\nu t) \int d\nu^2 a_1 a_2 u_L^2 \exp(i\Delta\psi) \\ & + \exp(-i2\pi\delta\nu t) \int d\nu^2 a_1 a_2 u_L^2 \exp(-i\Delta\psi), \end{aligned} \tag{B13}$$

where $\Delta\psi = \psi_1(\nu) - \psi_2(\nu)$ and $u_L^2 = |U_L(\nu)|^2$ is the power spectrum of the source. This expression can be put into the form of

$$i(t) = (I_1 + I_2)/2 + \text{Re}\{I_0 \exp(i2\pi\delta\nu t + \Psi)\}, \tag{B14}$$

with

$$\Psi = \arg|\int d\nu a_1 a_2 u_L^2 \exp(i\Delta\psi)|, \tag{B15}$$

where I_1 and I_2 correspond to the transmitted intensities for the two orthogonal modes. Finally, the measured phase Ψ from the beat signal at $\delta\nu$ is found to be explicitly

$$\tan\Psi = \frac{\int d\nu a_1 a_2 u_L^2 \sin(\Delta\psi)}{\int d\nu a_1 a_2 u_L^2 \cos(\Delta\psi)}. \tag{B16}$$

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Magneto-optic current sensor using a helical-fiber Fabry–Perot resonator

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The concept of a magneto-optic current sensor that employs a fiber-optic Fabry–Perot resonator shaped into a single-turn helix is presented. The helical shape permits construction of a compact device that has the required polarization properties. Compared with a single-pass sensor, a large increase in sensitivity is obtsined as a result of the finesse of the resonator. Experimental results confirm the theory and show that for a finesse of $F = 6$ the senaitivity is four times larger than for a single-pass sensor.

When an optical fiber is plunged into a properly oriented magnetic field, the Faraday effect on the fiber material produces a nonreciprocal circular birefringence in the core of the fiber. With this effect, ac and dc electric currents can be conveniently measured by placing a loop of fiber around the conductor.[1,2] Because the Verdet constant is very small in diamagnetic materials (e.g., $V \cong 3 \times 10^{-6}$ rad/A at $\lambda = 780$ nm for silica), optical fiber current monitors usually use fiber coils wrapped around the electrical conductor to increase the total optical rotation. In this Letter a new sensor is presented that uses a fiber-optic Fabry–Perot resonator shaped into a single-turn helix (Fig. 1). The increase in sensitivity is proportional to the finesse of the resonator, and the single-turn helix produces a short resonator that forms a complete loop and has the required polarization properties. For the measurement the optical frequency of a semiconductor laser ia locked to a resonance peak of the Fabry–Perot resonator, and the phase difference between the two eigenpolarizations of the resonator is then measured with heterodyne polarimetry.[3] Experimental data for a resonator of finesse $F = 6$ are reported. As expected, the measured sensitivity is four times greater than that for a single-pass configuration.

When an optical waveguide describes a nonplanar curve in space, the natural frame to describe the propagation of polarized light has been established by Tang,[4] who shows that the polarization does not follow the geometrical torsion of that curve. For a fiber of low intrinsic birefringence wound on a cylinder into a helical shape, the combination of the bend-induced birefringence, which follows the geometrical torsion, and the rotation of the polarization, which follows Tang'a frame, gives the same effect as a spun fiber with intrinsic birefringence (i.e., a fiber drawn from a rotating preform).[5,6] Detailed calculations show that in Tang's frame a piece of right-handed helical fiber of arc length s is represented in a circular-polarization basis by the Jones matrix

$$\mathbf{M}_c = \mathbf{N}_c \cdot \mathbf{R}_c = \begin{bmatrix} \cos \gamma s + i(\tau/2\gamma)\sin \gamma s & -i(\beta/2\gamma)\sin \gamma s \\ -i(\beta/2\gamma)\sin \gamma s & \cos \gamma s - i(\tau/2\gamma)\sin \gamma s \end{bmatrix} \times \begin{bmatrix} e^{-i\tau s} & 0 \\ 0 & e^{i\tau s} \end{bmatrix}, \quad (1)$$

where $\gamma = [\tau^2 + (\beta/2)^2]^{1/2}$. The geometric torsion τ of the helix is given by the pitch angle θ and the radius Q of the helix through the relation $\tau = \sin \theta \cos \theta/Q$. The bend-induced birefringence β is proportional to the square of tbe the radius of curvature $\kappa = \cos^2 \theta/Q$ of the helix and reads $\beta = (\pi n^3/2\lambda)(p_{11} - p_{12})(1 + \nu)(r \cos^2 \theta/Q)^2$, where λ is the wavelength and r is the radius of the fiber. The material parameters for silica fibers are $n = 1.45$, $(p_{12} - p_{11}) = 0.14$, and $\nu = 0.16$.[7] In the presence of a magnetic field the Faraday effect produces an additional circular birefringence that is nonreciprocal and modifies the matrix $\mathbf{N}_c$. In this matrix τ must then be replaced by $\tau - VH_s$, where H_s is the component of the magnetic field along the propagation direction of the light wave and is assumed to have a constant value along the fiber. For a Fabry–Perot resonator made from such a fiber the eigenpolarizations are given by the eigenvectors of the matrix for

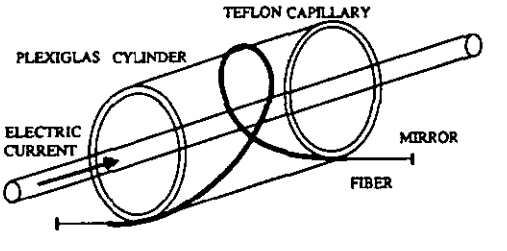

Fig. 1. Fiber-optic Faraday current sensor consisting of a Fabry–Perot resonator shaped into a single-turn helix. The cylinder has a diameter of 28 mm and a length of 52 mm.

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one total round trip, and their corresponding phases are obtained from the eigenvalues of that matrix. It is important to note that for $H = 0$ the eigenpolarizations of the resonator become degenerate when the condition $\gamma s = \pi$ is satisfied. In this particular case the phase difference after a round trip is exactly zero, and the eigenpolarizations of the single-pass matrix in Eq. (1) are circular. Provided that $(\beta/2)^2 \ll \tau^2$, the eigenpolarizations remain nearly circular for $H \neq 0$, and the eigenvalues for the round-trip matrix become $\exp(\pm i\Delta\gamma s)$, with

$$\Delta\gamma = [(\tau - VH_s)^2 + (\beta/2)^2]^{1/2} - [(\tau + VH_s)^2 + (\beta/2)^2]^{1/2} \cong -2(\tau/\gamma)VH_s, \quad (2)$$

where the approximation holds for $VH_s \ll \gamma$ and τ.

This sensor is designed to measure an electrical current I that flows through the helix, parallel to its axis. To be independent of its position with respect to the center, the length of the helix is chosen to make one complete loop, i.e., $s = 2\pi Q/\cos\theta$, so that the projection of the helix becomes a closed circle (Fig. 1). The errors introduced by the effects of different distributions of the electric current and the magnetic field are not addressed in this Letter. The accumulated phase difference between the two eigenpolarizations for one round trip in the resonator is $2\Delta\phi = 2\Delta\gamma s = 4(\tau/\gamma)VI$, since $H_s = I\cos\theta/2\pi Q = I/s$. The pitch angle of the helix is dictated by two conditions, $\gamma s = \pi$ and $s = 2\pi Q/\cos\theta$, which yield

$$4\sin^2\theta + [(\pi n^3/2)(p_{11} - p_{12})(1 + \nu)(r^2/\lambda Q)^2]^2 \times \cos^6\theta - 1 = 0. \quad (3)$$

For silica fibers of 80-μm outer diameter, a wavelength of 780 nm, and a helix diameter $Q > 10$ mm, the pitch angle θ is close to 30°.

At the output of the Fabry–Perot resonator the phase difference between the two transmitted eigenpolarizations can be written as

$$\Delta\psi = \psi_1 - \psi_2 = 2\tan^{-1}[\rho\tan(\Delta\phi/2)] + \delta, \quad (4)$$

with $\rho = (1 + 4F^2/\pi^2)^{1/2}$ (Ref. 3) and $\delta = 2\pi(1 - \tau/\gamma)$, which accounts for the single-pass phase shift between the transmitted eigenpolarizations according to Eq. (1). Near the resonances ($\Delta\phi = m\pi$) $\Delta\psi$ depends linearly on $\Delta\phi$, and the slope, or sensitivity, is readily seen to be $\Delta\psi/\Delta\phi = \rho$. The sensitivity is therefore increased by a factor ρ with respect to a single-pass senaor, e.g., $\rho = 7$ for $F = 10$, if one works close to the resonance.

The detection scheme for the Fabry–Perot sensor consists of two parts. The first part is a feedback loop, which locks the laser frequency to a resonance peak where the sensitivity is maximum.[8] The second part accurately measures the phase difference $\Delta\psi$. Because of the large amplitude variations near the resonance it is necessary to measure the phase difference between the two eigenpolarizations independently of their amplitudes, e.g., by heterodyne phase detection.[3] External perturbations affecting the length of the fiber are common mode and do not change the measured value of induced circular birefringence. Thus the sensor is insensitive to temperature changes of the Fabry–Perot cavity. However, the dynamic range of a Fabry–Perot sensor is limited to the resonance width, which is approximately $\Delta\psi = \pm 0.8$ rad (±45°). The dynamic range for the measurand $\Delta\phi$ becomes $\Delta\phi \cong \pm 1.2/F$, where F is the finesse of the Fabry–Perot resonator. At this point the sensitivity $\Delta\psi/\Delta\phi$ is still approximately 85% of its maximum value ρ at resonance. The relative resolution is determined by the accuracy $\delta\psi$ of the phase detection, which depends on the phase-measurement technique and the signal-to-noise ratio of the detector signals, compared to the dynamic range of $\Delta\psi$ given above. The finesse of the Fabry–Perot resonator can be chosen to match the desired dynamic range of the measurand. When silica fibers are used to measure electric currents with a dynamic range of ±2000 A, the gain factor should not exceed $\rho = 66$ [Eq. (4)], which gives a finesse of $F = 100$.

The experimental setup is shown in Fig. 2. Electric current measurements are made with a Fabry–Perot resonator (F-P) consisting of a 17-cm-long single-mode fiber of low intrinsic birefringence (York LB-800) and an outer diameter of 80 μm. As shown in Fig. 1, the fiber is bent into a single-turn helix of radius $Q = 14.3$ mm and pitch angle $\theta = 30°$. In order to avoid

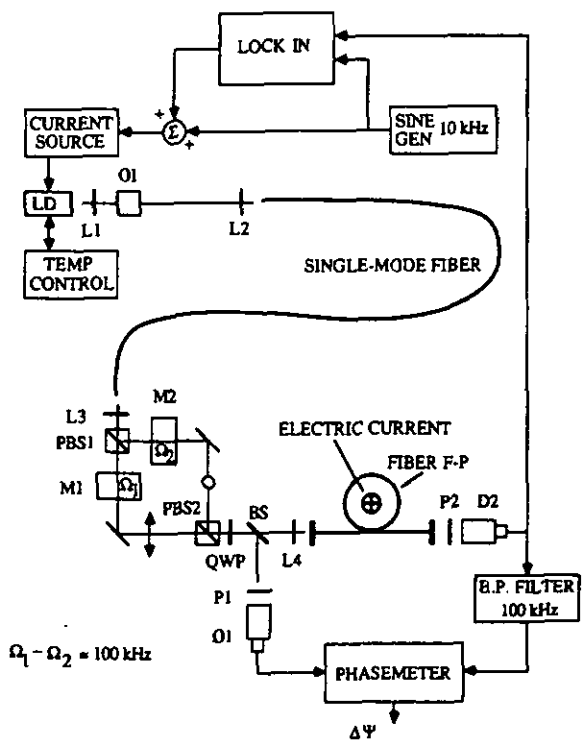

Fig. 2. Experimental setup that uses a feedback loop to lock the optical frequency of the laser to the Fabry–Perot resonance and beterodyne polarimetry to measure the phase difference between the two eigenpolarizations at the output of the resonator. LD, laser diode; L's, lenses; OI, optical isolator; PBS's, polarizing beam splitters; BS's, beam splitters; M's, acousto-optic modulators; P's, polarizers; QWP, quarter-wave plate; D's, detectors.

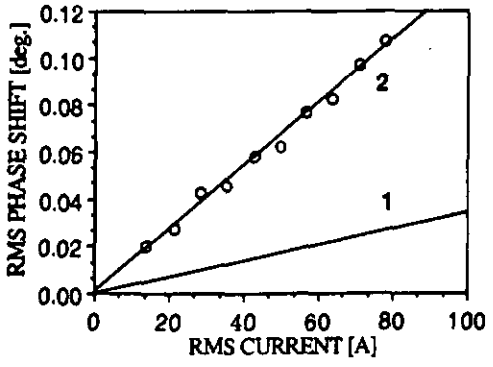

Fig. 3. Faraday-induced optical phase shift in the fiber-optic sensor. Curve 1, the response for a single pass through one loop of fiber; curve 2, the response for a Fabry–Perot resonator with a finesse of $F = 6.2$ compared with the experimental results (open circles).

mechanical torsion the fiber has its coating removed and is maintained in proper position by placing it inside a Teflon capillary that has been bent in the desired configuration and glued onto a Plexiglas cylinder. The geometric rotation rate provoked by the helix is $\tau = 30.2$ rad/m, and the bend-induced linear birefringence is $\beta = 4.4$ rad/m (i.e., $\beta/2\tau = 0.073$ and $\tau/\gamma \approx 0.997$). The linear retardation of the helix is measured to be smaller than 1° with the use of a high-resolution ellipsometer.[9]

The resonator is formed by a gold mirror of 58% reflectivity and a dielectric mirror of 97% reflectivity glued onto the well-polished end faces of the 17-cm-long fiber.[10] The free spectral range of the resonator is 600 MHz, and the finesse is measured to be $F = 6$, which is somewhat lower than the expected value. The laser diode is optically isolated from the Fabry–Perot resonator and emits a single line of 20-MHz width (FWHM) at 780 nm, which is approximately five times narrower than the Fabry–Perot resonance peaks. The current to be measured flows through the loop formed by the fiber resonator along a straight copper wire parallel to the axis of the helix (Fig. 1). Experimental results are shown in Fig. 3. The open circles represent experimental data points, and the straight lines correspond to the theoretical curves for a resonator with a finesse of $F = 6.2$ and a single pass through the same fiber coil (i.e., $F = 0$). At 43 Hz ac currents from 80 to 15 A rms are measured with a detection bandwidth of 1.45 Hz. It has been verified that the minimum measurable current corresponds to the noise floor produced by the amplifier of the output detector. This value is high because the large number of optical elements between the laser and the Fabry–Perot resonator add up to a low peak transmission of the resonator, so that the optical output power is only approximately 40 nW. Further improvements should increase the resolution by several orders of magnitude. The reproducibility is of the same order as the resolution, and careful electrical shielding of the electronics prevents any direct pickup from the current generator. The experimental points closely follow the theoretical curve for a Fabry–Perot resonator of finesse $F = 6.2$, and the slope is $\Delta\psi/I = 1.4 \times 10^{-8}$ deg/A, which is four times larger than the sensitivity of a single-pass sensor. This is in good agreement with the measured finesse of $F = 6$ for this resonator.

In conclusion, a helical-fiber Fabry–Perot resonator with circular eigenpolarizations in transmission has been constructed. It consists of a low-intrinsic-birefringence fiber wound into a single-turn helix of approximately 30-mm diameter and 30° pitch angle. The bend-induced birefringence is quenched by the rotation of the polarization owing to the geometrical torsion of the helical path. This setup has been used as a fiber-optic Faraday current sensor with high sensitivity. The detection scheme consists of locking the optical frequency of a laser diode to a resonance peak where the sensitivity is highest, and this scheme uses heterodyne detection to measure the phase difference between the eigenpolarizations. It has been verified experimentally that a resonator with a finesse $F = 6$ has a sensitivity of 1.4×10^{-3} deg/A, which is four times larger than for a single-pass configuration, as predicted by theoretical analysis.

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POLARIMETRIC RESONATOR SENSOR WITH HIGH SENSITIVITY AND LARGE DYNAMIC RANGE

Indexing terms: Optoelectronics, Birefringence, Optical sensors, Optical measurement

A new optoelectronic system which converts the variations of birefringence induced inside an optical resonator into an optical phase difference with a linear response over many periods is presented. This system has been applied to measure the static force by stress-induced birefringence in a fibre-optic Fabry–Perot resonator.

Introduction: In optical polarimetric sensors the sensitivity of the induced retardation to the external measurand is usually low (lateral force on a 125 μm-diameter fibre: 0·35 deg/g, Faraday rotation: $1{\cdot}7 \times 10^{-4}$ deg/A turn for silica at 780 nm).[1] The net phase shift between two eigenpolarisations which pass through a birefringent medium can be enhanced by placing it in an optical resonator. This technique has recently been demonstrated for optical fibre sensors.[1] However, the phase response is nonlinear and the dynamic range of direct phase detection is essentially limited to the width of the resonance. This letter presents a new optoelectronic system (Fig. 1), which converts the variations of the birefringence (linear or circular) inside an optical resonator (fibre or bulk, Fabry–Perot or other) into an optical phase difference with a linear response over a range of many periods.

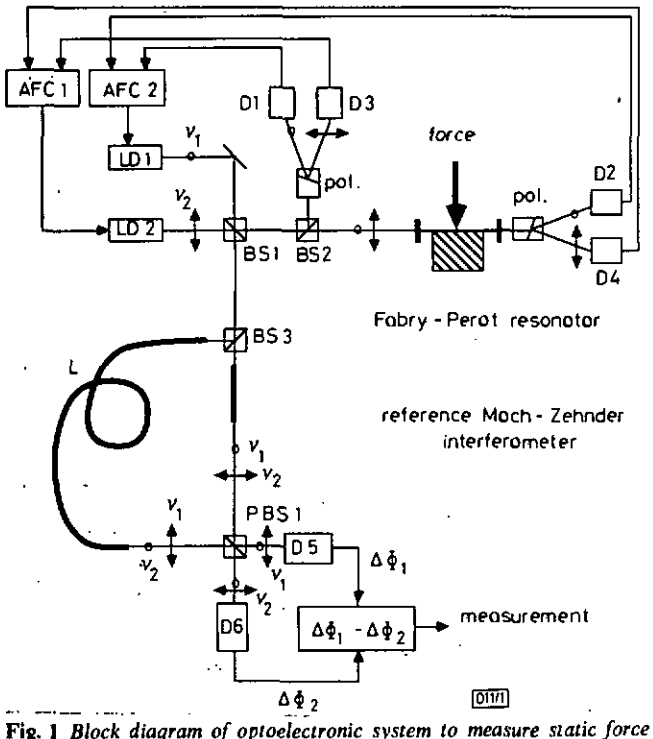

Fig. 1 *Block diagram of optoelectronic system to measure static force applied to fibre-optical Fabry–Perot resonator with high sensitivity and large dynamic range*

It combines high sensitivity and large dynamic range and it is therefore well suited for applications to high resolution transducers.

Principle: The system consists of three parts to perform the following steps:

(*a*) locking the frequencies of two lasers to two resonances of the two eigenpolarisations of the birefringent resonator.[1] As shown in Fig. 2, this procedure converts the relative position of the two resonances into an optical frequency difference.

(*b*) comparing the two optical frequencies in the same two-beam reference interferometer (e.g. Mach–Zehnder) with an appropriate optical path difference. This interferometer converts the optical frequency variations of each laser into phase variations with a linear response and identical scale factors.

Direct electronic detection of the beat frequency between the two lasers would avoid the reference interferometer, however the dynamic range is then limited by the bandwidth of the photodetector and the accuracy depends on the spectral width of both lasers.

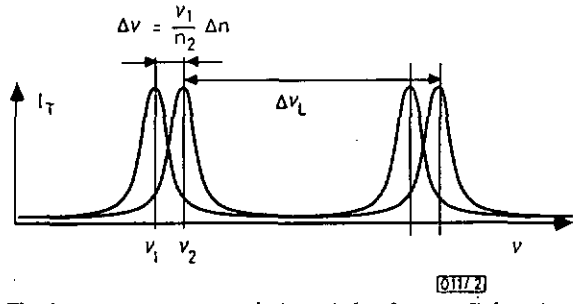

Fig. 2 *Intensity transmitted through birefringent Fabry–Perot resonator as function of optical frequency*

Retardation of single pass inside cavity is $\Delta n 2\pi d/\lambda$, where $\Delta n = n_1 - n_2$ is difference in refractive indices of eigenpolarisations, and d is length of resonator

(*c*) measuring the difference of the phase variations for the two optical frequencies at the output of the reference interferometer. A precise determination of the phase variations for both optical frequencies is accomplished by counting the periods and interpolating between them.

Fig. 1 shows a Fabry–Perot-type fibre-optical resonator and the optical arrangement for the conversion of the birefringence induced in the cavity into a linear phase difference. Note that the light reflected by the resonator could be used instead of the transmitted light. The system can be separated into two identical subsystems which are polarisation-encoded. The vertical linear polarisation is supplied by the laser source LD1. Its frequency ν_1 is locked to one of the two sensor resonance peaks by an automatic frequency control servo loop AFC1, which uses the detectors D1 and D2.[2] The beam splitter BS1 directs some of the light from LD1 to the reference Mach–Zehnder interferometer (BS3, PBS1). This interferometer has a nonzero optical path difference and converts the source frequency variation $\Delta\nu_1$ into a linear phase variation $\Delta\Phi_1 = (2\pi/c)L\,\Delta\nu_1$, where c is the speed of light and L is the optical path difference. For practical reasons, the Mach–Zehnder interferometer has to be made with polarisation-holding, high-birefringence fibres. To get exactly the same optical path difference for both eigenpolarisations, birefringence compensation[3] should be used. At the output of the two arms, the polarisations are adjusted so that the light of the source LD1 from the two arms of the reference interferometer is recombined at one output of the polarising beamsplitter PBS1. This can be done by rotating the output ends of the fibres so that the polarisations with the same optical frequency are orthogonal. An optoelectronic detection scheme D5, consisting of three detectors behind appropriate polarising elements to provide signals in quadrature,[4] is placed at this output to accomplish precise measurement of the phase $\Delta\Phi_1$ by counting the periods and interpolating the phase. The horizontal linear polarisation is supplied by the laser source LD2. Its frequency ν_2 is locked to the other resonance peak and produces finally a phase variation $\Delta\Phi_2$ at the other output of the reference interferometer, measured by the detector arrangement D6. The difference $\Delta\Phi_1 - \Delta\Phi_2 = (2\pi/c)L(\Delta\nu_1 - \Delta\nu_2)$, modulo 2π, corresponds exactly to the retardation produced by the birefringence inside the resonator cavity. Changes of the length of the Fabry–Perot resonator or of the reference interferometer have the same influence on $\Delta\Phi_1$ and $\Delta\Phi_2$; therefore they are compensated in the final result $\Delta\Phi_1 - \Delta\Phi_2$. The sensitivity of the frequency-to-phase conversion can be adjusted through the unbalance L of the reference interferometer.

Numerical example: An optical resonator with a finesse of 100 and an automatic frequency control system which locks the laser frequencies to within 1/100 of the resonance width allows one to determine the retardation, produced by the birefringence, with a resolution of $2\pi/10000$ over many periods. This

can be achieved with commercial single-frequency laser diodes as sources. For a resonator of finesse 100 and a free spectral range of 2 GHz (e.g. a 5 cm-long fibre Fabry–Perot) the resonance width is 20 MHz, which is about the same as the linewidth of CW single-mode AlGaAs lasers.[1] Stabilisation to 1/100 of the resonance then corresponds to 200 kHz. Assuming that the accuracy for the phases of the reference interferometer is 1°, an unbalance of $L = 4$ m is necessary to resolve a frequency change of $\Delta\nu_{1,2} = 200$ kHz. This optical path difference is still within the coherence length of the laser diodes. Since the two optical frequencies are not correlated, they do not interfere and the accuracy of the phase measurement is determined by the intensity cross-talk. The state-of-the-art for polarisation-maintaining fibres and couplers gives about −35 dB cross-talk, limiting the accuracy of 0·02°.

The dynamic range of the system depends on the number of periods of the resonator which can be covered by continuously tuning the laser frequency. Short resonators with a large free spectral range relax the requirement on the source coherence but increase the requirement on tuning range. The typical tuning range for Fabry–Perot-type single-mode laser diodes is 10 GHz by changing the current and 100 GHz by changing the case temperature.

Experiments: To demonstrate the principle of the transducer, the birefringence induced by a transverse force applied on a bare optical fibre was measured. A Fabry–Perot resonator with a free spectral range of $\Delta\nu_L = 850$ MHz and a finesse of 10 was constructed from a 125 μm diam. fibre with low intrinsic birefringence (York LB600) by gluing gold-coated mirrors on both ends.[5] Two well-isolated single-mode laser diodes (Hitachi HL 7801, $\lambda \cong 780$ nm) provided the two orthogonal linear polarisations corresponding to the eigenpolarisations of the resonator. The linewidth of each laser was measured to be about $\delta\nu = 20$ MHz at 5 mW, which gives a coherence length $L_c = c/n\delta\nu$ of 5 m. To reduce the effect of dispersion in the resonator and also in the reference interferometer, the lasers were carefully selected and operated at temperatures where they have the same wavelength. This was verified with a grating spectrum analyser of 0·05 nm resolution. PI controllers with about 20 kHz bandwidth maintained the optical frequencies within 1/100 of the resonance width (i.e. 0·85 MHz) under normal laboratory conditions. The unbalanced Mach–Zehnder reference interferometer uses polarisation-maintaining fibres (York HB600). At the output, the cross-talk between the two lasers was −20 dB, which limits the accuracy of the interpolation to $2\pi/600$. The optical path difference is 3 m, which gives a sensitivity for the frequency-to-phase conversion of $2\pi/100$ MHz. Fringe counting and interpolation to $2\pi/1000$ is performed by two synchronously triggered fringe counters (FT-Technologies 612 AS).

Fig. 3 shows experimental results obtained with this set-up. Open circles represent measured values of phase difference at the output of the Mach–Zehnder as a function of the weight applied to the fibre. The straight line corresponds to the theoretical value for a silica fibre with a diameter of 125 μm[1] and a Mach–Zehnder with an optical path difference of 3 m. The dashed line corresponds to the retardation induced inside the cavity. It shows that the frequency-to-phase conversion system increases the sensitivity by a factor of 15. The resolution was only about $2\pi/20$, mainly limited by mechanical instabilities of the recombination of the two beams at the output of the Mach–Zehnder. This problem can be avoided by using an all-fibre polarising coupler instead of the polarising beam splitter PBS1.

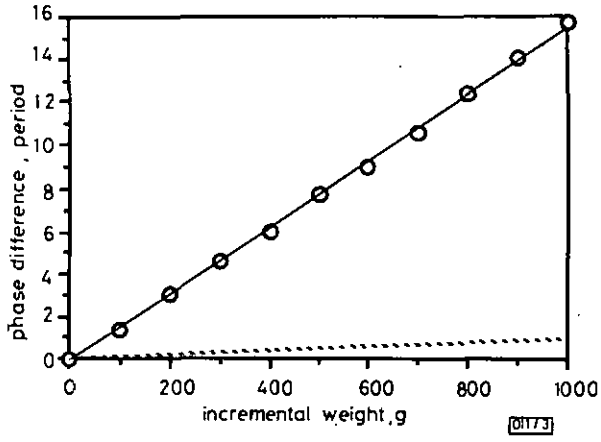

Fig. 3 *Experimental results*
----- induced retardation inside cavity for 125 μm silica fibre
——— theoretical response for system shown in Fig. 1 with 3 m optical path difference of the reference interferometer (○: measured values)

Conclusion: The presented concept of the optoelectronic detection scheme can be applied to any transducer based on the change of birefringence inside an optical resonator. For transducers with precision in the ppm range, all-fibre systems using high-birefringence fibres and all-fibre components can provide the required mechanical stability.

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