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Heterogeneous or homogeneous? Implications of simplifying heterogeneous streambeds in models of losing streams

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SUMMARY

A common approach in modeling surface water–groundwater interaction is to represent the streambed as a homogeneous geological structure with hydraulic properties obtained by means of model calibration. In reality, streambeds are highly heterogeneous, and there are currently no methodical investigations to justify the simplification of this geologic complexity. Using a physically based numerical model, synthetic surface water–groundwater infiltration flux data were generated using heterogeneous streambeds for losing connected, losing transitional and losing disconnected streams. Homogeneous streambed hydraulic conductivities were calibrated to reproduce these fluxes. The homogeneous equivalents were used for predicting infiltration fluxes between streams and the aquifer under different hydrological conditions (i.e. for different states of connection). Homogeneous equivalents are shown to only accurately reproduce infiltration fluxes if both the calibration and prediction are made for a connected flow regime, or if both the calibration and prediction are made for a disconnected flow regime. The greatest errors in flux (±34%) using homogeneous equivalents occurred when there was a mismatch between the flow regime of the observation data and the prediction. These errors are comparatively small when compared with field measurement errors for hydraulic conductivity, however over long river reaches these errors can amount to significant volumes of water.

Keywords
Groundwater/surface water interaction, Streambed heterogeneity, Losing streams, Disconnection, Numerical modeling, Inverse modeling

1. Introduction

The joint management of surface water (SW) and groundwater (GW) resources requires a solid and quantitative understanding of the interaction between the SW and GW (Woessner, 2000). Typically, these interactions are quantified using numerical models. Numerical models are powerful and versatile tools in the study of SW–GW interaction. As pointed out by (Fleckenstein et al., 2006; Doppler et al., 2007), most modeling approaches do not account for streambed heterogeneity. Instead, the streambed is often conceptualized as a homogeneous geologic structure with properties obtained through model calibration. This simplification is typically undertaken because quantifying streambed heterogeneity in the field is challenging. For example, it has been demonstrated that the hydraulic conductivity across a streambed can vary over several orders of magnitude (Calver, 2001), which poses significant practical problems for any field approach. The problem is further complicated, because erosion/deposition events (Hatch et al., 2010), biological activities (Treese et al., 2009) and temperature dependent material properties (Engeler et al., 2011) change the properties of the streambed in time.

While several sources of error in modeling surface water–groundwater interaction (such as neglecting the unsaturated zone) have been discussed in previous papers (e.g. Brunner et al., 2010), the implications of replacing the complexity of streambeds with homogeneous equivalents have so far not been explored.

Our investigation focuses on losing connected, losing transitional and losing disconnected flow regimes. In Fig. 1, the relation between depth to the water table and the infiltration flux is shown for a hypothetical stream–aquifer system. Losing connected flow is present if the stream loses water to the aquifer, and the flow between SW and GW remains saturated. In this regime, the relation between depth to the water table and the infiltration flux is approximately linear. If the water table is lowered, it is possible that an unsaturated zone develops beneath the stream (transitional regime, also referred to as the transition zone). By further lowering the water table, the infiltration flux asymptotically approaches a maximum value. Once this value is reached, the infiltration flux is independent of a further decrease of the groundwater level, and the SW–GW system is considered disconnected.

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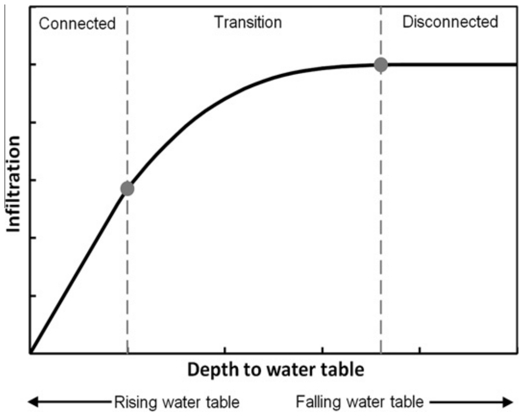

Fig. 1. Flow regime and infiltration flux plotted for different depths to the water table. For a connected regime, the infiltration flux will increase linearly as a function of the change in head. The flow regime becomes transitional when the flow beneath the streambed becomes variably saturated. In this flow regime, the relationship between the infiltration flux and depth to water is non-linear. Finally, a point is reached where further lowering the water table no longer significantly influences the infiltration flux. At this point the surface water and groundwater are disconnected.

A range of studies on the physics of disconnection have been carried out using homogeneous streambeds (e.g. Fox and Durnford, 2003; Brunner et al., 2009a,b; Banks et al., 2011). Fox and Durnford (2003) investigated the formation of an unsaturated zone beneath a partially penetrating stream caused by nearby groundwater extraction. Banks et al. (2011) analyzed the impact of land clearance and revegetation on the state of connection for perennial streams. Brunner et al. (2009a) outlined a criterion which can be used to determine whether a 1D flow system can become disconnected. The criterion uses the hydraulic conductivity of the streambed (K_{sb}), thickness of the streambed (h_{sb}), the hydraulic conductivity of the aquifer (K_a) and the depth of ponded water (d) to identify whether or not disconnection is possible. Disconnection can occur if the following criterion is met:

$$\frac{K_{sb}}{K_a} \leqslant \frac{h_{sb}}{d + h_{sb}} \tag{1}$$

A detailed discussion on the geometric, hydrogeological and temporal controls of the transition from a connected to a disconnected flow regime was presented by Brunner et al. (2009b).

Studies that consider heterogeneity in disconnected rivers were presented by Fleckenstein et al. (2006) and Frei et al. (2009) who investigated the influence of heterogeneity on the SW–GW interactions of the Consumnes River in California. These studies demonstrated that in losing perennial rivers, heterogeneity allows for the simultaneous occurrence of saturated and unsaturated flow, independent of the depth to the adjacent water table. It is important to point out that a disconnection can also occur in heterogeneous systems where saturated and unsaturated zones can be present simultaneously. In the review by Brunner et al. (2011) on disconnection, this simultaneous occurrence of saturated and unsaturated flow was discussed in the context of disconnection.

While the papers of Fleckenstein et al. (2006) and Frei et al. (2009) have illustrated how saturated and unstaturated flow can occur jointly in heterogeneous environments, a systematic analysis of the implications of representing a heterogeneous streambed with a homogeneous equivalent has not yet been carried out. Here, we conduct synthetic experiments to investigate the role that streambed heterogeneity plays in SW–GW interactions for losing connected, transitional and disconnected streams. We further quantify the implications of simplifying the streambed through the use of homogeneous equivalents in predicting infiltration fluxes under changing hydrological conditions. As we will show, the transitional regime plays a key role in determining the implications of replacing a heterogeneous streambed with a homogeneous equivalent. We will also demonstrate that determining the state of connection between streams and aquifers can allow the identification of the maximum errors associated with representing a heterogeneous streambed with a homogeneous equivalent.

2. Numerical modeling

Including the effects of unsaturated flow is a prerequisite to accurately simulate the transition from connected to disconnected flow regimes (Brunner et al., 2011). We have chosen HydroGeoSphere (Therrien et al., 2006) for this study as this code is able to simulate three-dimensional variably saturated subsurface flow. The full capabilities of HydroGeoSphere are outlined in a review by Brunner and Simmons (in press). To represent streambed heterogeneity, we used GCOSIM3D (Gómez-Hernández and Journel, 1993) to generate $\log_{10}$ normally distributed MultiGaussian geostatistical K-fields.

2.1. Conceptual model and spatial discretization

To isolate the effect of streambed heterogeneity on the SW–GW exchange, we deliberately chose a simple conceptual model. The conceptual model (Fig. 2) is a 20 m long section of a stream that is 20 m wide. The stream is represented by a constant head boundary, where $d = 0.5$ m, $h_{sb} = 0.5$ m. To isolate the influence of heterogeneity, we chose a rectangular shape for the river, because sloping banks can significantly influence SW–GW exchange (Doble et al., in press). This rectangular shape of the river and the streambed is also necessary to ensure that the depth of surface water is constant throughout the stream. No regional gradient was imposed and our analyses were conducted in steady state.

Throughout the study, typical van Genuchten parameter values for loam ($\alpha = 3.6\ \text{m}^{-1}$ and $\beta = 1.56$) were used for the streambed, and typical values for sand ($\alpha = 14.5\ \text{m}^{-1}$ and $\beta = 2.68$) were used for the aquifer (Carsel and Parrish, 1988). While K_a and aquifer heterogeneity will also influence SW–GW interaction (e.g. K_a appears in Eq. (1)), the properties of the aquifer are held constant at $1\ \text{m day}^{-1}$ throughout the study, as the focus here is on simplification of the streambed heterogeneity only.

To ensure grid independent results, the three-dimensional model domain was discretized using variable rectangular elements that allow for a sufficiently fine grid surrounding the stream, with 76 elements across the model domain (x), 40 elements along the length of the stream (y), and 73 elements with depth (z), totaling 221,920 elements. The streambed was uniformly discretized into 28,160 elements with element dimensions of 0.5 m, 0.5 m, and 0.03125 m in the x, y and z directions respectively.

2.2. Generation of synthetic data and calibration process

The flow of stream water to the aquifer was forced by lowering the lateral constant head boundaries (Fig. 2). The boundary conditions used in our simulations are as follows: no flow boundaries for the top, bottom, and both x–z faces. The lateral constant head boundaries were lowered in 0.5 m increments, and the steady state hydraulic head distribution and the corresponding infiltration flux were calculated for each 0.5 m increment. The process of systematically lowering the lateral constant head boundaries was repeated

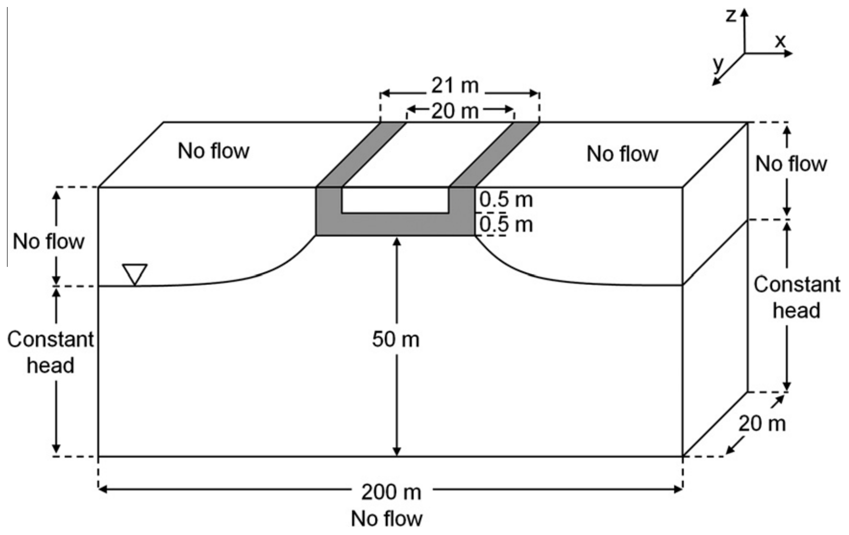

Fig. 2. Conceptual model with boundary conditions and dimensions (example shown for a hydraulically connected system). The streambed is shown in gray. The dimensions and boundary conditions are symmetrical. Note that the figure is not drawn to scale.

until the water table was lowered by 20 m at the boundary. The simulations were summarized in graphs plotting the infiltration flux (L^3T^{-1}) through the streambed as a function of the depth to the water table (DTW) at the lateral model boundaries. These plots are from now on referred to as infiltration curves.

There are numerous methods for representing spatial heterogeneity, for extensive reviews refer to Koltermann and Gorelick (1996) or de Marsily et al. (1998). Heterogeneity in streambeds can be represented through Gaussian statistics, although some researchers have used more advanced approaches that consider the connectivity of streambed deposits (e.g. Fleckenstein et al., 2006). In our study, we represented the streambed using Gaussian statistics and argue that the method chosen to represent heterogeneity does not affect our conclusions. We will show that a key to understanding the simplification of heterogeneity is whether or not an unsaturated zone can develop under the entire streambed, or if some areas remain saturated. The first order control on this key feature of streambeds is not the spatial distribution but the absolute values of hydraulic conductivity (equation 1).

Synthetic and unconditional realizations of streambed $\log_{10}K$ were generated to represent a range of possible streambed structures. In all cases, we adopted a spherical variogram, but with different sill and range values. Two different sill values were considered; one corresponding to a mild heterogeneity ($\sigma^2_{\log_{10}\mathrm{K}} = 0.1886$) and one corresponding to a comparatively stronger heterogeneity ($\sigma^2_{\log_{10}\mathrm{K}} = 1$). For both degrees of heterogeneity, variations of the correlation lengths in the horizontal (τ_h, i.e. $\tau_x = \tau_y$) direction with values of 2.5 m and 10 m were considered; as well as variations of the correlation length in the vertical (τ_z) direction with values of 0.25 m, and vertically homogeneous layers (i.e. $\tau_z = \infty$). For the stronger heterogeneity scenarios, we also tested an intermediate horizontal correlation length (τ_h = 5 m) and a smaller vertical correlation length (τ_z = 0.125 m). The combinations of τ_h/τ_z used ranged between 0 (where $\tau_z = \infty$) to 80. See Table 1 for an overview. For each geostatistical scenario, 10 realizations were simulated. The restriction of the number of realizations and number of correlation lengths used in the study does not allow for a full stochastic analysis; however it does allow us to demonstrate the influence of streambed heterogeneity on possible flow behaviors.

Table 1
The considered geostatistical simulation scenarios. For each scenario, 10 stochastic realizations were considered. L denotes low variance scenarios and H denotes high variance scenarios.

Scenario	Mean K (m day^{-1})	$\sigma^2_{\log_{10}\mathrm{K}}$	τ_h (m)	τ_z (m)	τ_h/τ_z
L1	0.1	0.1886	2.5	0.25	10
L2	0.1	0.1886	2.5	∞	0
L3	0.1	0.1886	10	0.25	40
L4	0.1	0.1886	10	∞	0
H1	0.1	1	2.5	0.25	10
H2	0.1	1	2.5	∞	0
H3	0.1	1	10	0.25	40
H4	0.1	1	10	∞	0
H5	0.1	1	10	0.125	80
H6	0.1	1	5	∞	0

To determine a homogeneous equivalent for a given DTW, the K_{sb} of a homogeneous and isotropic streambed was adjusted until it reproduced the infiltration flux for a given DTW. This calibration method mimics calibration methods which use field data of infiltration fluxes (infiltration fluxes can be estimated on the basis of two stream gauging stations) and the DTW in a nearby observation borehole. In all cases, the models were considered to be calibrated once the infiltration fluxes differed by less than 1×10^{-4}% between the heterogeneous realization and the homogeneous equivalent.

The calibration process (matching infiltration flux with a homogeneous K_{sb} for a given DTW) was repeated for a range of DTWs for each heterogeneous realization. The homogeneous K_{sb} obtained from the calibrations were used in forward simulations to generate the infiltration curves. This procedure was performed for each of the 10 realizations for each geostatistical scenario. The infiltration curves obtained using the homogeneous equivalents were compared to the infiltration curves based on the heterogeneous streambeds. Any mismatch between the infiltration curves represents an error in the prediction of fluxes using the homogeneous equivalent.

3. Results and discussion

3.1. Comparison between infiltration curves of homogeneous equivalents and heterogeneous streambeds

Examples of the predicted infiltration fluxes from the calibration of homogeneous equivalents of a heterogeneous streambed are presented in Fig. 3. The homogeneous K_{sb} value that reproduces

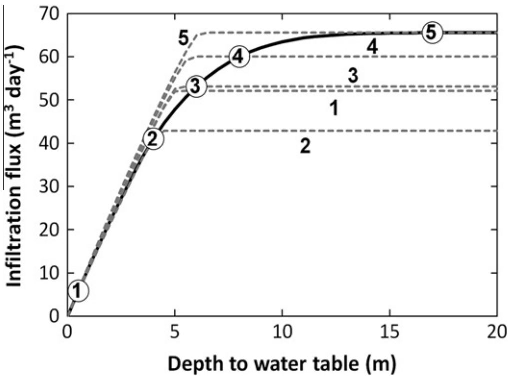

Fig. 3. Infiltration curve for a heterogeneous streambed (solid black line) with $\tau_h = 10$ m and $\tau_z = 0.25$ m and $\sigma^2_{\log K} = 1$ (H3 scenario, realization 1). The circled values coincide with calibrations performed at different DTWs. The numbered dashed gray lines are the infiltration curves of the homogeneous equivalents for each of these calibrations.

the infiltration flux for a given DTW was calibrated at five different DTWs. The homogeneous equivalent K_{sb} values obtained via calibration differed depending on the DTW that was used to generate the data point for the calibration. For the realization presented in Fig. 3, the K_{sb} values calibrated ranged between 0.0488 m day^{-1} for an observation taken at a DTW of 4 m, to 0.0750 m day^{-1} for an observation taken at a DTW of 17 m.

Fig. 3 demonstrates that depending on the DTW (from which the observation was taken and used in model calibration), the resulting infiltration curves can be above or below the infiltration curve based on the heterogeneous streambed. Also, in some cases the homogeneous equivalent can reproduce parts of the original data. For example, if the K_{sb} was calibrated for a DTW of 4 m (Fig. 3, point 2), chosen at a depth where the system is connected, but close to transition, and predictions were made for a rising water table (i.e. the value on the *x*-axis decreases), the homogeneous equivalent will represent the infiltration flux accurately. However, if the prediction for a falling water table is based on the observation at point 2, the calibrated model will under-predict the infiltration flux, by as much as 34.7% for the conditions employed in this instance.

Similarly, if the homogeneous K_{sb} value was calibrated at a DTW of 17 m (Fig. 3, point 5, disconnected flow regime), and the predictions are made for a falling water table, the error is negligible. However, if predictions based on this observation are made for a rising water table, the calibrated model only predicts the infiltration flux accurately as long as the heterogeneous streambed remains disconnected. Once the water table is shallow enough that the heterogeneous streambed moves into a transitional regime, the infiltration curves deviate.

In Table 2, the influence of flow regime, the direction of the movement of the water table and error in prediction are summarized across all of the heterogeneous scenarios.

Table 2 illustrates the following important points:

(1) If the observation data are obtained for a connected or disconnected flow regime, and the flow regime does not change for the prediction, the homogeneous equivalents perform extremely well.

(2) If the observation data are obtained for a connected flow regime, and a falling water table is simulated, the prediction of fluxes becomes erroneous. Likewise, if the observation was obtained for a disconnected regime, and a rising water table is simulated, the prediction of fluxes also becomes erroneous.

(3) If the observation data were obtained for a transitional flow regime, errors are expected for both rising and falling water tables.

Fig. 4 presents an alternative way to summarize how well the homogeneous equivalents perform. In this figure, the mean maximum over- and under-predictions as well as the maximum and minimum values themselves are shown for each geostatistical scenario.

Table 2
Range of under- and over-predictions of infiltration fluxes according to the flow regime while calibrated and direction of water table movement.

Stream connectivity when calibration performed	Water table movement	Prediction error range (%)	Mean prediction error (%)
Connected	Rising	<0.1	<0.1
	Falling	−27.71 to 34.53	−6.10 to 5.14
Transition	Rising	−11.91 to 16.70	−1.46 to 0.78
	Falling	−34.67 to 7.71	−9.04 to 0.21
Disconnected	Rising	−10.91 to 22.21	−0.87 to 5.45
	Falling	<0.1	<0.1

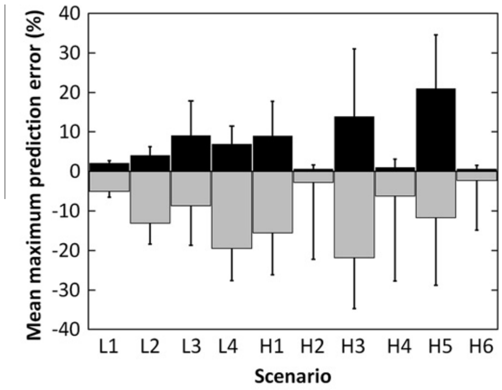

Fig. 4. Mean maximum over-prediction and under-prediction (%) of infiltration flux for the 10 different geostatistical scenarios. The whiskers represent the maximum errors obtained from each geostatistical scenario.

For example, the geostatistical scenarios H2, H4 and H6, (highly heterogeneous and uniform with depth), had the smallest over-predictions. Similarly, the over-predictions for case L1 with a weaker heterogeneity and a layered structure were also small. The results presented in Fig. 4 also demonstrate that both the over- and under-predictions were greatest for the cases H1, H3 and H5, where the streambeds were both highly heterogeneous and had a layered structure (i.e. $\tau_z < h_{sb}$). Fig. 4 illustrates that in addition to the findings pertaining to Table 2, the geostatistical properties of the streambed influence how well a homogeneous equivalent performs. Before we discuss how the combination of the findings summarized in Table 2 and the properties of the streambeds affect the performance of homogeneous equivalents, the effect of streambed heterogeneity on the infiltration curves has to be analyzed in more detail.

3.2. The influence of streambed heterogeneity on infiltration curves

In Fig. 5, the three different types of heterogeneous streambeds (5a, 5b and 5c) are presented together with their homogeneous equivalent, and the associated infiltration curves. The gray area in the infiltration curves and hollow triangles shown in Fig. 5 indicate the width of the transition zone.

The three streambeds are fundamentally different. The heterogeneous realization shown in Fig. 5a is an example where an unsaturated zone can develop under the entire streambed (corresponding to scenarios L1 or L3). We subsequently refer to the term “type I” for these kinds of streambeds. The case shown in Fig. 5b (and subsequently called “type II”) is an example of a streambed where practically no unsaturated areas can develop because the hydraulic conductivity of the streambed is large (and therefore the condition in equation 1 is not met). In Fig. 5c (corresponding to scenarios H3 or H5), parts of the flow between SW and GW remain saturated, even at full disconnection.

The different types of streambeds profoundly influence the infiltration curves, and as we will show the minimum and maximum errors made using a homogeneous equivalent. The infiltration curves of a type II streambed (5b) are essentially linear for all DTW we tested. Theoretically, an upper bound of infiltration also exists for these streambed types: once the infiltration rate is equal to the hydraulic conductivity of the aquifer, an additional drop in the water table will not increase the infiltration flux. However, for perennial rivers such a situation is unrealistic. A very different infiltration curve is associated with type I streambeds (5a). Recall that for this type, a sufficient drop of the water table causes an unsaturated zone to develop under the entire streambed. Importantly, only a very small drop in the water table is required to change the flow regime from connected to disconnected.

In Fig. 5c, a streambed is shown that is between the two end members (type I and type II): lowering the water table will result in the simultaneous presence of both saturated and unsaturated areas. As opposed to the type II streambed shown in 5b, an increased depth to groundwater will result in a transitional or a disconnected flow regime. The presence of saturated areas, however, significantly extends the transition zone as compared to type I streambeds.

The absolute values of hydraulic properties of the streambed determine whether a SW–GW flow system is of the type I or type II, or between these two end members. The hydraulic properties are therefore also the first order control on the width (and, in case of Fig. 5b the existence) of the transition zone.

A systematic comparison between infiltration curves based on homogeneous equivalents and heterogeneous streambeds reveals that the maximum errors made can be clearly associated with the type of streambed. Homogeneous equivalents perform well in simulating infiltration curves of type II systems where the flow between the SW and GW remains saturated. In the realization shown in Fig. 5b, the minimum and maximum deviations between the infiltration curves were as small as a 0.44% over-prediction to a 0.43% under-prediction. Errors associated with type I streambeds were larger, and the errors for the realization shown in Fig. 5a range between a 2.3% over-prediction to a 6.4% under-prediction. The largest

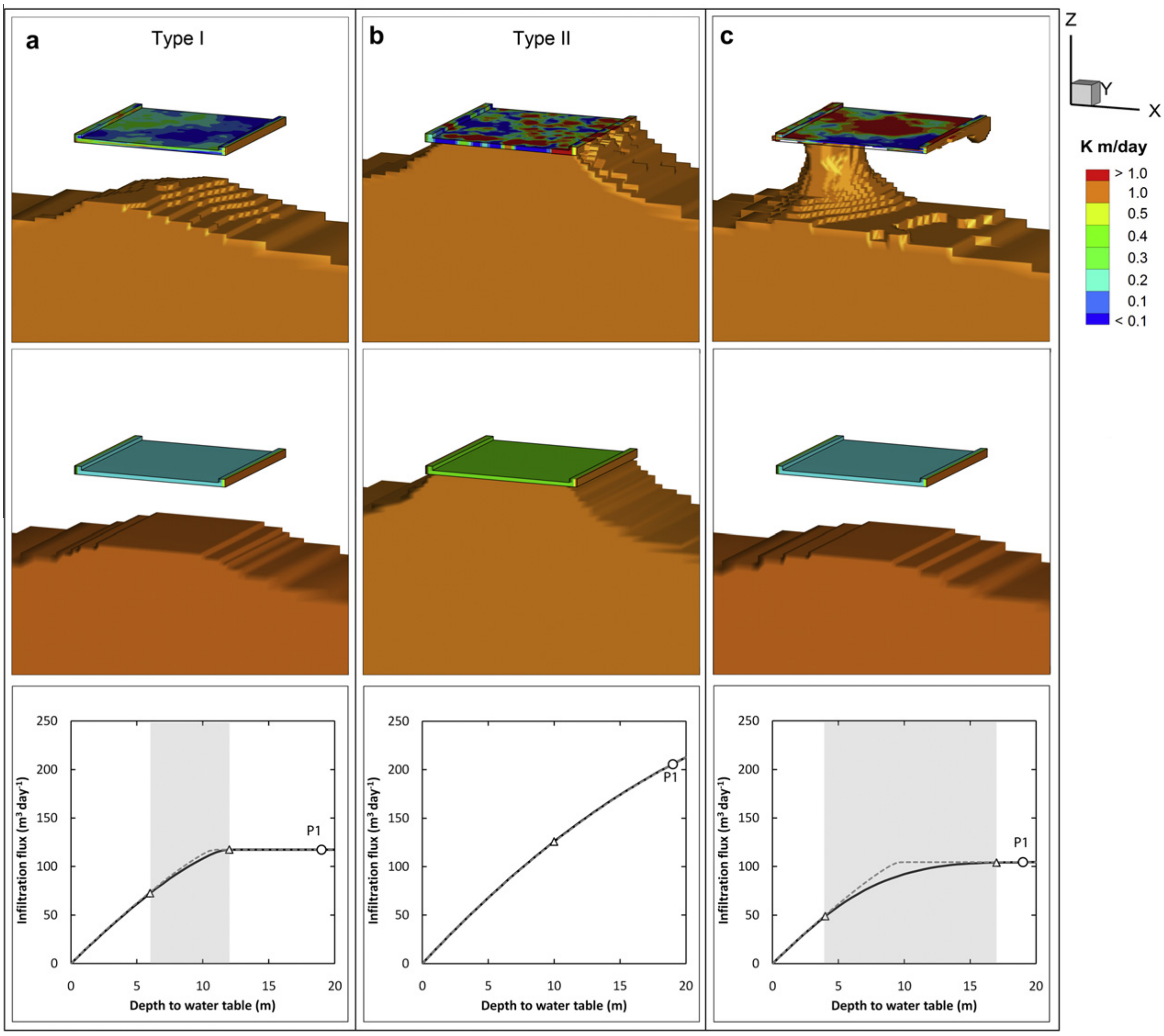

Fig. 5. Three surface water-groundwater systems are shown (top row). Fig. 5a corresponds to scenario L3 realization 1 (Type I, see text for details), Fig. 5b corresponds to H2 realization 1 (Type II, see text for details) and Fig. 5c corresponds to H5 realization 7 respectively. Their homogeneous equivalents are shown in the middle row. In the first two rows, regions where saturation exceeds 0.99 are shown, and color represents hydraulic conductivity. The bottom row corresponds to the infiltration curves of both the heterogeneous scenario (black line) and the homogeneous equivalent calibrated at point P1 (gray dashed line, bottom row). The transition zone is denoted by the hollow triangles and shaded regions.

errors were found in streambeds that are between the two end members. Here the largest errors found in the realization shown in Fig. 5c were up to a 15.9% over-prediction to a 28.9% under-prediction.

The comparison of the largest errors associated with the different types of streambeds suggests that the width of transition zone is the key control on how well a homogeneous equivalent performs. In this context, it is important to mention that a homogeneous equivalent will either be of type I or type II, but will never behave as shown in Fig. 5c. Therefore, the width of the transition zone of a homogeneous equivalent is always smaller compared to their heterogeneous counterparts.

3.3. *Understanding the process of simplification*

In Fig. 6, we explain why both the width of the transition zone as well as the flow regime that the observation is based on are important for the accuracy of predictions with a homogeneous equivalent. In this schematic figure, we only discuss the case where the observation of infiltration was obtained from a disconnected flow regime (we discuss errors for cases where observations were obtained from a connected or transitional regime later on).

Let us assume that the infiltration flux was obtained at point P1 (at disconnection, Q_{max}). As the system is already disconnected, a further lowering of the water table will not influence the infiltration flux, therefore the homogeneous equivalent will always correctly simulate infiltration fluxes for a falling water table. This explains the small errors shown in Table 2 associated with the simulation of a drop of the water table (for models that are based on observations of disconnected systems). The situation becomes less favorable if the homogeneous equivalent is used to simulate a rise of the water table. The mismatch for the simulation of a rising water table for any given depth to groundwater (e.g. DTW3) is given by the difference between the homogeneous equivalent and the original infiltration curve (indicated with line ab in Fig. 6). Even though the original streambed and thus the real infiltration curve are unknown in reality, the maximum error can be easily estimated (eg. for explanation, see Fig. 6) by drawing a straight line between the coordinate origin and the point where the infiltration flux was obtained (provided we know that the system is disconnected at this point).

The fact that no infiltration curve will ever fall below the line defined through the origin and the point of disconnection (P2 in Fig. 6) can also be illustrated using Fig. 6. Let us assume that an infiltration curve could actually fall below this line. For example, such a hypothetical infiltration curve could be defined through the line between the coordinate origin, point P^* and P1 in Fig. 6. Let us assume that P^* marks the point where the streambed begins to desaturate, and therefore the slope of the infiltration curve changes. As shown in Fig. 6, the slope of the infiltration curve to the right of P^* must increase in order to reach the flux given at full disconnection at P2. However, this is not possible as the underlying aquifer desaturates, and the slope of the infiltration curve will therefore be reduced. Thus, we can safely conclude that no infiltration curve will ever fall below the line between the origin and the point of disconnection.

If the infiltration flux was obtained exactly at the point between transitional and disconnected flow (P2 in Fig. 6), this maximum over-prediction will be smaller than a calibration based on the observation at P1, and we know that the over-prediction is limited to a maximum determined by the line $\overline{ab}$-$\overline{bc}$. In Fig. 7, we discuss examples where observations were obtained from a connected regime (e.g. P1 or P2). If the homogeneous equivalent calibrated on the basis of either P1 or P2 is used to simulate a rise in the water table, the error is negligible (see Table 2). This can be explained by recalling that the connected range of an infiltration curve can be described with a linear relation between depth to groundwater and infiltration flux. A linear relation is defined through 2 points; in this case one of them is the observation employed in calibration, and the second one is the coordinate origin (if the hydraulic gradient is zero there is no flow, thus defining the coordinate origin as the second point). This linear behavior can unambiguously be reproduced by the homogeneous equivalent. This also explains why homogeneous equivalents for type II systems work well, because in such systems, the infiltration curve is approximately linear for a wide range of DTW. However, for streambeds other than type II, problems arise if a falling water table is simulated. As illustrated in Fig. 7, the homogeneous equivalent might predict a very different behavior depending on the depth to groundwater (within the connected regime) the observation was obtained from. For cases where calibration is performed with fluxes near the coordinate origin (point P1), it is possible that the flux estimated for the disconnected state is either over-estimated (sometimes significantly) or under-estimated (point P2). We analyzed all of the calibrations of the heterogeneous realizations and found that a significant over-estimation of the infiltration at disconnection was consistently associated with scenarios L3, H1, H3 and H5,

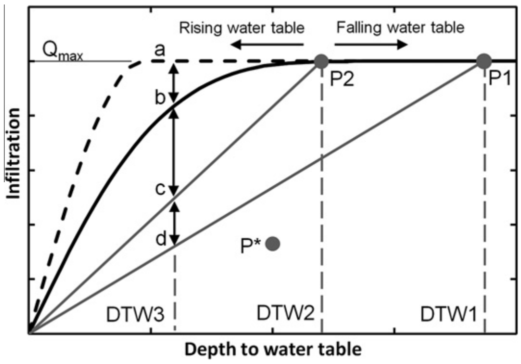

Fig. 6. A schematic diagram of an infiltration curve (solid black line) and the homogeneous equivalent (dotted black line) for a system between the streambed type I and II. The homogeneous equivalents obtained from an observation at P1 or P2 (corresponding to DTW1 or DTW2) will be identical because in both cases the stream is disconnected. Line ab represents the over-prediction of fluxes if predictions were made at DTW3. The largest possible over prediction occurs for calibrations performed at P1, and corresponds to the lines $\overline{ab}$-$\overline{bc}$-$\overline{cd}$. Maximum errors in predictions made from a calibration performed at P2 correspond to the lines $\overline{ab}$-$\overline{bc}$. The point P^* can be used to demonstrate that the minimum infiltration flux cannot fall below the line joining a point where the SW–GW system is known to be disconnected, and the origin of the infiltration curve.

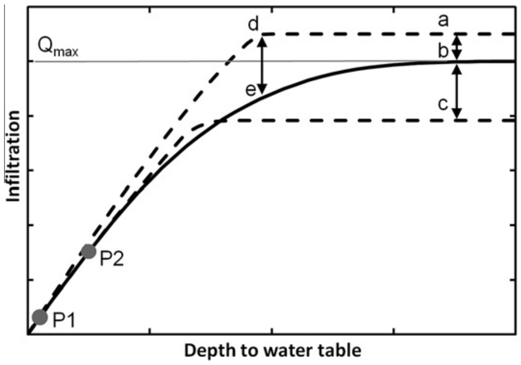

Fig. 7. A schematic diagram of an infiltration curve (solid black line) and homogeneous equivalents (dotted black lines) calibrated for a connected flow regime for two water table depths at points P1 and P2. The predictions made from the model calibrated at P1 passes through the origin and points d and a. The predictions made from the model calibrated at P2 passes through the origin, and through point c. Line $\overline{de}$ represents the greatest over-prediction, and line $\overline{ab}$ represents the over-prediction at disconnection Q_{max}. The line $\overline{bc}$ represents the largest under-prediction for this case.

which were scenarios with layered streambeds where $\tau_z < h_{sb}$. However, in these cases the over-estimation of the flux at disconnection occurred only if the observation was taken at a small depth to groundwater. This is not unexpected, because the closer the observation is to the co-ordinate origin, the less information it contains on the streambed properties. No information on the streambed properties can be inferred from an observation taken at a DTW of zero.

Finally, the mismatch for rising and falling water tables is discussed for cases where the observation was obtained from the transitional zone. As for the simulations based on observations in the connected regime, the position of the observation within the transitional regime is also important. An example is illustrated in Fig. 3: The closer the observation point is to the point where the system disconnects, the smaller the error associated with the simulation of a falling water table is. On the other hand, the closer the observation point is to the connected regime, the smaller the error associated with the simulation of a rising water table. The opposite is also true. The largest errors related to a falling water table are found for the observation points close to the connected regime. The difference to the previously discussed cases (observations obtained at either a connected or a disconnected regime) is that an error for both a rise and a fall of the water table is expected and that the magnitude of this error is related to how close the observation was taken to either a connected or disconnected regime. In systems with a short transition zone, an observation within the transition zone is always close to either a connected or disconnected regime. This is not the case for systems with a wide transition zone. Therefore, the propensity for error increases with the width of the transition zone. As discussed in Section 3.2, streambeds that allow for the simultaneous occurrence of saturated and unsaturated flow for all DTW have large transition zones.

4. Summary and Conclusions

We have presented a synthetic modeling study to investigate the implications of representing a heterogeneous streambed with homogeneous properties obtained via model calibration. Our analyses were based on heterogeneous streambeds underlain by a homogeneous aquifer. We did not consider transience of flows in the stream or aquifer. We tested the relationship between infiltration flux and the depth to the water table for 10 heterogeneous geostatistical scenarios (with 10 realizations in each scenario), totaling 100 heterogeneous realizations. In summary, our work demonstrates the following key points:

- Homogeneous equivalents accurately reproduce infiltration fluxes if both the calibration and prediction are made for a connected flow regime, or if both the calibration and prediction are made for a disconnected flow regime. If the observation is taken from a transitional regime, a homogeneous equivalent will result in errors for simulations of both a rising and falling water table. These findings are true for all possible streambeds.
- If the observation is taken from the transition zone, or if the flow regime of the prediction differs from the flow regime of the observation, the homogeneous equivalent will not accurately predict infiltration fluxes. The magnitude and sign of errors made when predicting infiltration fluxes using homogeneous equivalents depends on the properties of the streambed.
- We have identified two important end members of streambeds: Streambeds where lowering the water table does not cause unsaturated flow, and streambeds where lowering the water table results in an unsaturated zone under the entire streambed. We have shown that if an unsaturated zone can develop under the entire streambed, the transition zone is smaller compared to streambed types where both saturated and unsaturated flow can occur. Streambeds where no unsaturated zone can develop have an infiltration curve that is linear for a wide range of DTW and consequently the errors associated with a homogeneous equivalent are very small.
- The wider the transition zone of a streambed is, the larger the deviations between the homogeneous equivalent and the heterogeneous streambed and vice versa. The largest transition zones can be expected for streambeds where saturated and unsaturated flow can occur for all DTW.
- If the SW–GW system is known to be disconnected, the maximum possible over-prediction of fluxes from the use of a homogeneous equivalent can be estimated geometrically using a measured infiltration flux and the depth to the water table where this flux was observed.

The largest error we observed by using a homogeneous equivalent was 34%. This volume may be a small component of a groundwater balance, but in the case of streams with low flows, errors of this magnitude can be significant. Moreover, over long stream reaches these errors could amount to significant volumes of water. It is important to point out that the largest error we observed is only representative for the conditions we tested within our conceptual framework. Different conditions (e.g. shallower stream depth or heterogeneity in the aquifer) are likely to cause greater errors.

The study illustrates the importance of understanding and correctly identifying the state of connection of the SW–GW system. Understanding the state of connection is the key to quantify possible errors produced by numerical models based on homogeneous equivalents obtained by means of model calibration. Moreover, correctly identifying the state of connection also allows the identification of situations where a homogeneous equivalent can be employed without losing any significant predictive power compared to a heterogeneous streambed.

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