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ABSTRACT

CPPIs are popular medium- to long-term investment products that dynamically allocate between a risk-free asset and a risky portfolio, with the objective of combining upside potential with a capital guarantee. This paper uses a block-bootstrap evaluation approach to study whether combining smart beta and portfolio insurance is mutually beneficial under various scenarios. Our results show that the improvement in performance is most apparent for CPPIs combined with a low-risk equity portfolio. This finding is consistent with the negative ***vega*** of CPPIs and with path-dependency of the CPPI protection against portfolio losses between rebalancing dates.

1. Introduction

Recent financial engineering and marketing by investment firms led to the growing market share of so-called *smart beta* equity portfolios (Bloomberg, 2014). This commercial success has triggered the launch of capital guaranteed products with a smart beta equity solution as the underlying risky asset. The rationale is that while the alternative weighting schemes and selections used in smart beta tend to lead to out-of-sample gains in terms of higher risk-adjusted returns over sufficiently long investment horizons compared

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with the buy-and-hold investment in the market capitalization weighted benchmark (see, e.g., Arnott et al., 2005; Baker and Haugen, 2012), they do not offer protection against the large unexpected drawdowns due to the undiversifiable and systematic risk of market downturns (Theiler, 2011)]. In this paper, we study the impact of the choice of equity index on the performance of a *constant proportion portfolio insurance* (CPPI) investment product, introduced by Perold (1986) and Black and Jones (1987).

Under the CPPI framework, the portfolio is dynamically invested in a (almost) risk-free fixed-income instrument (providing protection against market losses) and a risky reference asset (offering upside potential). The balance between the two components is determined in such a way that the terminal portfolio wealth equals at least the guaranteed capital. The CPPI strategy includes the stop-loss rule as a special case. As emphasized in Black and Jones (1987), a critical feature of a CPPI is that it has negative *vega*: as the volatility of underlying risky asset increases, the CPPI payoffs decline. It is therefore expected that, *ceteris paribus*, a low-risk smart beta equity solution with the same expected return as a competing smart beta solution, will lead to an improved CPPI performance.

In this paper, we examine the impact of the choice of smart beta equity index on the CPPI performance. Due to the non-linear and multi-horizon nature of a CPPI investment, it is not immediately obvious how the risk and return properties of the underlying equity portfolio affect the compound performance of the CPPI investment. Complicating factors in the impact analysis are the price-path dependency of the performance of CPPI investment, the impact of downside risk on the CPPI value, the asymmetric impact of volatility on cumulative returns, and the volatility costs of buying high and selling low. Indeed, as shown by Black and Perold (1992), price reversals cause the CPPI to increase the exposure at high prices and decrease the exposure at low prices, making the CPPI strategy more costly.

Using a block-bootstrap simulation-based evaluation framework to generate historical performance of CPPIs invested in the S&P 500 universe, we show that the choice of equity portfolio can have economically important effects on risk-adjusted performance and portfolio stability. Overall, the smart beta CPPIs report improved risk and return characteristics compared with the traditional market capitalization CPPI. While the improvement for the fundamental value and equally-weighted CPPIs is rather small, we find a substantial improvement in terms of risk-return trade-off and downside protection for the low-risk CPPI. The difference in performance is more pronounced in the bearish and high volatility markets, when protection is most needed.

We further show how the performance of the CPPI can be improved by scaling the underlying equity solution through a volatility target overlay that aims at ensuring that the risky portfolio has the same volatility as the low-risk equity portfolio. When using such a volatility target overlay, the differences between the equity allocation approaches partly diminish, but remain substantial. In particular, we find that the CPPI strategies with a volatility target exhibit a lower number of gap events compared with their pure equity counterparts. We also show that a CPPI strategy with a low-risk pure equity solution outperforms both in terms of upside potential and capital protection.

This research has important implications for both designers of CPPI investment products, investors and regulators. Typically, a CPPI investment is commercialized with a focus on the performance chart of the underlying equity index, compared to a benchmark index. As we show, this may be misleading because of the highly non-linear and path dependent nature of the dependence of the CPPI performance on the underlying equity index performance. When constructing and commercializing, approving and deciding to invest in a CPPI product, the issuer, regulator and investor should thus carefully consider the choice of the equity portfolio. The use of the block-bootstrap evaluation approach is a useful tool to do so.

The remainder of this paper is organized as follows. Section 2 elaborates on the CPPI framework. Section 3 discusses the data and equity portfolios construction. Section 4 describes the simulation framework and presents the performance measures. Section 5 presents the results. Section 6 concludes.

2. CPPI framework

The impact of the choice of risky reference asset on CPPI performance depends on the specific implementation of the strategy. This section first describes the various design parameters of the CPPI strategy. We then discuss the determinants of the CPPI value, with a focus on the impact of the volatility of the risky reference asset on the CPPI end-value.

2.1. Definition

The objective of CPPI investors is twofold: to participate in the upside potential of the risky reference portfolio, while ensuring that the value of the portfolio at maturity V_T is higher than a guaranteed amount G ($V_T \geq G$). The guarantee G is defined as a proportion p of the initially invested amount V_0. These two objectives are realized by dynamically allocating between a fixed-income asset and an equity portfolio. At the rebalancing date, the investor starts by determining the floor F_t which is the lowest acceptable portfolio value to ensure the capital guarantee at the end of the investment horizon T. The floor F_t is given by:

$$F_t \equiv \frac{1}{(1+r_f)^{T-t}} G,$$

with r_f the fixed-income rate over the remaining time to maturity $(T - t)$. In a next step, the investor computes the cushion C_t, which is the difference between the portfolio value V_t and the floor F_t. Then the investor determines the exposure E_t to the risky reference portfolio. The exposure E_t is the nominal amount allocated to the risky reference portfolio given by a multiplier m times the cushion C_t:

$$E_t \equiv \max\{\min\{m \underbrace{(V_t - F_t)}_{\equiv C_t}, bV_t\}, 0\}.$$

To avoid excessive equity positions, E_t is often bounded to be at most bV_t, such that, if $b > 1$, the maximum allowed leverage in the CPPI portfolio is $b - 1$. Typically, no leverage is allowed in CPPI portfolios (see, e.g., Do, 2002; Annaert et al., 2009; Dichtl and Drobetz, 2011). The remaining funds $(V_t - E_t)$ are invested in the fixed-income asset.

As such, when the CPPI contract is created, the following design parameters need to be determined: i) the risky reference asset, ii) the fixed-income asset, iii) the protection level p, iv) the multiplier m, v) the investment horizon T and vi) the rebalancing frequency. Table 1 presents an overview of these choices made in the literature.

Table 1. Implementation of CPPI strategies in the literature
This table presents an overview of the implementation parameters used in the literature. Horizon *T* is in years. Rebalancing is daily (d), monthly (m) or yearly (y).

Reference	Risky reference asset	Fixed-income asset	Proportion p	Horizon T	Multiplier m	Rebalancing
Annaert et al. (2009)	AU, CAN, JP, US, UK	Euro interbank rate	95%	1	14	d
Ben Ameur and Prigent (2014)	S&P 500 comp	2%	95%, 100%	5	3, 5, 6, 7, 9	d, w, m, a
Bertrand and Prigent (2011)	S&P 500 comp	Fed Fund rate	95%-100%	1	5-8	d
Dierkes et al. (2010)	S&P 500 comp	US Treasury bond	95%	$\frac{1}{12}$, 1-7	5-8	d
Hamidi et al. (2014)	Dow Jones Index	–	90%	1-15	1-13	d
Theiler (2011)	Stoxx Europe 50	1.5%	80%	2	5	d
Zieling et al. (2014)	S&P 500 comp	3-month T-Bill	90%	$\frac{1}{12}$, 1	1, 4, 6	d

In earlier empirical research, the risky reference portfolio is usually a fully invested market capitalization portfolio. The (riskless) fixed-income asset is typically a government bond and offers a fixed rate. Most CPPIs are implemented with a capital guarantee between 80% and 100% of the initial portfolio value. CPPIs are medium to long-term contracts with in practice an investment horizon of five years on average (Ben Ameur and Prigent, 2014).

In a standard CPPI strategy, the multiplier is typically set to an integer between one and eight. The value of the multiplier depends on the investor's risk tolerance as it determines the exposure to the equity portfolio. It is a trade-off between seeking yield and offering capital protection. Cesari and Cremonini (2003) show that the multiplier has to be bounded by the inverse of the maximum loss of the equity portfolio between two rebalancing dates. For instance, assume the maximum loss between two subsequent rebalancing dates is 20%. To guarantee that the portfolio value is above the floor, the upper bound on the multiplier should be five.

Theoretical research on CPPI often assumes continuous rebalancing (see, e.g., Leland, 1980; Bookstaber and Langsam, 2000; Bertrand and Prigent, 2005). However continuous rebalancing is infeasible in practice as potential profits will be eroded by transaction costs. Therefore, CPPIs are usually implemented with discrete time rebalancing at a fixed frequency. Alternatives for this so-called *time* discipline are the *market move* and *lag* disciplines. As described by Etzioni (1986), the latter approaches rebalance when the equity portfolio value moves with a pre-specified amount or when the portfolio weights deviate with a pre-specified amount from the target portfolio weights implied by the CPPI strategy. The practical implementation of the *market move* and *lag* discipline approaches thus require *ad hoc* choices in terms of maximum allowed move of the equity portfolio or the deviation of the portfolio weights. The most popular approach in the literature is the time discipline with daily rebalancing (see Table 1). We follow this standard in our study.

In the application, we set the initial portfolio value V_0 at \$100. The base results are for a protection level $p = 90\%$, an investment horizon of $T = 5$ years and a static multiplier $m = 6$. The CPPI portfolios are rebalanced daily. As common in CPPI products we do not allow for leverage, hence the maximum investment quote $b = 1$.

2.2. The property of negative vega

The final value V_T of the CPPI investment is a result of the stochastic evolution of the risky reference portfolio, and how that interacts with the rule-based CPPI trading strategy, as described above. The actual composition of the risky reference portfolio only matters indirectly for the CPPI investor by affecting the aggregate price dynamics of the risky portfolio. Generally speaking, we have that the drift of the underlying log-price diffusion of the risky portfolio affects positively the end-value of the CPPI portfolio, while the value of the volatility has a negative impact. The partial derivative of the CPPI value with respect to the volatility of the risky portfolio is called the *vega* of the CPPI portfolio. A more formal treatment of the *vega* of CPPI portfolios is given in Bertrand and Prigent (2005). Under the assumption of a Brownian semi-martingale, they derive an explicit expression for the value of the CPPI portfolio and then show that the partial derivative of the CPPI value with respect to the (time-varying) volatility of the underlying is negative.

A crucial feature of the CPPI portfolio is thus that, in a non-directional market, a higher volatility of the risky reference portfolio is detrimental to the compound CPPI performance. Intuitively, this property of negative *vega* is due to two reasons. First, the higher the volatility, the more frequently the portfolio shifts between the risk-free and risky asset. This leads to transaction costs and the adverse timing cost of buying when the risky reference portfolio is expensive, and selling when it is cheap. Second, the volatility cost of reversals is amplified by the asymmetric effect of positive and negative returns on compound returns. Assume for instance that a risky equity faces a loss of 50%. In order to revert to pre-loss price level, the price of the risky equity has to increase by 100%.

From the property of negative *vega* of CPPI performance, it follows mechanically that the CPPI performance is improved when the risky reference asset is replaced with an asset for which the volatility is lower but all other characteristics of the performance distribution are identical. In practice, there are always confounding factors and such a *ceteris paribus* modification does not exist. It follows that the performance impact of substituting the market capitalization weighting equity index with a low-risk equity portfolio is ultimately an empirical question for which we propose in the next sections a block-bootstrap evaluation methodology.

2.3. Downside risk between rebalancing dates

A primary reason to invest in portfolio insurance strategies is to protect the investment capital against large losses. This is handled in an asymmetric way by setting the weight of the risky investment as a positive multiplier of the cushion, defined as the spread between the current portfolio value and the nominal value of the risk-free investment that has end-value equal to the capital guarantee G (i.e., the floor). The exposure to large equity losses thus depends on prior performance. It also depends on the frequency of rebalancing. If rebalancing were continuous and price movements sufficiently smooth, the allocation rule that the equity investment is a positive multiplier of the cushion ensures that losses are bounded in such a way that the portfolio value does not fall below the guarantee (Cont and Tankov, 2009; Balder et al., 2009; Hamidi et al., 2014). In practice, rebalancing is discrete and prices can jump, implying that there is a non-negligible probability that $V_T < G$. The risk of failing to achieve the desired capital protection is the gap risk, which has two components, namely the probability of a gap event and the severity of the loss when a gap event occurs.

Gap risk is closely connected to downside risk, defined as the risk of large losses between two rebalancing dates. From the construction of the CPPI it follows that the downside risk of the CPPI investment depends on the price-path of the CPPI investment and also on the downside risk of the underlying equity investment. When, for instance, the presence of jumps in asset prices, the risky investment returns are non-normally distributed, the volatility of the risky investment is insufficient to describe downside risk and the choice of risky investment should also consider the magnitude of extreme losses on the equity return, as described by the asset returns' Value-at-Risk and expected shortfall.

3. Smart beta definitions, data and buy-and-hold performance

In our analysis, we apply the CPPI strategies to four types of equity portfolios, constructed on the S&P 500 investment universe over the period 1985-2014. We discuss the portfolio construction methodology of the portfolio, introduce the data and discuss the buy-and-hold performance of the considered risky reference assets.

3.1. Portfolio construction and data

We consider the following four equity portfolios. Each portfolio is rebalanced at month-end.

Market capitalization portfolio. The market capitalization portfolio sets stock weights proportional to their market capitalization. As the market capitalization weighted portfolio is the standard portfolio, we consider this as the benchmark in our analysis.

Fundamental value portfolio. The fundamental value portfolio is a composite portfolio of a book value, dividend, net operating cash flow and sales portfolio (Arnott et al., 2005). In a first step, four portfolios are created in which the stocks' weights are proportional to a single fundamental metric. Next, a composite portfolio is created by aggregating the four single-metric portfolios in equal proportions. When a company does not distribute dividends, the composite weight is the averaged weight of the remaining fundamental measures. The fundamental value data are annual, but the dataset is updated at every month-end to deal with differences in reporting dates. The fundamental data are lagged by one quarter to ensure data availability. The cash flow, dividend and sales measures are five-year rolling averages.

Equally-weighted portfolio. The equally-weighted portfolio assigns an equal weight to each constituent of the investment universe.

Low-risk portfolio. The low-risk portfolio is invested in the one hundred stocks reporting the lowest volatility and the weights of the selected stocks are proportional to the inverse of their volatility.[5] The volatilities are estimated using the 252 daily returns prior to the rebalancing date.

Total returns, prices, number of stocks and fundamental value data are retrieved from the COMPUSTAT database and expressed in USD. As a risk-free investment, we take the investment in a five-year U.S. Treasury bond, for which the bond rates vary between 0.26% in 2013 and 12% in 1985 over the 1985-2014 sample.

3.2. Buy-and-hold performance

We first examine the impact of the portfolio construction for a buy-and-hold investor. Panel A of Table 2 presents the differences in return and risk of a buy-and-hold investment in each of the four equity portfolios. The smart beta portfolios all report a higher Sharpe ratio compared with the market capitalization portfolio. The fundamental value and equally-weighted portfolios report higher average returns, while the low-risk portfolio reports, as expected, a substantially lower annualized volatility. Furthermore, the 1% and 5% quantiles of the daily returns are less extreme for the low-risk portfolio, i.e., it exhibits lower downside risk. The last two columns indicate the annual occurrence of a 2% and 5% loss, respectively. A 2% loss occurs almost three times more often for the market capitalization, fundamental value and equally-weighted portfolios. A loss of at least 5% occurs every year for the fundamental value and equally-weighted portfolios.

Panel A of Figure 1 displays the cumulative value of the four portfolios (in logarithmic scale). We see that the evaluation period is characterized by several booms and busts. The choice of equity portfolio does not offer protection against the large unexpected drawdowns due to the undiversifiable

5. Low-risk equity portfolios exploit risk-based anomalies (Scherer, 2011). As emphasized for instance by Blitz and van Vliet (2007) and Chow et al. (2014), benefiting from the low volatility anomaly requires shrinking the investment universe to a pool of low-risk stocks. These approaches are also consistent with the construction methodologies used in practice (see, e.g., S&P Dow Jones index, 2014).

and systematic risk of market downturns, and thereby makes the analysis of portfolio insurance interesting. Over the entire sample period, the three smart beta portfolios report a substantial higher cumulative return than the market capitalization portfolio.

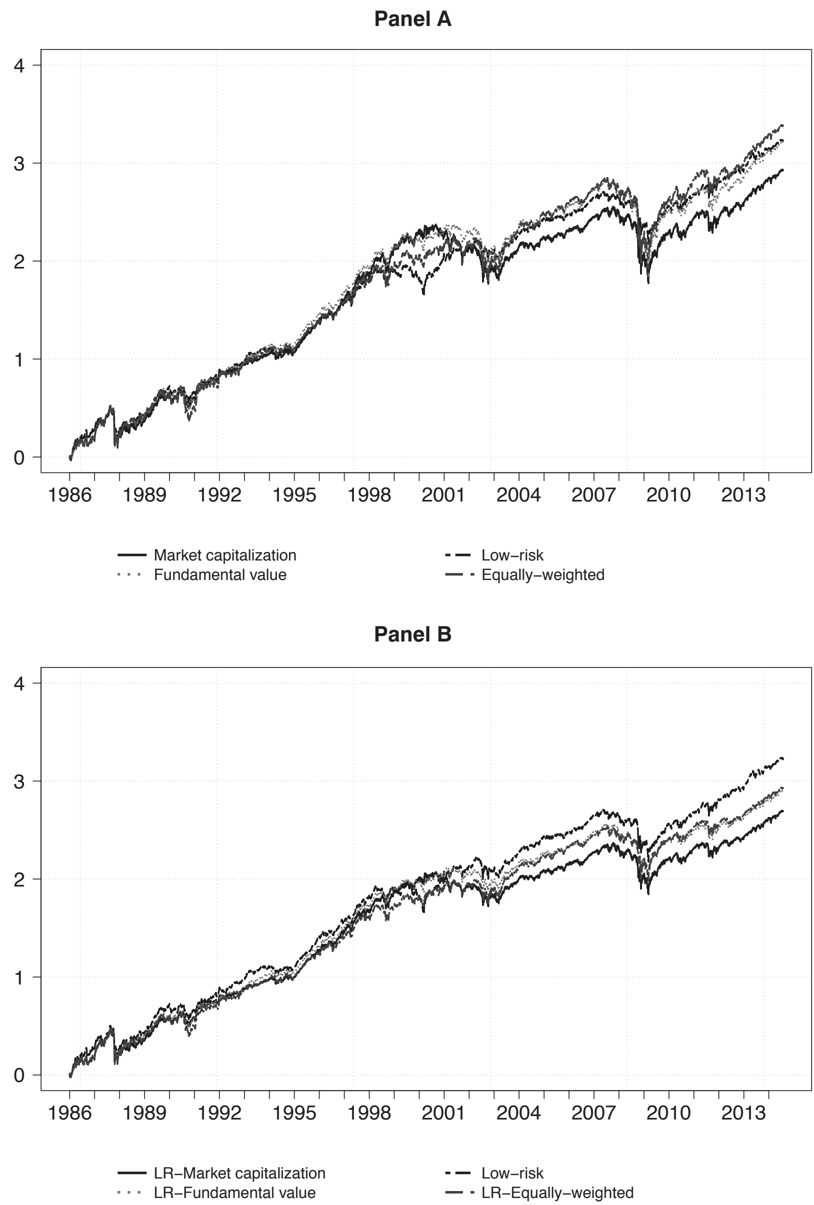
Table 2. Performance of the risky reference assets

This table presents the performance results of the risky reference assets. We report the average annualized returns (Mean, in percent), annualized standard deviation (Sd, in percent), annualized Sharpe ratio (SR), the 95% and 99% historical daily Value-at-Risk (VaR_{95} and VaR_{99}, in percent) and the number of days per year that the daily return is below –2% and –5% over the period January 1986 to July 2014. The risky reference assets are the market capitalization (MC), fundamental value (FD), equally-weighted (EW) and low-risk (LR) equity portfolios, volatility-target market capitalization (LR-MC), fundamental value (LR-FD) and equally-weighted (LR-EW) portfolios.

	Mean	Sd	SR	VaR_{95}	VaR_{99}	$\#(R_t < -2\%)$	$\#(R_t < -5\%)$
Panel A: Equity portfolios							
MC	10.78	18.31	0.30	–1.71	–3.11	8.88	0.70
FD	11.89	18.83	0.35	–1.66	–3.28	8.60	0.98
EW	12.55	19.17	0.37	–1.74	–3.25	9.27	1.01
LR	11.92	13.44	0.49	–1.22	–2.25	3.67	0.21
Panel B: Volatility-target portfolios							
LR-MC	9.87	13.44	0.34	–1.26	–2.24	3.71	0.21
LR-FD	10.67	13.44	0.40	–1.23	–2.25	3.60	0.24
LR-EW	10.78	13.44	0.40	–1.21	–2.25	3.88	0.28

We aim at providing a better insight to the impact on CPPI performance resulting from the choice of the equity portfolio. The smart beta portfolios report difference in risk and return characteristics (i.e., higher average returns for the fundamental value and equally-weighted portfolios and lower volatility for the low-risk portfolio). Because of the complex nature of the product and the path dependence of the return evolution over the investment horizon of the product, there is no straightforward answer. We thus need to evaluate how the equity portfolios affect the performance and how the impact differs under various market conditions.

Figure 1. Cumulative performance of the equity portfolios
This figure presents the cumulative buy-and-hold investment returns of the equity portfolios over the 1986–2014 period (in logarithmic scale).



3.3. Smart beta with volatility target

The largest differences in the smart beta portfolios are observed in terms of risk, with the low-risk equity portfolio offering a substantially lower volatility than the alternatives considered. From the negative *vega* characteristic of the CPPI portfolio, it thus follows that we expect the low-risk CPPI portfolios to outperform other CPPIs. These differences in volatility are of importance because of the asymmetric impact of volatility on the compound returns, and the volatility costs of buying high and selling low.

We therefore consider also the alternative of improving the risky reference portfolios with a volatility target overlay in which the volatility of the risky reference portfolio is targeted to be equal to the volatility of the low-risk portfolio. This is achieved by combining a position in the equity portfolio with a position in the risk-free asset.

More precisely, we construct volatility-target portfolios in which the investor allocates across an equity portfolio and a risk-free asset, such that the estimated conditional volatility of the mixed portfolio is proportional to the estimated conditional volatility of the low-risk portfolio. We implement this by setting the weights on the equity portfolio ($w_{eq,t}$) as:

$$w_{eq,t} \equiv k \frac{\sigma_{low-risk,t|t-1}}{\sigma_{eq,t|t-1}},$$

where $\sigma_{\text{low-risk}|t,t-1}$ is the predicted one-period ahead volatility of the time t return of the low-risk equity portfolio, $\sigma_{\text{eq}|t,t-1}$ is the corresponding predicted volatility of the considered equity portfolio (i.e., the market capitalization, fundamental value or equally-weighted portfolio). We compute the predicted volatility as the standard deviation over the 252 trading days preceding day t. The adjustment factor k ensures that the unconditional sample volatility of the mixed portfolio equals the unconditional sample volatility of the low-risk portfolio. The value of k is of course close to unity. For our sample, k equals 0.984, 0.974 and 0.969, for the mixed market capitalization, fundamental value and equally-weighted portfolio, respectively.

Panel B of Table 2 shows the performance results of the volatility-target portfolios. All portfolios exhibit lower volatility, but this comes at the cost of lower average returns. The loss occurrence is less frequent compared with the full equity portfolios. However, the low-risk portfolio faces a loss of over 5% less often. Furthermore, the Value-at-Risk is lower. It is important to note that while these differences in downside risk seem small for a full equity investment, they are nevertheless amplified in the CPPI context through the leveraged position it takes. The larger the multiplier is, the larger the leverage and thus the more important these differences. Over the sample period, the cumulative returns are lower compared with a buy-and-hold investment in the low-risk equity portfolio, as shown in Panel B of Figure 1.

These volatility-target portfolios have the disadvantage of being dependent on the conditional volatility. They are therefore rebalanced with a lag, and are thus possibly exposed to sudden unexpected jumps. Sudden unexpected downward jumps can cause gap events (i.e., the CPPI violates the capital guarantee, as discussed in Section 2.3). Therefore, we expect an improvement of the CPPI strategy with smart beta volatility-target portfolios because of the lower volatility, but the gap risk is dependent on the severeness of the jumps in the risky reference asset value.

4. Evaluation framework

In this section, we first discuss the simulation method, followed by a description of the metrics used to evaluate the CPPI performance.

4.1. Simulation method

The simulated baseline CPPI is implemented over an investment horizon of five years. The drawback of such a long investment horizon is that the historical data provide only a small number of independent evaluation samples. To overcome this problem, CPPI performance is evaluated by backtesting the strategy on M artificially generated price-paths of the equity portfolios (i.e., pseudo-samples).

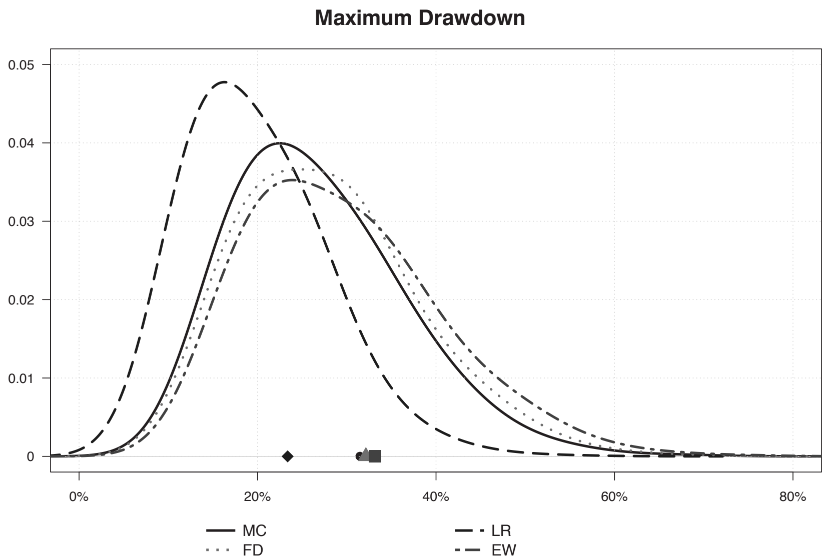
Several approaches to simulate price-paths are proposed in the literature and can be categorized into parametric and non-parametric approaches. Cesari and Cremonini (2003) and Caliman et al. (2013) use a Monte Carlo simulation assuming a normal distribution for the returns. Others allow for jumps in the data (see, e.g., Weng, 2013) or account for regime switching (see, e.g., Zieling et al., 2014). To avoid the model risk inherent of parametric simulation techniques, we generate the pseudo-samples using a non-parametric approach which samples blocks of returns from the original sample in such a way that the pseudo-samples have similar statistical properties as the original sample.

We apply the stationary block-bootstrap approach of Politis and Romano (1994) and sample blocks of returns of various length in two steps. First, the block lengths are drawn from a geometric distribution, with a minimal block length of one day and an average block length of 15 days.[6] The latter is estimated to be optimal for the returns of the market capitalization weighted portfolio in terms of the mean square error criterion in Politis and Romano (1994). Under this setup, around 10% (respectively 1%) of the block lengths are longer than 30 (respectively 60) days. We thus draw a variety of relatively longer block lengths (needed to capture the broad market trends) and relatively shorter block lengths (ensuring the heterogeneity in the generated pseudo-samples compared with the initial sample). Second, we draw a random starting date for the return series and bind the series together. We repeat this until we have a complete return series of 1,260 days (i.e., five years) for which we construct a price-path. In total, we create $M = 10{,}000$ price-paths. The sampling is equal for all equity portfolios thus ensuring the same dependence structure between the simulated equity indices as in the original sample. The risk-free rate is the U.S. Treasury bond rate at the sampled starting date of the investment period. We apply the CPPI strategies to each of the simulated price-paths.

We verified that the moments of the returns in the pseudo-samples match with the actual returns observed over the period January 1986 to July 2014. To save space, we focus our discussion on the empirical validity of the maximum drawdowns of the generated price-paths. These are crucial features in the price process, since portfolio insurance is designed to offer protection against them. In Figure 2, we compare the distribution of the maximum drawdowns over the simulated paths and the actual, historical five-year average maximum drawdown. We find that for all portfolios, the actual average maximum drawdown is located at the center of the maximum drawdowns computed for the simulated price-paths. Note also that the shape of the simulated maximum drawdown distribution is similar for the market capitalization, fundamental value and equally-weighted portfolios, all exhibiting a heavy right tail indicating larger drawdowns. The low-risk portfolio has a more concentrated distribution. The actual average maximum drawdown of the low-risk portfolio is lower (i.e., around 20%) compared with the maximum drawdowns of the other alternative portfolios (i.e., around 35%).

6. We also studied the CPPI performance under alternative simulation methods. We implemented the block-bootstrap method with an average block length of five and 30 days, and implemented a more naive strategy where we randomly draw the block length from a uniform distribution between one and 60 weeks. The results are qualitatively similar to those presented in Section 5 and are available from the authors upon request.

Figure 2. Distribution of the maximum drawdown of the equity portfolios
This figure presents the distribution of the maximum drawdowns for the simulated price-paths of the equity portfolios together with the actual, historical average five-year maximum drawdown. Distributions are computed from 10,000 simulations generated with the block-bootstrap methodology described in Section 4.1. The equity portfolios are the market capitalization (MC, ●), low-risk (LR, ◆), fundamental value (FD, ▲) and equally-weighted (EW, ■) portfolios.



4.2. Performance measures

The objective of portfolio insurance is twofold: benefiting from the upward potential of the equity portfolio and offering capital protection. Therefore, the performance of the CPPIs should be evaluated both in terms of the upside potential and the risk of not meeting the capital guarantee. The performance evaluation is done on the basis of the M simulated price-paths of the CPPI strategies (see, e.g., Dichtl and Drobetz, 2011). We standardize the total compounded performance by expressing it as a yearly growth rate, $R_T \equiv \left(\frac{V_T}{V_0}\right)^{\frac{1}{T}} - 1$, and report the average return and the standard deviation, as well as the Sharpe ratio and Sortino ratio of R_T, as in Zieling et al. (2014). The Sharpe ratio is computed as:

$$\text{SR} \equiv \frac{\mathbb{E}[R_T - R_{\text{f},T}]}{\sigma_{R_T}},$$

in which $R_{f,T}$ is the yearly growth rate of the risk-free investment and σ_{R_T} is the standard deviation of the yearly growth rates (calculated over the simulated end-values). The Sortino ratio proposed by Sortino and Price (1994) explicitly takes the non-normality in the CPPI return distribution into account and is defined as the average excess return (relative to the risk-free investment), divided by the downside deviations benchmarked to the risk-free investment:

$$\text{SoR} \equiv \frac{\mathbb{E}[R_T - R_{f,T}]}{\sqrt{\mathbb{E}[\min\{R_T - R_{f,T}, 0\}^2]}}.$$

The second objective of portfolio insurance strategies is to offer protection against losses. One of the aspects of CPPI performance that needs to be evaluated is the risk of not meeting the capital guarantee. Recall that the CPPI investor wants to ensure that the end-value is higher than the guaranteed amount, i.e., $V_T \geq G$. If rebalancing were continuous and price movements sufficiently smooth, the allocation rule that the equity investment is a positive multiplier of the cushion would guarantee this almost surely (Cont and Tankov, 2009; Balder et al., 2009; Hamidi et al., 2014). In practice, rebalancing is discrete and prices can jump, implying that there is a non-negligible probability that $V_T < G$. The risk of failing to achieve the desired capital protection is the gap risk. The expected shortfall of a CPPI is the average percentage amount which is lost if a gap event occurs:

$$\text{ES} \equiv \mathbb{E}\left[\frac{G - V_T}{G} \mid V_T < G\right].$$

A final point of relevance is the stability of the CPPI portfolio allocation and the impact of transaction costs. To gauge this, we report the portfolio turnover defined as the average size of the trades across the risky and risk-free asset (DeMiguel et al., 2009). To account for the transaction costs due to the portfolio turnover, we report all performance results for the net return, where we follow Annaert et al. (2009) and take a proportional transaction cost of 0.1% into account.

5. Results

This section evaluates the performance of CPPIs with different underlying risky reference portfolios. We first present the results over all simulated price-paths. Then we discuss the results for different market regimes based on the *ex-post* average return, the *ex-post* volatility and the level of the risk-free rate.

5.1. CPPI performance for various reference portfolios

We start our analysis with the comparison of the sensitivity of the CPPI strategies to the equity portfolio. Results are presented in Table 3. The first four rows compare the CPPI performance implemented with a market capitalization, fundamental value, equally-weighted and low-risk equity portfolio as the risky reference asset. We find that the differences in return and risk properties of the underlying portfolios have a direct impact on the performance of the CPPI portfolios. The smart beta portfolios combined with CPPI outperform the market capitalization portfolio. The improvement is most clear for the low-risk CPPI portfolio. The equally-weighted CPPI reports the highest average return, but also the highest standard deviation in terms of the possible outcomes over the five-year investment horizon. The low-risk CPPI outperforms the other portfolios in terms of Sharpe and Sortino ratios. Furthermore, it reports a substantially lower gap risk (i.e., 0.54%) compared with the market capitalization portfolio (i.e., 4.59%). The expected shortfall relative to the guaranteed capital is similar for all CPPI portfolios.

Turnover is substantially lower for the low-risk CPPI because of the lower volatility of the equity portfolio: A highly volatile underlying leads to more shifts in the exposure and hence to higher turnover and transaction costs.

The outperformance of the low-risk CPPI strategy raises the question of whether these performance gains are fully determined by the lower volatility of the low-risk portfolio and thus the reduced negative *vega* drag of the CPPI portfolio. We answer this question by comparing the performance of the CPPI portfolio with low-risk equity solution as underlying with CPPI portfolios that are invested in the equivalent volatility target portfolios. These volatility target portfolios combine a position in an equity portfolio (i.e., in the market capitalization, fundamental value or equally-weighted equity portfolios) with a variable position in the risk-free asset. The precise construction of the volatility-target overlay portfolios is explained in Section 3.3. Comparing the last three rows in Table 3, we see that the CPPI strategies with volatility-target portfolios as an underlying exhibit similar volatility as the low-risk CPPI. The gap risk is lower compared with the CPPI invested in the equivalent portfolio but that has no volatility target. The impact of reducing the volatility on the gap risk depends on the equity portfolio. Gap events occur as a consequence of both discrete time rebalancing and jumps in the reference asset's value. For the market capitalization and fundamental value portfolios with volatility target, the probability of a gap event occurring is five times larger compared with the low-risk equity portfolio. The equally-weighted portfolio with volatility target is exposed to a lower gap risk, but the expected shortfall is greater.

Table 3. Performance of the CPPI strategies

This table presents the performance results of the CPPI strategy with various underlying risky reference portfolios. Results are obtained from the block-bootstrap simulation method described in Section 4.1, where we generate five-year price-paths, for which we compute the return $R_T \equiv (V_T / V_0)^{1/T} - 1$. Overall, we generate 10,000 paths and compute various (end-of-period) performance measures. Mean: average return (in percent). Sd: standard deviation (in percent). SR: Sharpe ratio. SoR: Sortino ratio. Gap: gap risk (i.e., $V_T < G$) (in percent). ES: expected shortfall when a gap event occurs (in percent). Exposure: average exposure to risky asset (in percent). Turnover: average annual turnover (in percent) when proportional transaction costs of 0.1% are taken into account. The CPPI parameters are $T = 5$, $V_0 = 100$, $p = 0.90$, $m = 6$, and $b = 1$. See Table 2 for details on the risky reference assets.

	Mean	Sd	SR	SoR	Gap	ES	Exposure	Turnover
Panel A: CPPI with equity portfolios								
MC	10.07	8.42	0.62	2.61	4.59	–1.23	89.01	71.77
FD	11.16	9.03	0.69	3.21	4.55	–1.27	89.20	67.01
EW	11.88	9.75	0.70	3.59	4.51	–0.73	88.66	69.88
LR	11.52	6.63	0.93	4.22	0.54	–0.01	95.63	36.60
Panel B: CPPI with volatility-target portfolios								
LR-MC	9.55	6.14	0.73	2.59	2.77	–0.06	94.66	42.55
LR-FD	10.41	6.39	0.82	3.23	2.65	–0.57	95.15	39.03
LR-EW	10.62	6.74	0.81	3.28	0.13	–0.02	94.78	42.12

Panel A of Figure 3 displays the cumulative distribution functions of the CPPI values at maturity. The market capitalization CPPI underperforms the three smart beta CPPIs. The low-risk CPPI has the lowest probability of reporting gap events, while the equally-weighted and fundamental value CPPIs have a higher probability of reporting an end-value above \$175 and \$200, respectively. The right Panel displays the cumulative distribution functions around the portfolio guarantee (i.e., \$90). The low-risk CPPI has a very low probability of violating the capital guarantee, while the other CPPIs report a higher gap risk combined with a higher expected shortfall.

Panel B of Figure 3 compares the cumulative distribution functions of the low-risk and volatility-target portfolios. The low-risk CPPI outperforms the volatility-target market capitalization and fundamental value portfolios as its cumulative distribution function lies below the volatility-target's cumulative distribution functions for all CPPI end-values. On the other hand, it underperforms the volatility-target equally-weighted CPPI for values close to the guarantee.

To further evaluate these differences, Table 4 presents the frequency (in percent) with which a certain CPPI strategy yields a higher value at maturity. At least 60% of the smart beta CPPIs at maturity exceed the portfolio value of the market capitalization CPPI at maturity. The volatility-target portfolios tend to underperform the pure equity solutions in the right tail of the distribution.

Table 4. Outperformance frequencies of the CPPI strategies

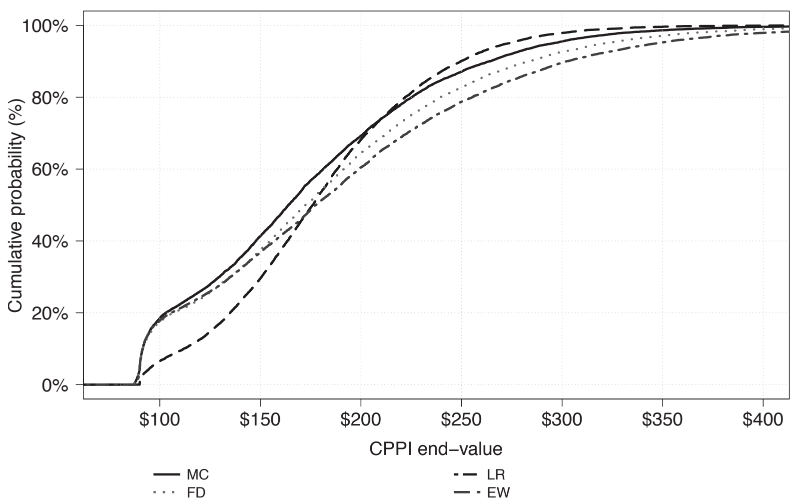
The table presents the frequency (in percent) for which the end-value of the CPPI strategy with underlying equity portfolio in the row exceeds the end-value of the CPPI strategy with underlying equity portfolio in the column. Frequencies are computed from 10,000 simulations generated with the block-bootstrap methodology described in Section 4.1. See Table 2 for details on the risky reference assets.

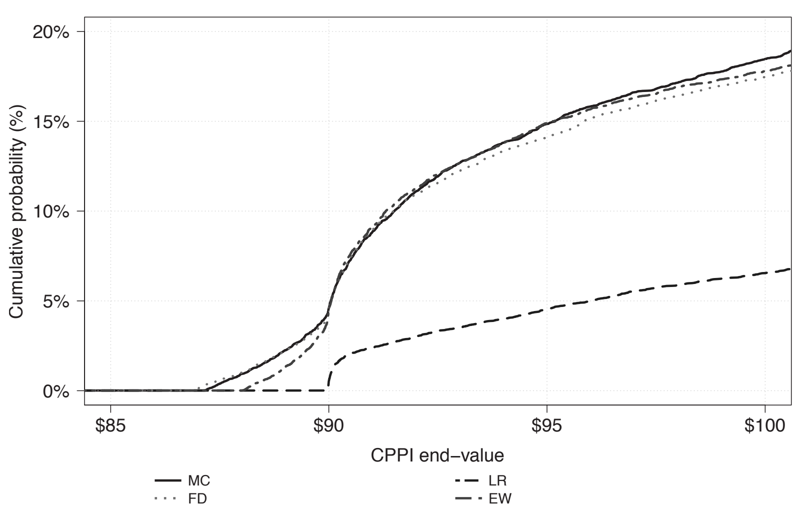
	MC	FD	EW	LR	LR-MC	LR-FD	LR-EW
MC		35	28	37	58	46	43
FD	65		38	45	63	61	55
EW	72	62		51	65	63	62
LR	63	55	49		72	65	60
LR-MC	42	37	35	28		31	28
LR-FD	54	39	37	35	69		44
LR-EW	57	45	38	40	72	56	

Figure 3. Cumulative distribution function of the CPPI strategies end-values

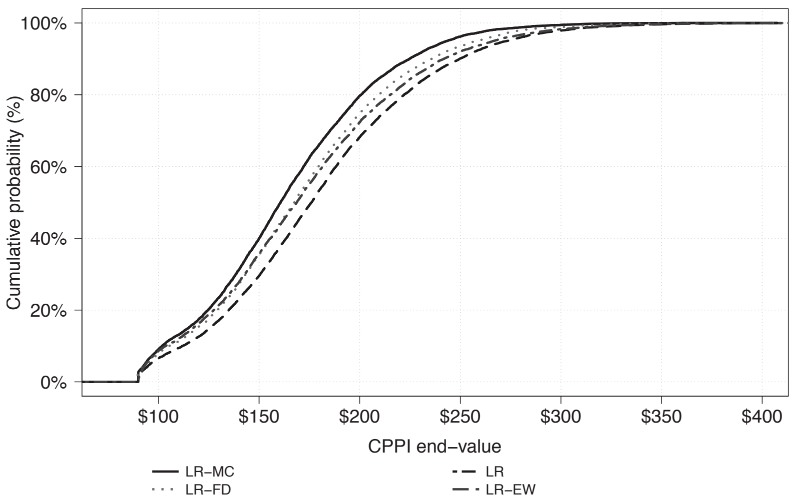
This figure presents the cumulative distribution function of the CPPI portfolio values at maturity. Cumulative distribution is computed from 10,000 simulations generated with the block-bootstrap methodology described in Section 4.1. Left plots display the complete distribution. Right plots zooms in on the gap events (i.e., $V_T < G$, where $G = \$90$). The CPPI underlying equity portfolios are the market capitalization (MC), fundamental value (FD), equally-weighted (EW) and low-risk (LR) portfolios (Panel A), and the volatility-target market capitalization (LR-MC), fundamental value (LR-FD) and equally-weighted portfolio (LR-EW) (Panel B). See Table 3 for details on CPPI settings.

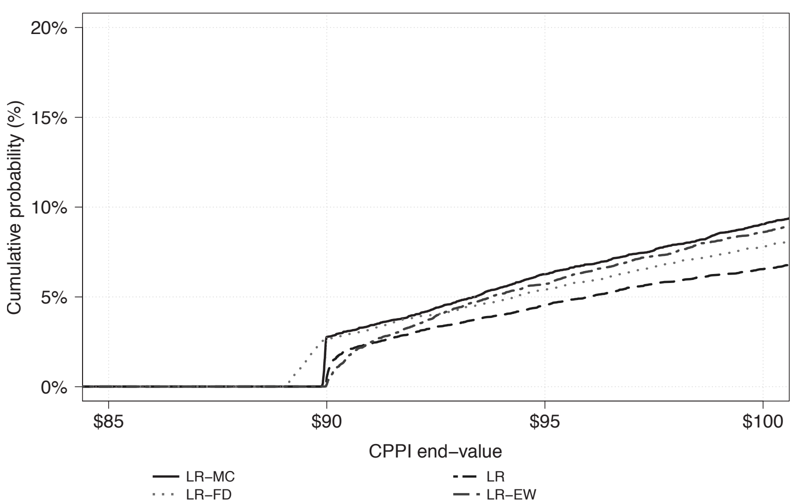
Panel A





Panel B





5.2. CPPI performance under different market conditions

The effect of the choice of equity portfolio on the CPPI performance depends on the market conditions. The average performance, volatility and the value of the interest rate are clearly three key state variables determining the performance of the CPPI investment. To evaluate the differences in

CPPI performance under different market regimes, we split the simulated price-paths into tercile groups based on *ex-post* mean returns, volatility and level of the risk-free rate. We report results on the two extreme groups per characteristic in Table 5; bullish and bearish markets in Panel A, low and high volatility markets in Panel B, and high and low risk-free rate regimes in Panel C.

In bullish markets, all CPPI strategies are highly exposed to the underlying equity portfolio, leading to a higher average end-of period return. The importance of the choice of the equity portfolio seems to increase in bearish markets. The low-risk CPPI clearly dominates the traditional market capitalization CPPI portfolio on all considered performance characteristics. The difference in CPPI performance compared with both the fundamental value and equally-weighted CPPI is limited.

In low volatility markets, all CPPI portfolios report similar results. The impact of the choice of the equity portfolio increases in high volatility markets. In high volatility markets, the low-risk and volatility-target CPPI outperform both in terms of upside potential and capital protection, and the low-risk portfolio exhibits a lower gap risk.

Finally, the interest rate is also a state variable of the market regime affecting the composition and performance of the CPPI investment. It determines the value of the floor needed to ensure the capital guarantee. Panel C of Table 5 shows that the CPPI strategies differ in terms of exposure and turnover over high and low risk-free rate regimes. The higher the risk-free rate is, the higher the exposure to the reference asset at inception, by construction. The exposure at the inception date is 12.5% when the risk-free rate is at its lowest (i.e., 0.56%) and 50% when the risk-free rate is at its maximum level over our sample (i.e., 11.70%). As the exposure to the reference asset is bounded, the CPPIs are more often fully exposed to the reference asset in periods of high interest rate, leading to a substantially lower turnover. For example, the differences in average annual turnover increase up to 76% for the market capitalization CPPI.

Regarding the effect of the interest rate on the financial performance of the CPPI investment, we find, in Panel C of Table 5, that the value of the interest rate seems to have a minor effect on the absolute performance statistics (average return and volatility). However, it has a significant impact on the relative performance statistics that compare the investment performance with the risk-free rate; the Sharpe and Sortino ratios are substantially higher in the low interest rate regime. In the case of the market capitalization and low-risk weighted portfolio as choice for the reference asset, the Sharpe ratio almost doubles from 0.42 to 0.80, and from 0.64 to 1.19, respectively. In both regimes, the choice of the low-risk equity portfolio leads to the best risk-adjusted performance for the CPPI.

Table 5. Impact of the market regime on the performance of the CPPI strategies
This table presents the performance results of the CPPI strategies under different market regimes. Based on the *ex-post* mean returns and volatility of each simulated series, we sort the 10,000 simulated series and divide these into three groups of equal size. We report results on the two extreme groups per characteristic. Panel A: top (*bullish market*) and bottom (*bearish market*) terciles based on *ex-post* returns. Panel B: top (*low volatility market*) and bottom (*high volatility market*) terciles based on *ex-post* volatility. Panel C: top and bottom terciles based on the level of the risk-free rate. See Table 2 for details on the risky reference assets and Table 3 for details on the performance measures and CPPI settings. When no gap event occurs, ES is not defined. We report this by × in the table.

	Mean	Sd	SR	SoR	Gap	ES	Exposure	Turnover
Panel A: Top and bottom terciles based on *ex-post* returns								
MC bullish	19.37	4.63	2.35	57.88	0.12	–1.96	98.83	18.46
MC bearish	0.97	3.32	–1.29	0.05	4.08	–1.17	72.90	143.52
FD bullish	20.69	5.49	2.13	71.92	0.08	–1.86	98.70	19.90
FD bearish	1.81	4.20	–0.76	0.14	4.04	–1.21	73.90	129.17
EW bullish	22.18	6.02	2.08	100.46	0.07	–1.32	98.69	19.93
EW bearish	1.85	4.38	–0.70	0.16	4.01	–0.67	72.68	134.03
LR bullish	17.42	4.50	2.21	59.54	0.00	×	99.38	10.89
LR bearish	5.42	4.74	0.11	0.69	0.54	–0.01	89.52	75.48
LR-MC bullish	16.13	3.17	2.77	83.49	0.05	–0.07	99.46	9.51
LR-MC bearish	2.85	3.26	–0.73	0.14	2.48	–0.06	86.37	95.00
LR-FD bullish	17.02	3.58	2.63	90.94	0.04	–0.79	99.47	9.46
LR-FD bearish	3.70	3.73	–0.38	0.26	2.35	–0.56	87.81	85.98
LR-EW bullish	17.57	3.90	2.50	208.10	0.00	×	99.49	10.02
LR-EW bearish	3.58	3.85	–0.39	0.27	0.13	–0.02	86.75	91.87

Panel B: Top and bottom terciles based on *ex-post* volatility								
MC low vol	13.90	7.32	1.09	7.39	0.00	×	96.98	40.76
MC high vol	5.56	7.70	0.15	0.92	4.24	–1.19	78.55	103.85
FD low vol	15.04	7.73	1.14	9.07	0.00	×	97.12	37.35
LR low vol	14.13	5.51	1.44	12.61	0.00	×	99.05	16.56
LR high vol	8.15	6.64	0.49	1.73	0.54	–0.01	90.63	61.19
LR-MC low vol	12.25	5.12	1.27	7.49	0.00	×	98.96	18.55
LR-MC high vol	6.21	6.01	0.23	0.97	2.59	–0.06	88.52	70.46
LR-FD low vol	12.96	5.34	1.32	8.99	0.00	×	99.02	17.87
LR-FD high vol	7.16	6.41	0.36	1.30	2.45	–0.57	89.61	63.80
LR-EW low vol	13.43	5.68	1.30	9.29	0.00	×	98.95	18.30
LR-EW high vol	7.00	6.60	0.33	1.25	0.13	–0.02	88.72	71.94
Panel C: Top and bottom terciles based on the level of the risk-free rate								
MC high r_f	10.66	8.19	0.42	1.56	1.00	–1.12	93.90	39.87
MC low r_f	9.34	8.73	0.80	5.33	2.23	–1.21	83.04	116.29
LR high r_f	11.79	6.37	0.64	2.27	0.16	–0.02	98.24	14.96
LR low r_f	11.16	6.93	1.19	9.83	0.21	–0.01	92.14	66.27
FD high r_f	11.78	8.76	0.51	2.01	0.98	–1.21	94.11	36.34
FD low r_f	10.40	9.41	0.83	6.28	2.22	–1.32	83.22	108.00
EW high r_f	12.42	9.55	0.54	2.27	1.04	–0.61	93.24	41.14
EW low r_f	11.20	10.06	0.84	7.11	2.17	–0.79	83.11	108.48
LR-MC high r_f	9.86	5.93	0.39	1.28	0.43	–0.08	97.68	18.35
LR-MC low r_f	9.13	6.45	1.03	6.37	1.65	–0.05	90.65	76.68
LR-FD high r_f	10.72	6.16	0.50	1.67	0.42	–0.56	98.01	16.45
LR-FD low r_f	9.98	6.74	1.09	7.56	1.62	–0.54	91.19	71.73
LR-EW high r_f	10.86	6.59	0.50	1.71	0.07	–0.02	97.52	20.03
LR-EW low r_f	10.28	7.00	1.07	8.09	0.02	–0.01	91.14	73.69

Overall, we find an improved risk-adjusted performance for the smart beta CPPIs with a clear outperformance for the low-risk CPPI. Importantly, in bearish and high volatility markets, both the low-risk and volatility-target CPPI portfolios report a lower gap risk. This result is of primary importance in the context of CPPIs since these are the equity market regimes when capital protection is most needed. Since the market condition is unknown when making the investment decision, protection buyers should carefully consider the equity portfolio they choose as an underlying.

5.3. CPPI performance under different CPPI implementations

This section reports the performance measures for different CPPI implementations. First, we show that performance is improved by allowing the multiplier to vary over time. Second, we set the floor such that capital gains over the investment horizon are (partly) protected. Third, we discuss the results for alternative CPPI implementation parameters.

5.3.1. Dynamic multiplier

Alternatively to a constant multiplier, the multiplier can be dynamically adjusted depending on market conditions. Lee et al. (2008) set a multiplier which evolves with the asset's price, i.e., increases (decreases) in up markets (down markets). Jiang et al. (2009) and Hamidi et al. (2014) implicitly propose a dynamic multiplier by allocating the budget over a fixed-income and an equity portfolio such that the portfolio's Value-at-Risk or expected shortfall is constant over time. Bonelli and Mantilla-Garcia (2014) illustrate empirically the performance of a time-varying multiplier with improved return estimates.

Because of the difficulty of forecasting the expected returns, the dynamic multiplier is often set as proportional to the inverse of the equity volatility.[7] In order to avoid unrealistically low values of the multiplier, we follow the recommendation of Caliman et al. (2013) and parameterize the dynamic multiplier as proportional to the product between the target average value for the dynamic multiplier (m) and the historical average volatility $\overline{\sigma}$:

$$m_t \equiv m\frac{\overline{\sigma}}{\hat{\sigma}_t},$$

where $m = 6$, $\hat{\sigma}_t$ is the daily volatility estimated over a 252-day rolling window[8] and $\overline{\sigma}$ is the average daily volatility of the S&P 500 index over the sample (i.e., about 1% in our case).

7. As shown by Merton (1971) and Zieling et al. (2014), this calibration is optimal for a mean-variance utility maximizing agent with constant risk-aversion.
8. Recall from Section 4 that the performance evaluation is done through historical simulation in which the simulated return series is a chain of independently chosen blocks of daily returns. In order to respect the serial dependence in the data when estimating the dynamic multiplier, we use of course the historically observed 252 days preceding the rebalancing date for which the multiplier is calibrated (instead of the 252 simulated days).

Panel A of Table 6 presents the performance for strategies implemented with a dynamic multiplier. The dynamic adjustment of the multiplier allows the CPPI to adjust to changing market conditions. All CPPI strategies report no gap risk. We notice an improved capital protection, especially for the market capitalization, fundamental value, equally-weighted and volatility-target CPPIs. Overall, the low-risk CPPI outperforms on a risk-adjusted basis, but differences in performance are substantially smaller compared with the strategies based on a constant multiplier.

Table 6. Impact of alternative multiplier and floor on the performance of the CPPI strategies

This table presents the performance results of the CPPI strategies implemented with an alternative multiplier and floor calculation. Panel A: the multiplier is dynamic; see Section 5.3.1. Panel B: the floor is set such that 90% of the portfolio gains over the investment horizon are protected and the gap risk Gap(η) and expected shortfall ES(η) (both in percent) are computed relative to the protected gains for the TIPP strategies; see Section 5.3.2. See Table 2 for details on the risky reference assets and Table 3 for details on the performance measures and CPPI settings. When no gap event occurs, ES is not defined. We report this by × in the table.

	Mean	Sd	SR	SoR	Gap	ES	Gap(η)	ES(η)	Exposure	Turnover
Panel A: Dynamic multiplier										
MC	6.91	3.27	0.46	2.46	0.00	×			30.31	401.28
FD	7.33	3.49	0.55	3.12	0.00	×			31.38	413.78
EW	7.52	3.65	0.57	3.47	0.00	×			29.57	392.28
LR	7.81	3.58	0.66	3.94	0.00	×			41.04	460.26
LR-MC	6.88	3.13	0.48	2.36	0.00	×			39.54	442.58
LR-FD	7.29	3.33	0.56	2.98	0.00	×			40.82	449.55
LR-EW	7.38	3.45	0.57	3.15	0.00	×			39.43	432.30
Panel B: TIPP										
MC	6.71	3.36	0.41	2.20			15.70	–0.39	25.87	277.43
FD	7.08	3.57	0.48	2.75			15.70	–0.48	26.01	277.29
EW	7.47	3.81	0.55	3.41			15.70	–0.26	25.69	283.91
LR	7.70	3.25	0.71	4.15			0.06	–0.37	34.82	281.18
LR-MC	6.81	2.95	0.49	2.41			15.70	–0.02	33.31	266.68
LR-FD	7.15	3.08	0.58	3.00			15.70	–0.20	33.95	269.83
LR-EW	7.37	3.19	0.62	3.46			0.02	–6.48	33.70	268.33

5.3.2. Alternative floor calculation

In the main application, the floor is set to the discounted value of the minimum acceptable portfolio value at maturity and thus grows at the risk-free rate. Depending on the investor's preferences, alternative floor definitions can be considered, e.g., the benchmark portfolio protection, maximum drawdown floor and trailing performance floor (Amenc et al., 2010). In this section, the CPPI floor is adjusted dynamically such that interim gains are (partially) locked in (Estep and Kritzman, 1988). Such a portfolio strategy is known as time-invariant portfolio protection (TIPP). At every rebalancing point, the floor $F_t^*(\eta)$ is defined as:

$$F_t^*(\eta) \equiv \max\left\{\frac{1}{(1+r_{\mathrm{f}})^{T-t}}G, \eta \sup_{s \le t}\{V_s\}\right\},$$

in which η is the percentage of the cumulated wealth, which is locked in.

Clearly, the higher floor limits the exposure to the equity portfolio. Thus, the impact of the choice of the equity portfolio on the total performance is less apparent. In Panel B of Table 6, we illustrate this for a TIPP strategy, which locks 90% of the interim capital gains. Comparing the results with the basic CPPI strategies, the TIPPs report improved downside protection of the initially guaranteed capital. Sharpe and Sortino ratios are similar for both the CPPI and TIPP strategies. The gap risk with respect to 90% of the past gains is similar for the market capitalization, fundamental value, equally-weighted and volatility-target TIPPs. The low-risk TIPP and equally-weighted with volatility target report a lower number of floor violations. As the TIPP strategy is more restrictive, the exposure to the risky reference portfolio is limited.

5.3.3. Alternative parameters

The performance of the CPPI portfolios depends on the chosen implementation parameters. The described CPPI parameters in Section 2 have an impact on the exposure to the equity portfolio and thus on portfolio performance. In this section, we redo our analysis with a higher multiplier, a higher protection level, a shorter investment horizon of one year, weekly rebalancing and 30% leveraged portfolios (i.e., b = 130%). Overall, our conclusions are robust to the changes in the parameters of the CPPI portfolios. We find that the low-risk CPPI performs similar or better as the best performing CPPI with a volatility-target equity solution as the risky reference asset.

In Panel A of Table 7, we report the results for CPPIs with a multiplier set to eight. Since currently the risk-free rate is at a low, investors seeking for yield set the multiplier often higher. The multiplier determines the participation in the upward movements of the equity portfolio, but also the exposure to unanticipated drops in the value of the portfolio. The higher the multiplier, the more the CPPI strategy behaves like a stop-loss strategy. All CPPI portfolios report a higher gap risk compared with the CPPI with the multiplier set to six. The Sharpe and Sortino ratios are comparable for both levels of the multiplier. The low-risk CPPI outperforms in terms of stability, capital protection and turnover. The volatility-target portfolios exhibit similar performance characteristics, except for the expected shortfall, which is lower for the equally-weighted equity portfolio combined with the volatility target.

In Panel B, we consider the impact of increasing the protection level from 90% to 100%. The higher the protection level, the lower the exposure to the equity portfolio and therefore the lower the upward potential of the CPPI portfolios. Results report Sharpe and Sortino ratios similar to the 90% protected portfolios. However, the CPPIs report a larger gap risk with a lower expected shortfall. In general, the performance characteristics indicate a similar impact of the reference portfolio on CPPI performance and we find an outperformance of the low-risk CPPI and equally-weighted portfolio combined with a volatility target.

Panel C of Table 7 presents the results for CPPIs with a shorter investment horizon of one year. The results are very similar compared with the CPPIs with an investment horizon of five years, although the gap risk increases with the investment horizon.

In Panel D of Table 7 we lower the rebalancing frequency from daily to weekly. Overall the results are comparable for daily and weekly rebalanced portfolios, with the advantage of substantially lower turnover. The low-risk CPPI reports a higher gap risk, but it is still lower compared with other portfolios. The market capitalization and fundamental value portfolios, and their equivalents with volatility target, report a slightly lower gap risk indicating that these portfolios are able to recover from a drawdown between two rebalancing dates.

In Panel E of Table 7, we set the maximum investment quote to 130%. Leveraged CPPIs report similar Sharpe and Sortino ratios, but also an increased gap risk. The low-risk and equally-weighted equity portfolio with a volatility target CPPI benefits from most of the upward movements in the market while offering the best protection.

Finally, consistent with actual investment practice (see, e.g., the methodology description of the S&P 500 low volatility index, as published by S&P Dow Jones index (2014)), we consider the low-risk portfolio as a combination of a selection step (investing in the one-hundred lowest volatility stocks) and a weighting step (based on inverse volatility). We show in Panel F of Table 7 that eliminating the selection step, and thus using the low-risk portfolio constructed on the full S&P 500 investment universe, leads to a combination of a low-risk portfolio with CPPI that is still superior to the market capitalization CPPI. Without the selection step, the gains are, however, partly diminished. The impact is most apparent in terms of capital protection with an increase in the gap risk. Overall, the low-risk CPPI on the full investment universe outperforms the market capitalization portfolio both in terms of upside potential and capital protection.

Table 7. Impact of parameters choice on the performance of the CPPI strategies
This table presents the performance results of the CPPI strategies implemented with alternative parameters. Panel A: $m = 8$. Panel B: $p = 1$. Panel C: $T = 1$. Panel D: weekly rebalancing. Panel E: $b = 130\%$. Panel F: low-risk portfolio including all S&P 500 stocks. See Table 2 for details on the risky reference assets and Table 3 for details on the performance measures and CPPI settings.

	Mean	Sd	SR	SoR	Gap	ES	Exposure	Turnover
Panel A: $m = 8$								
MC	10.12	8.50	0.63	2.60	5.90	–1.98	89.37	55.95
FD	11.19	9.12	0.69	3.18	6.24	–1.87	89.39	51.56
EW	11.88	9.86	0.70	3.52	6.40	–1.31	88.58	54.75
LR	11.58	6.64	0.94	4.22	1.98	–2.17	96.23	27.01
LR-MC	9.62	6.16	0.74	2.61	2.68	–2.12	95.47	30.93
LR-FD	10.48	6.40	0.83	3.25	2.55	–2.64	95.87	28.05
LR-EW	10.66	6.79	0.81	3.26	2.52	–1.69	95.35	31.56
Panel B: $p = 1$								
MC	9.24	8.13	0.54	2.43	7.99	–1.03	77.98	124.11
FD	10.25	8.80	0.60	3.01	7.69	–1.09	78.69	116.90
EW	10.98	9.48	0.62	3.44	7.57	–0.67	78.50	117.37
LR	10.81	6.71	0.82	3.87	1.13	–0.02	87.59	83.01
LR-MC	8.88	6.11	0.62	2.35	5.86	–0.05	85.26	95.03
LR-FD	9.72	6.41	0.71	2.94	5.56	–0.44	86.37	88.51
LR-EW	9.97	6.71	0.71	3.08	0.44	–0.01	86.16	90.48

Panel C: $T = 1$								
MC	9.91	15.84	0.30	1.25	3.19	–1.43	77.31	368.52
FD	11.09	17.18	0.34	1.41	3.09	–1.60	77.71	352.90
EW	12.22	18.52	0.38	1.58	3.01	–0.95	77.51	353.15
LR	11.02	12.95	0.45	1.58	0.43	–0.01	82.90	290.75
LR-MC	9.07	11.75	0.33	1.21	3.10	–0.07	81.62	310.67
LR-FD	9.91	12.36	0.38	1.36	3.09	–0.54	82.22	299.88
LR-EW	10.44	12.97	0.40	1.46	0.01	–0.02	82.10	294.09
Panel D: Weekly rebalancing								
MC	10.23	8.34	0.64	2.70	4.08	–1.97	89.89	34.08
FD	11.28	8.97	0.70	3.31	4.05	–2.25	89.85	32.20
EW	11.91	9.76	0.70	3.59	6.12	–1.82	88.72	33.35
LR	11.59	6.56	0.95	4.35	1.14	–2.14	96.12	17.63
LR-MC	9.65	6.05	0.75	2.69	1.11	–3.13	95.42	20.49
LR-FD	10.50	6.29	0.84	3.35	1.05	–3.27	95.83	19.28
LR-EW	10.64	6.73	0.81	3.30	1.57	–2.66	94.99	20.74
Panel E: $b = 130\%$								
MC	10.47	10.72	0.55	2.70	7.60	–1.61	106.09	148.61
FD	11.80	11.61	0.60	3.36	7.30	–1.70	106.70	138.73
EW	12.77	12.54	0.62	3.89	7.07	–1.05	106.35	140.45
LR	12.72	8.82	0.84	4.31	0.91	–0.02	118.40	89.43
LR-MC	10.15	8.03	0.66	2.62	5.24	–0.07	115.94	103.79
LR-FD	11.27	8.42	0.74	3.28	4.89	–0.71	117.10	95.33
LR-EW	11.60	8.84	0.74	3.46	0.26	–0.02	116.75	99.16
Panel F: Low-risk portfolio with 500 stocks								
LR (S&P 500)	12.43	8.86	0.80	4.18	3.47	–0.65	91.75	54.73

6. Conclusion

This paper contributes to the literature on the design and evaluation of capital guaranteed investment products by scrutinizing the effect of the equity portfolio choice on the performance of portfolio insurance strategies. The motivation for our research comes from the increasing popularity of CPPI strategies that use a smart beta equity index as the underlying risky index. Since the 2008-financial crisis, these products have become attractive for investors, when the track record of the smart beta index reports improved risk-adjusted returns, compared to the standard market capitalization weighted equity index, and when the investor wants to avoid the broad market downturns to which also the smart beta equity portfolios are exposed. In this paper, we emphasize that the impact of their use on CPPI performance is not straightforward because of the price-path dependency of the performance of portfolio insurance strategies, the impact of jumps on the CPPI value, the asymmetric impact of volatility on cumulative returns and the volatility cost inherent to the strategy.

Through a block-bootstrap evaluation method, we study the performance of CPPI strategies with different equity portfolios as the reference underlying index. Our results show that, for the combination of the CPPI with the low-risk portfolio, the whole is greater than the sum of its parts. The combination dominates the buy-and-hold investment in the low-risk portfolio by offering capital protection. Furthermore, it dominates the market capitalization CPPI both in terms of upside potential and capital protection. The improvement in performance for the fundamental value and equally-weighted CPPI is limited. We further show that this result of outperformance is not completely explained by the smaller exposure to the negative *vega* of CPPI portfolios when a low-risk portfolio is used. In fact, when alternative underlying portfolios are constructed that target the same volatility as the low-risk portfolio, we still find that the low-risk portfolio outperforms both in terms of upside potential and in terms of capital protection, namely it has the smallest expected shortfall risk.

For investors, regulators and designers of CPPI investment products, our main take-away messages are threefold. First, the choice of equity portfolio matters. Second, a mere analysis of the historical performance is too simplistic to describe this dependence. This is due to the path dependence and the non-linear features of the CPPI performance. We recommend to use block-bootstrap to study the impact of the choice of equity portfolio on the distribution of CPPI performance. Third, as rule of thumb, when the investor has no view on market direction, the low-risk equity investment is preferred over market capitalization, fundamental value and equal weighting, because it avoids part of the drag in performance of the negative *vega* in CPPI performance and also has lower downside, and thus gap risk, between rebalancing dates.

References

Amenc, N., Goltz, F., Grigoriu, A., 2010. Risk control through dynamic core-satellite portfolios of ETFs: Applications to absolute return funds and tactical asset allocation. J*ournal of Alternative Investments* 13 (2), 3–41.

Annaert, J., Van Osselaer, S., Verstraete, B., 2009. Performance evaluation of portfolio insurance strategies using stochastic dominance criteria. *Journal of Banking & Finance* 33 (2), 272–280.

Arnott, R. D., Hsu, J., Moore, P., 2005. Fundamental indexation. *Financial Analysts Journal* 61 (2), 83–99.

Baker, N. L., Haugen, R. A., 2012. Low risk stocks outperform within all observable markets of the world. *Working paper*.

Balder, S., Brandl, M., Mahayni, A., 2009. Effectiveness of CPPI strategies under discrete-time trading. *Journal of Economic Dynamics & Control* 33 (1), 204–220.

Ben Ameur, H., Prigent, J.-L., 2014. Portfolio insurance: Gap risk under conditional multiples. *European Journal of Operational Research* 236 (1), 238–253.

Bertrand, P., Prigent, J.-L., 2005. Portfolio insurance strategies: OBPI versus CPPI. *Finance* 26 (1), 5–32.

Bertrand, P., Prigent, J.-L., 2011. Omega performance measure and portfolio insurance. *Journal of Banking & Finance* 35 (7), 1811–1823.

Black, F., Jones, R., 1987. Simplifying portfolio insurance. *Journal of Portfolio Management* 14 (1), 48–51.

Black, F., Perold, A. F., 1992. Theory of constant proportion portfolio insurance. *Journal of Economic Dynamics & Control* 16, 403–426.

Blitz, D. C., van Vliet, P., 2007. The volatility effect: Lower risk without lower return. *Journal of Portfolio Management* 34 (1), 102–113.

Bloomberg, August 2014. Smart beta: The investing buzzword that won't – and needn't – die. Retrieved from www.bloomberg.com.

Bonelli, M., Mantilla-Garcia, D., 2014. Should a skeptical portfolio insurer use an optimal or a risk-based mutliplier? *Working paper*.

Bookstaber, R., Langsam, J. A., 2000. Portfolio insurance: Trading rules. *The Journal of Futures Markets* 20 (1), 41–57.

Caliman, T., D'Hondt, C., Petitjean, M., 2013. Determining an optimal multiplier in dynamic core-satellite strategies. *Journal of Asset Management* 14(4), 210–227.

Cesari, R., Cremonini, D., 2003. Benchmarking, portfolio insurance and technical analysis: A Monte Carlo comparison of dynamic strategies of asset allocation. *Journal of Economic Dynamics & Control* 27 (6), 987–1011.

Chow, T.-m., Hsu, J. C., Kuo, L.-I., Li, F., 2014. A study of low-volatility portfolio construction methods. *Journal of Portfolio Management* 40 (4), 89–105.

Cont, R., Tankov, P., 2009. Constant proportion portfolio insurance in the presence of jumps in asset prices. *Mathematical Finance* 19 (3), 379–401.

DeMiguel, V., Garlappi, L., Uppal, R., 2009. Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy? *Review of Financial Studies* 22 (5), 1915–1953.

Dichtl, H., Drobetz, W., 2011. Portfolio insurance and prospect theory investors: Popularity and optimal design of capital protected products. *Journal of Banking & Finance* 35 (7), 1683–1697.

Dierkes, M., Erner, C., Zeisberger, S., 2010. Investment horizon and the attractiveness of investment strategies: A behavioral approach. *Journal of Banking & Finance* 34 (5), 1032–1046.

Do, B. H., 2002. Relative performance of dynamic portfolio insurance strategies: Australian evidence. *Accounting & Finance* 42 (3), 279–296.

Estep, T., Kritzman, P., 1988. Time-invariant portfolio protection: Insurance without complexity. *Journal of Portfolio Management* 14 (4), 38–42.

Etzioni, E. S., 1986. Rebalance disciplines for portfolio insurance. *Journal of Portfolio Management* 13 (1), 59–62.

Hamidi, B., Maillet, B., Prigent, J.-L., 2014. A dynamic autoregressive expectile for time-invariant portfolio protection strategies. *Journal of Economic Dynamics & Control* 46, 1–29.

Jiang, C., Ma, Y., An, Y., 2009. The effectiveness of the VaR-based portfolio insurance strategy: An empirical analysis. *International Review of Financial Analysis* 18 (4), 185–197.

Lee, H.-I., Chiang, M.-H., Hsu, H., 2008. A new choice of dynamic asset management: The variable proportion portfolio insurance. *Applied Economics* 40 (16), 2135–2146.

Leland, H. E., 1980. Who should buy portfolio insurance? *Journal of Finance* 35 (2), 581–594.

Merton, R. C., 1971. Optimum consumption and portfolio rules in a continuous-time model. *Journal of Economic Theory* 3 (4), 373–413.

Perold, A. F., 1986. Constant proportion portfolio insurance. Harvard Business School.

Politis, D. N., Romano, J. P., 1994. The stationary bootstrap. *Journal of the American Statistical Association* 89 (428), 1303–1313.

Scherer, B., 2011. A note on the returns from minimum variance investing. *Journal of Empirical Finance* 18 (4), 652–660.

Sortino, F. A., Price, L. N., 1994. Performance measure in a downside risk framework. *Journal of Investing* 3 (3), 59–64.

S&P Dow Jones index, 2014. S&P low volatility index methodology. http://us.spindices.com/indices/strategy/sp-500-low-volatility-index.

Theiler, U., 2011. Risk-minimising investment strategies - Embedding portfolio optimisation into a dynamic insurance framework. *Journal of Risk Management in Financial Institutions* 4 (4), 334–369.

Weng, C., 2013. Constant proportion portfolio insurance under a regime switching exponential Lévy process. *Insurance: Mathematics and Economics* 52 (3), 508–521.

Zieling, D., Mahayni, A., Balder, S., 2014. Performance evaluation of optimized portfolio insurance strategies. *Journal of Banking & Finance* 43, 212–225.