Imputation of income variables in a survey context and estimation of variance for indicators of poverty and social exclusion

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Outline

Motivation & Introduction
- Goal
- Nonresponse in SILC
- SDAP at the SFSO
- IVEware
- Swiss SILC09 : match with a register file

Imputation of Income Data with Generalized Calibration and GB2 Distribution : Illustration with SILC Data

Variance Estimation Using Linearization for Laeken Indicators

Goal

Through a household survey, we collect some income variable y.
We intend to do inference to the population in publishing estimated values with confidence intervals for several indicators of poverty and social exclusion.

Press release

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in this country, the Gini index
is of 29% ± 1.5%.
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- Diff. levels of nonresponse & sources of variance.

Need of procedure to impute the missings and estimate the variance.

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- European project yielding comparable indicators on poverty, social welfare and exclusion and generally living conditions.
- In CH : about 7,500 responding households and 12,500 responding individuals, rotative panel on 4 years.
- Among the missing values, those for income variables are the most problematic :
  - On individual level : >100 income components are surveyed,
  - on HH-level : 30 income components.
  - Without proper treatments (re-weighting and/or imputation), inequality measures are biaised.
  - Informative knowledge about the income distribution is a great advantage.

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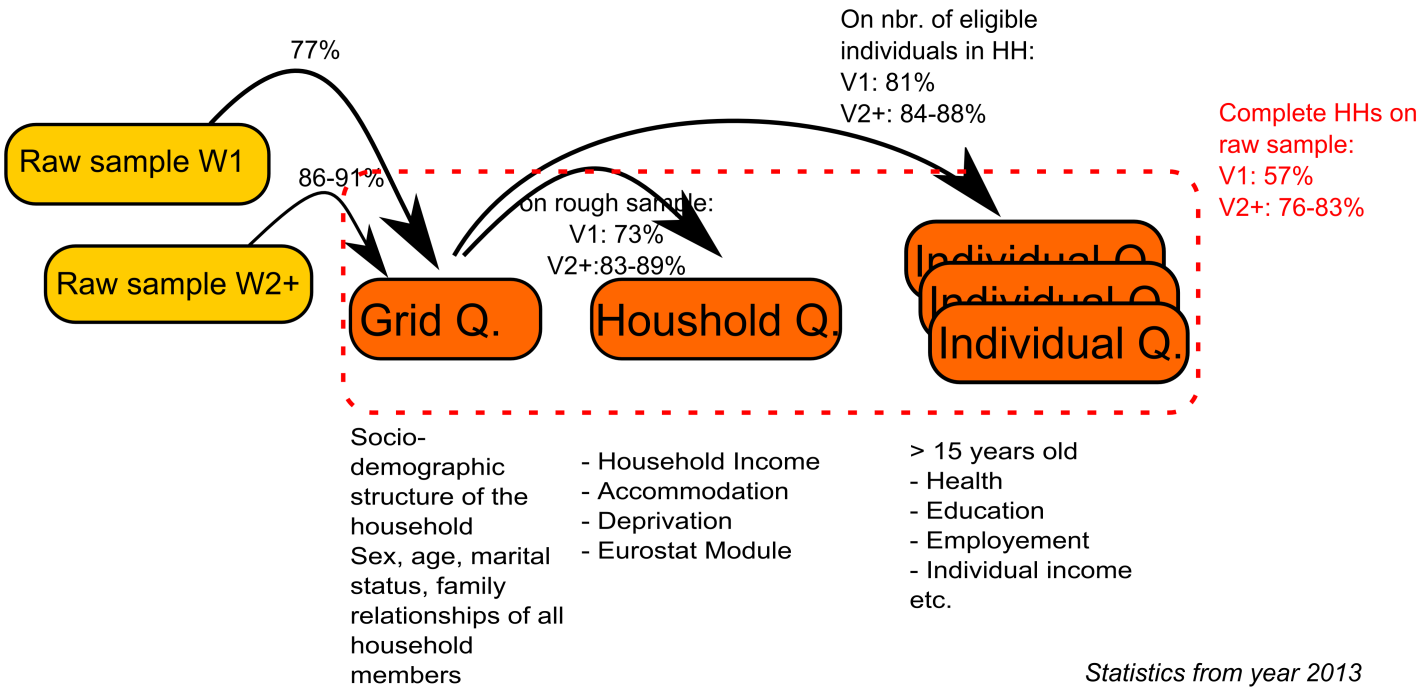
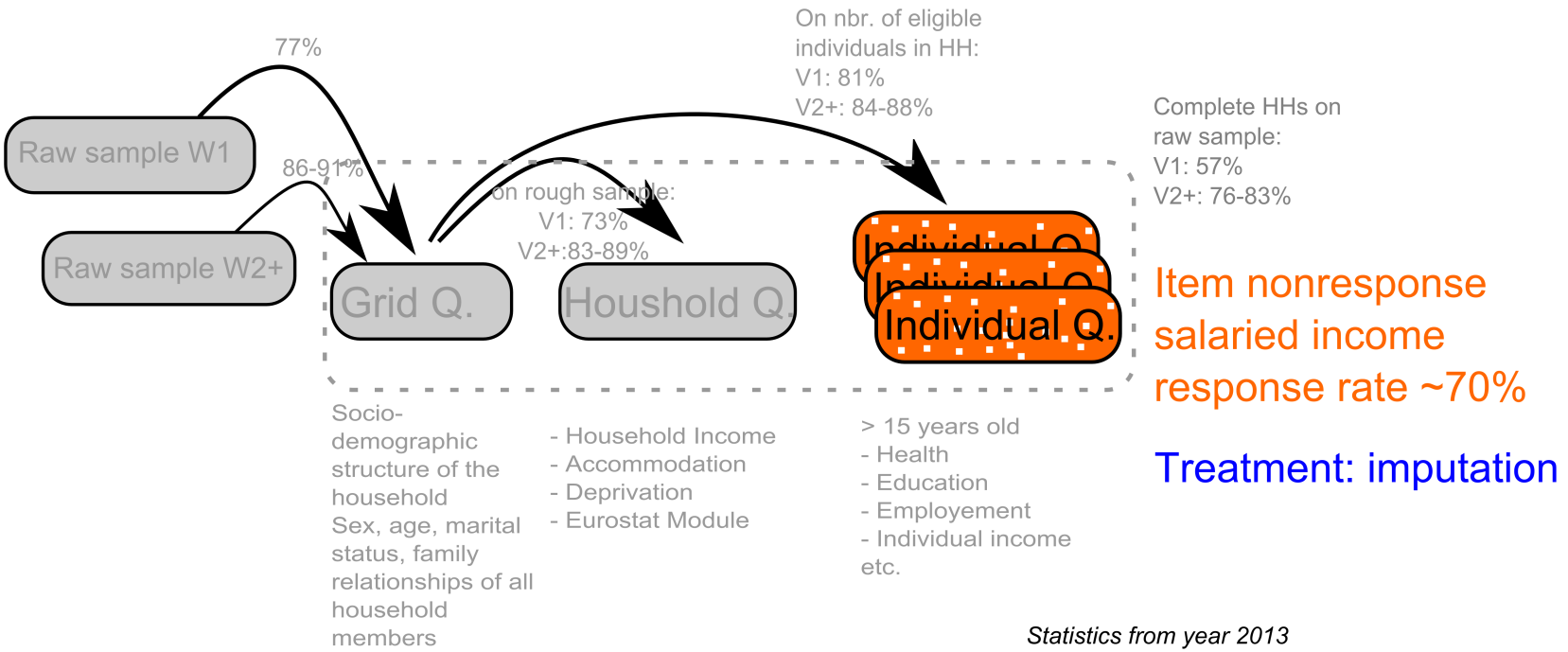
Response rates in SILC

Questionnaire NR appears at many different stages in the survey process, treated by reweighting.



Response rates in SILC. Example : salaried income



Statistics from year 2013

Statistical Data Preparation at the SFSO (regarding nonresponse to monetary variables)

- Quality control & editing of survey data (coherence control rules, validity rules, comparison with other sources at micro-/macro-level, matching with registers)
- Treatment for extreme-values and/or outliers
- Total/questionnaire NR : **re-weighting** (model NR via segmentation trees, calibrations),
- Item NR : multiple **imputations** (50 values) via regression model with IVEware software (Raghunathan et al., 2001) among imputation classes
- Imputation rate in SILC : 1.4 - 48.8% depending on the income component.

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Problems met with IVEware

- The income distribution is not normal, neither log-normal.
- Cannot cope with NMAR nonresponse (Little and Rubin, 2002).
- Lack of transparency, insufficient output about adjusted models.
- Extreme values can destabilize the adjusted models : lack of robustness (Chambers, 1986 ; Hulliger, 1999).
- Cannot take the survey weights into account.
- Choice of number of multiple imputations to be conducted not easy.

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SILC09 : match with a register file

CATI survey

salary
15,000
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2,506
45,000
110,000
.
84,100

CCO register

salary
15,900
67,140
3,050
40,152
115,000
33,000
84,100

- The SILC CATI-data could be matched with the register of the Central Compensation Office (CCO).

- We applied the true NR mechanism observed for CATI-data -salaried income- to the register-data. $\Rightarrow$ 30% missing values. We impute and compare to the true known values.
- That NR mechanism is non ignorable (NMAR).
- $corr(y_{CATI}, y_{CCO}) = 84\%$.

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Imputation of Income Data with Generalized Calibration and GB2 Distribution : Illustration with SILC Data

- Imputation strategy
- GB2 distribution and inequality (Laeken) indicators
- Generalized calibration
- How to choose & combine auxiliary/instrumental variables
- Results
- Conclusions - Imputation

Variance Estimation Using Linearization for Laeken Indicators

Income distribution, GB2 & imputation strategy

Two main ideas :

1. use the good fit and properties of the generalized beta distribution of the second kind *GB2* fitted on the income data in a regression imputation strategy,
2. use generalized calibration to compensate for NMAR nonresponse.

Imputation procedure I

1. For the respondents to y, produce adjusted weights, w_k^{cal}, through generalized calibration. These weights can compensate even for NMAR NR. A GB2-adjustment based on these weights is used to evaluate their relevance.
2. Order the income variable by increasing weighted ranks $R_m^w \in [0, n_r]$.
3. Compute normal scores by using the van der Waerden's method (Conover, 1999), based on ranks :

$$Q^w = \Phi^{-1}\left(\frac{R^w}{n_r + 1}\right)$$

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Imputation procedure II

4. Imputation : adjust a classical weighted multiple linear regression model on the normally shaped Q^{wcal} and predict $\hat{Q}_o^{wcal}$ for the nonrespondents.
5. Back transformation : obtain imputed ranks from the predicted normal scores (quantiles) : $\hat{R}_o = \Phi(\hat{Q}_o^{wcal}) \in]0, 1[$.
6. The imputed values $\hat{R}_o$ of each nonrespondents intercalate between the respondent's ranks, by renumbering all these ranks from 0 to $n = n_r + n_o$, one defines the imputed ranks. The predicted income is then estimated for example by simple interpolation.

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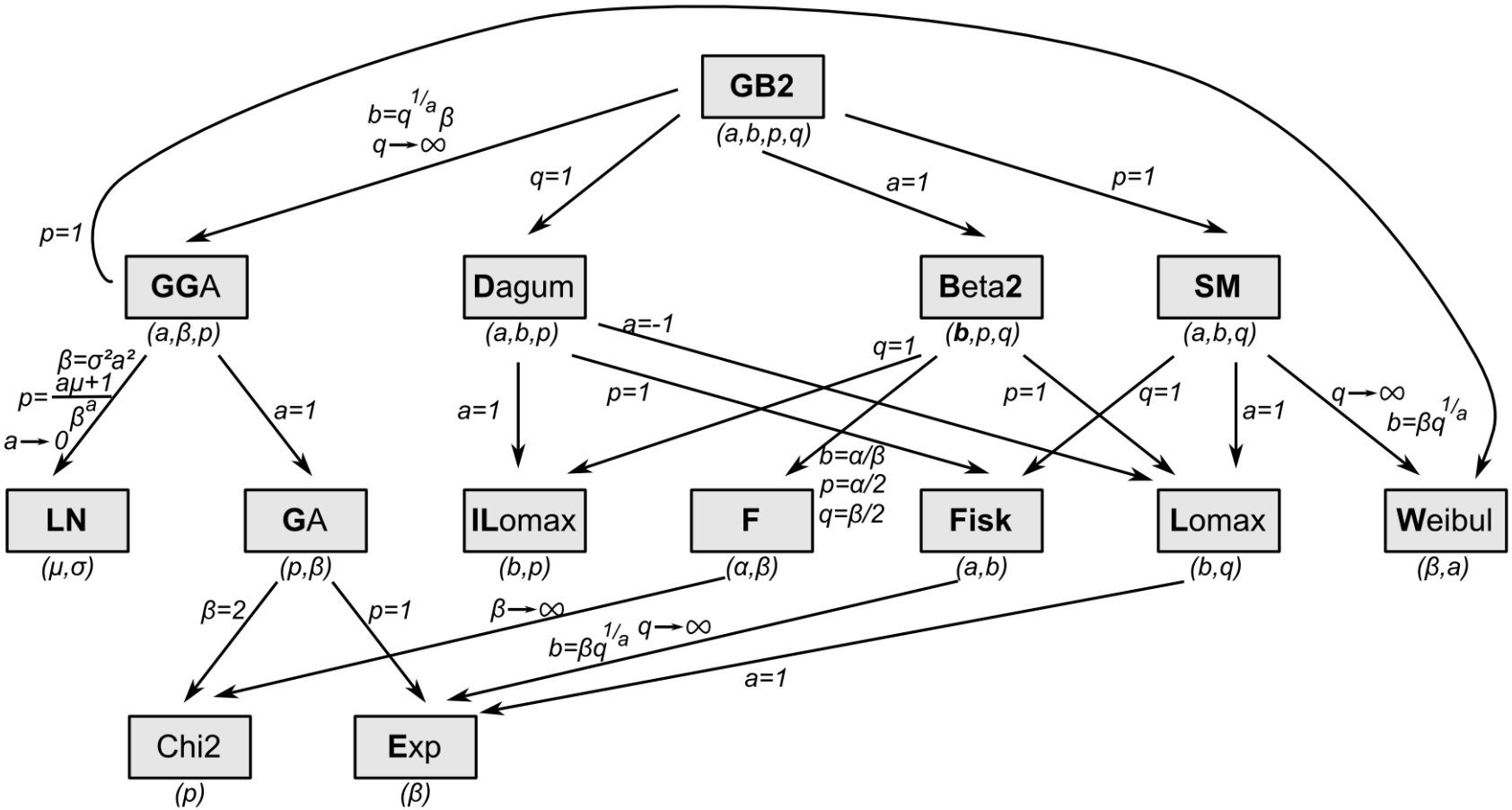
Generalized beta distribution of the second kind (GB2)

- four parameters distribution : $GB2(a, b, p, q)$ (McDonald, 1984).
- Empirical studies on income - see for ex. Kleiber and Kotz (2003) ; Jenkins (2007) ; Dastrup et al. (2007) ; Sepanski and Kong (2008) ; Jones et al. (2011) ; McDonald et al. (2013) - show that the GB2 fits well with such data and it is often more suitable than other four-parameter distribution.
- Results from AMELI 2011 confirms for EU-SILC.
- Explicit formulae for the inequality measures, McDonald (1984) ; Graf (2009) ; AMELI 2011.

Many other probability distributions can be seen as special cases of the GB2



Generalized calibration I

- Calibration methods have been introduced by the reference articles of Deville and Särndal (1992) and Deville et al. (1993).
- Generalized calibration has been presented and applied by Le Guennec and Sautory (2002) ; Sautory (2003) ; Deville (2002).
- Kott (2006) ; Osier (2013) ; Lesage and Haziza (2013b,a) ; Park and Kim (2014) studied generalized calibration and its ability to correct nonresponse.
- The objective is to estimate a population total $t_y = \sum_U y_k$, ideally by $\hat{t}_y = \sum_s w_k y_k$ if all the units of s are available,
- more realistically based on the respondents' subset : $\hat{t}_y^{cal} = \sum_{k \in r} w_k^{cal} y_k$, where w_k^{cal}, $k \in r$, are the calibrated weights satisfying to the constraints $\sum_{k \in r} w_k^{cal} \mathbf{x}'_k = \hat{\mathbf{t}}_x$.

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Generalized calibration II

- the calibrated weights are of the form $w_k^{cal} = w_k F(\mathbf{z}_k.\boldsymbol{\lambda})$
  F : calibration function from $\mathbf{R}^J \rightarrow \mathbf{R}$ (can even be observation-dependent), usual choices are available : linear, raking ratio, logit, truncated (Deville et al., 1993).
- Simplest case : $F(u) = 1 + u$ (linear), leads to

$$\hat{t}_{ylinG} = \hat{\mathbf{t}}_x' \hat{\mathbf{B}}_{rzx} + \sum_{k \in r} w_k e_k^g,$$

where $e_k^g = y_k - \mathbf{x}_k.\hat{\mathbf{B}}_{rzx}$ are the residuals of the instrumental regression of y on the J auxiliary variables $\mathbf{x}_k.$ on sample r, with the J instrumental variables $\mathbf{z}_k.$ (Deville, 2000, 2002).

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Choice of auxiliary/instrumental variables combination

- z_k. only needs to be known on the sub-sample r of respondents to y.
- Asymptotically, in the case of full response, all the choices of calibration functions are equivalent to the linear.
- But : in the presence of nonresponse, the choice of calibration function reflects implicit assumptions on the nonresponse model (Lesage and Haziza, 2013a,b).
- Although we could verify this statement, praxis has shown that a good choice of the combination of auxiliary/instrumental variables is much more influential !

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Why generalized calibration ?

- calibration $\leftrightarrow$ WLS regression
  generalized calibration $\leftrightarrow$ WIV regression,
- If the NR mechanism is imperfectly modelled, or if NMAR NR, or errors in some aux. var. : calibration estimator is biased,
- generalized calibration can be better if one finds a good combination of auxiliary and instrumental variables.
- ↯ no procedure available to identify good instruments a priori, only a posteriori checks are possible.

WLS regression ($\leftrightarrow$ calibration) :

$$
\begin{array}{rl}
U-level: & y_k = \mathbf{x}_k.\hat{\mathbf{B}}_U + e_k^U,\ k \in U, \\
p(s)\downarrow & \text{sampling} \\
s \subset U: & y_k = \mathbf{x}_k.\hat{\mathbf{B}}_s + e_k^s,\ k \in s, \\
\Psi(r|s)\downarrow & \text{NR mechanism} \\
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$$\hat{\mathbf{B}}_r = (\mathbf{X}_r'\hat{\mathbf{D}}_r\mathbf{X}_r)^{-1}\mathbf{X}_r'\hat{\mathbf{D}}_r\mathbf{y}$$
$$\hat{\mathbf{D}}_r = \mathrm{diag}(1/\pi_k \cdot 1/\widehat{\psi}_k)$$

Problem : ψ_k unperfectly modelled / NMAR NR / errors in X_r
Consequence : $\hat{\mathbf{B}}_r$ biased estimator of $\hat{\mathbf{B}}_U$.

WIV regression ($\leftrightarrow$ generalized calibration) :

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Conditions on instruments

However, if we dispose of some other variables grouped in matrix $\mathbf{Z}_r$, such that, on r :

(i) $\frac{1}{n_r}\sum_{k\in r}\frac{1}{\pi_k}\frac{1}{\widehat{\psi}_k}z_{kj}e_k^U = O(n_r^{-\frac{1}{2}})$ for all $j = 1, ..., J$: the instrument $\mathbf{z}_{.j}$ is valid or *exogenous*.

(ii) $\left(\frac{1}{n_r}\mathbf{Z}_r'\hat{\mathbf{D}}_r\mathbf{X}_r\right)^{-1}$ exists and is finite.

Then $\hat{\mathbf{B}}_U$ can still be estimated via the so-called *weighted instrumental variable regression estimator* $\hat{\mathbf{B}}_{rzx}$ and e_k^g can be taken as an estimate of e_k^U, $k \in r$.

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(i) $\frac{1}{n_r}\sum_{k\in r}\frac{1}{\pi_k}\frac{1}{\widehat{\psi}_k}z_{kj}e_k^U = O(n_r^{-\frac{1}{2}})$ for all $j = 1, ..., J$: the instrument $\mathbf{z}_{.j}$ is valid or *exogenous*.

(ii) $\left(\frac{1}{n_r}\mathbf{Z}_r'\hat{\mathbf{D}}_r\mathbf{X}_r\right)^{-1}$ exists and is finite.

Then $\hat{\mathbf{B}}_U$ can still be estimated via the so-called *weighted instrumental variable regression estimator* $\hat{\mathbf{B}}_{rzx}$ and e_k^g can be taken as an estimate of e_k^U, $k \in r$.

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Novel idea : check via GB2-fit

Best possible GB2-adjustment = the one obtained on the full dataset (without NR)

Iterative process until satisfactory GB2-fit obtained, given two lists :

- auxiliary variables : explain y & total known on s.
- instruments : explain the NR & observed at least on r.

1. With some combination of aux. and instr. variables produce weights compensating for NR through generalized calibration.
2. Relying on these weights, adjust a GB2 on the income data.
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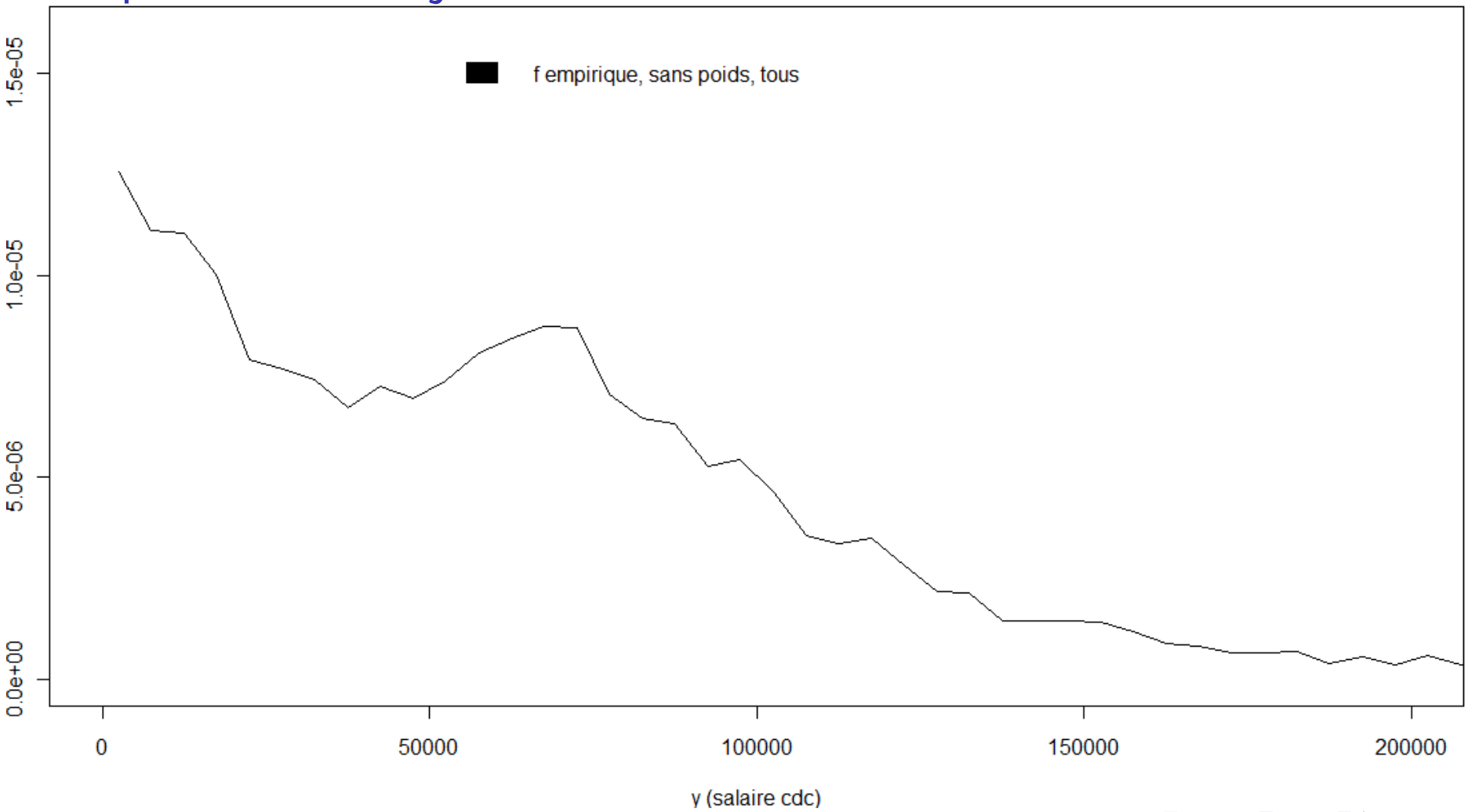
Example : GB2 adjustments on SILC09, salaried income



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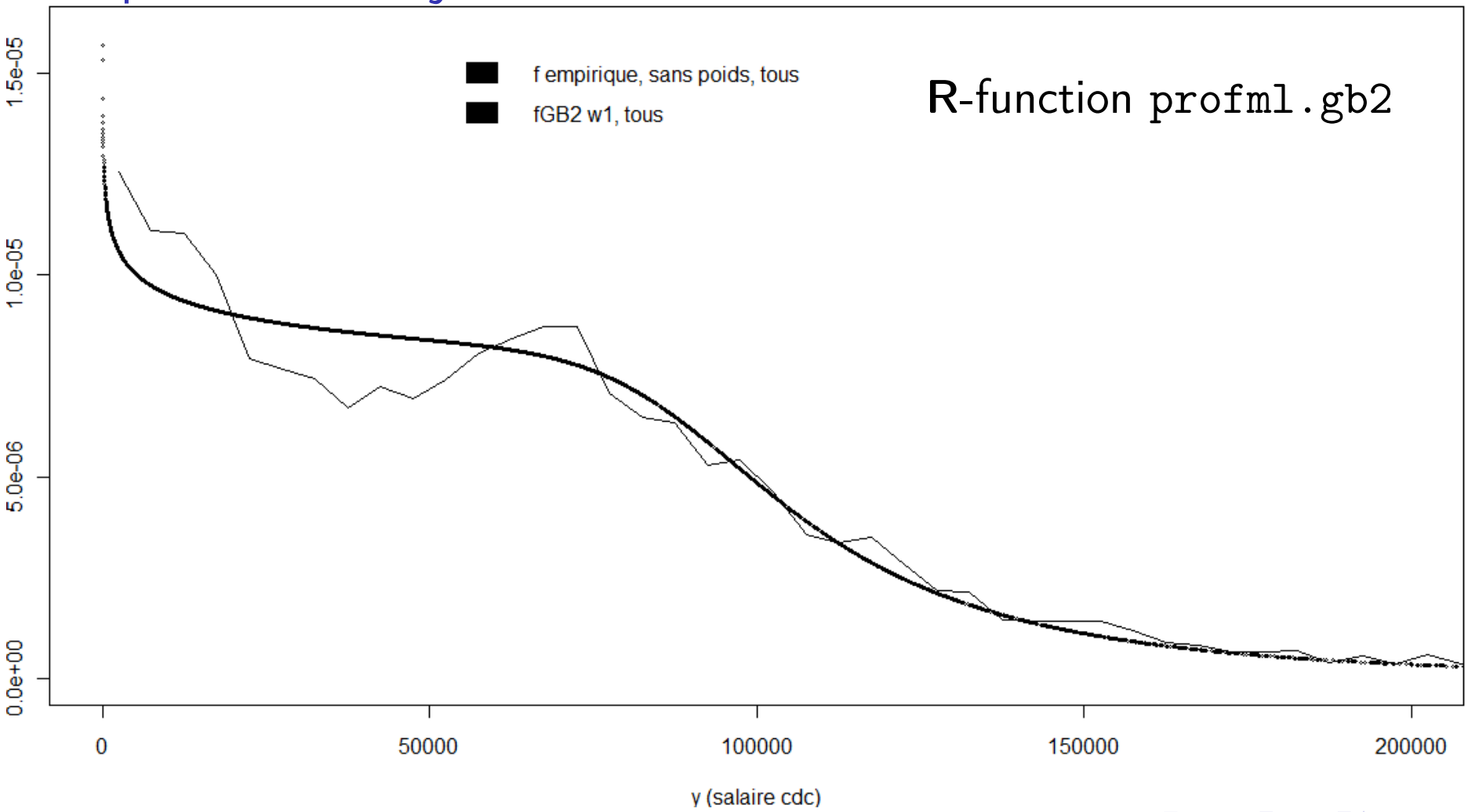

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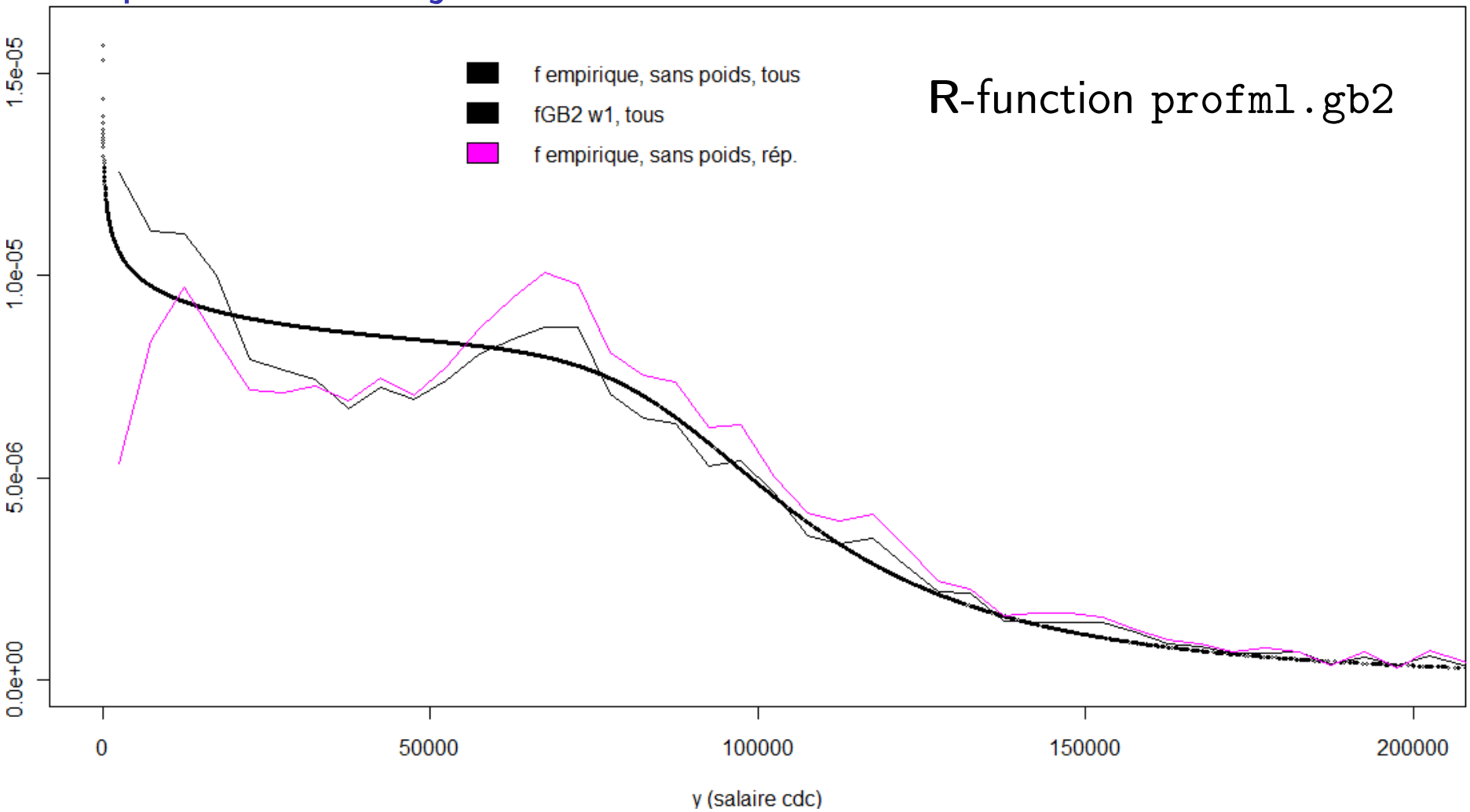

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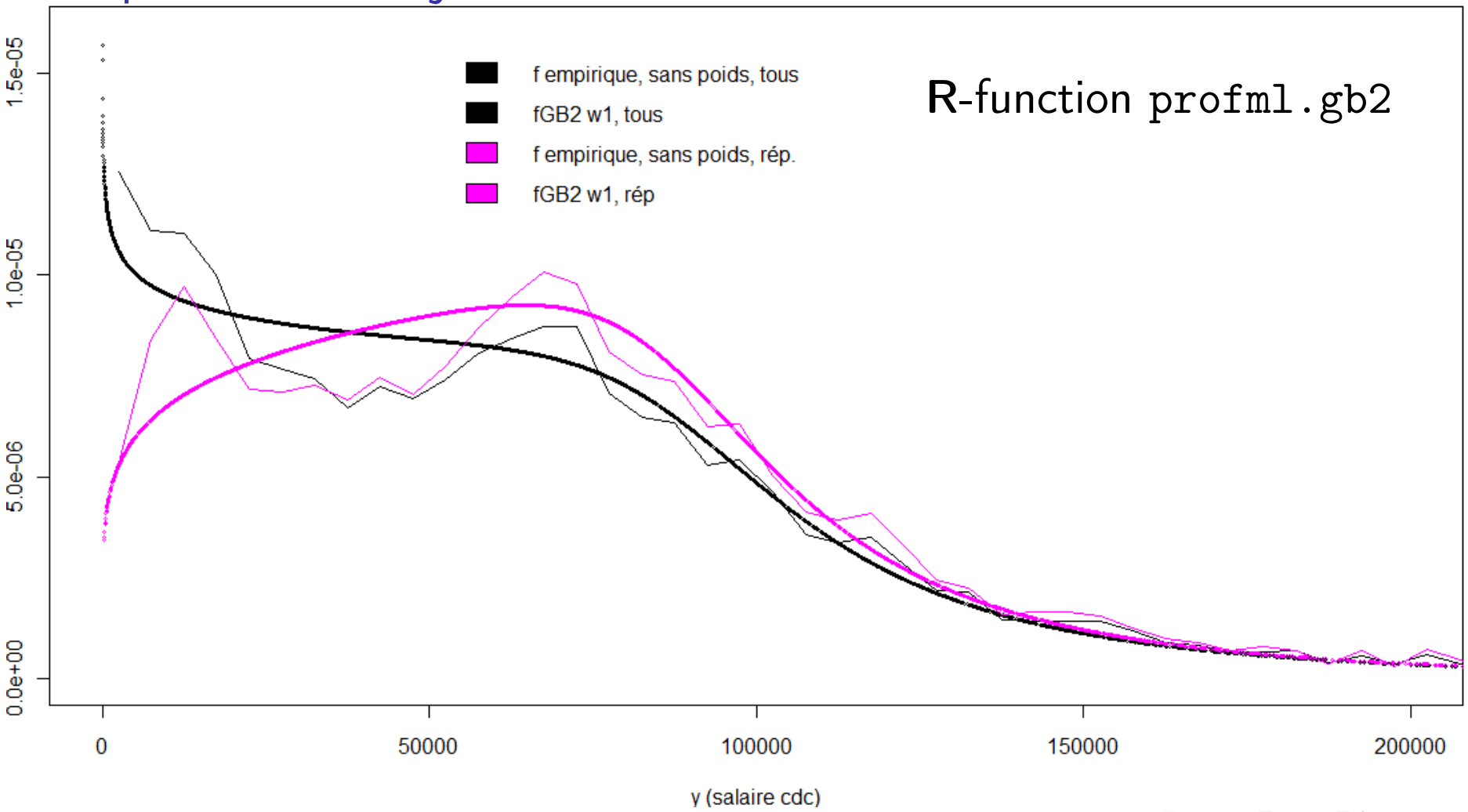

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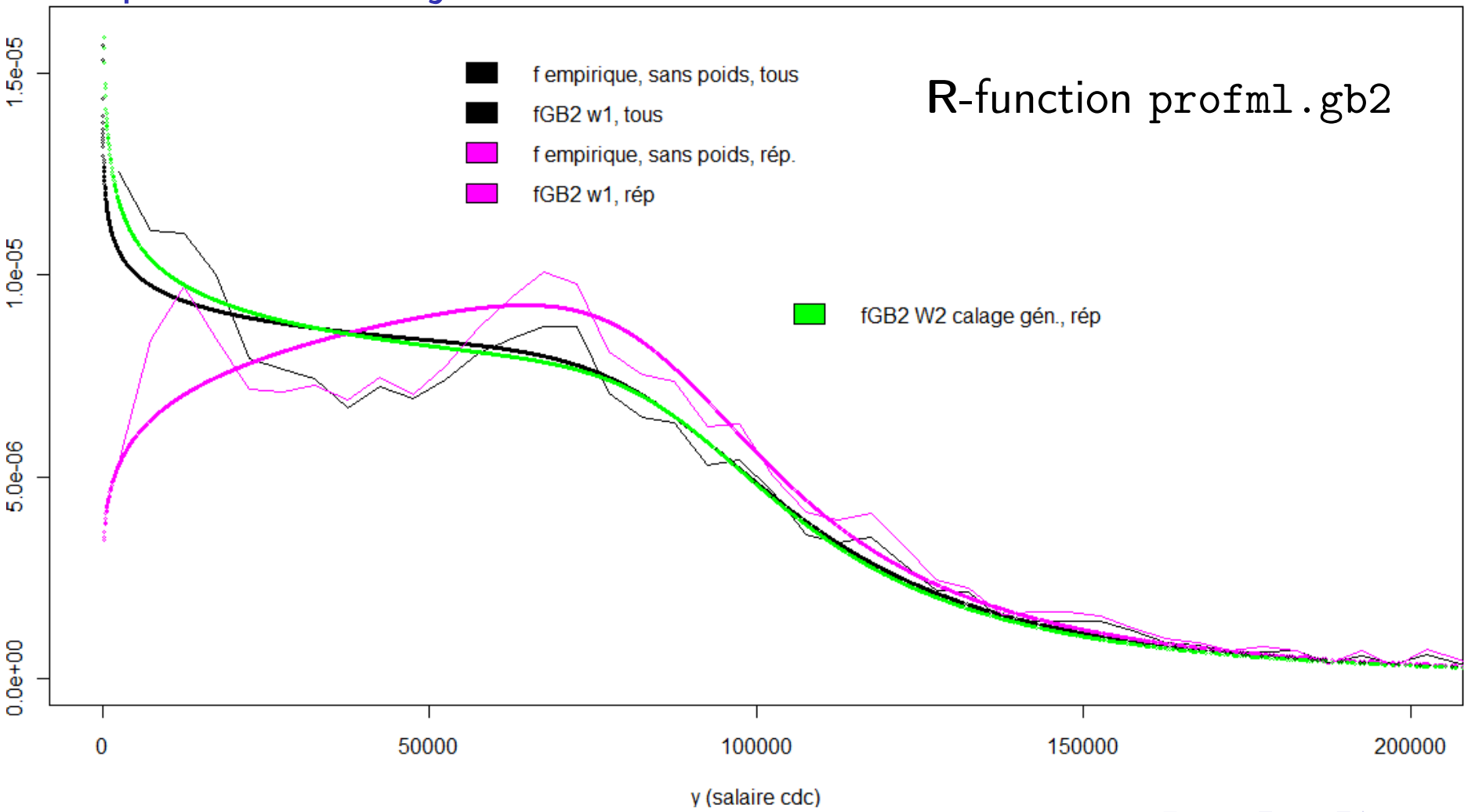

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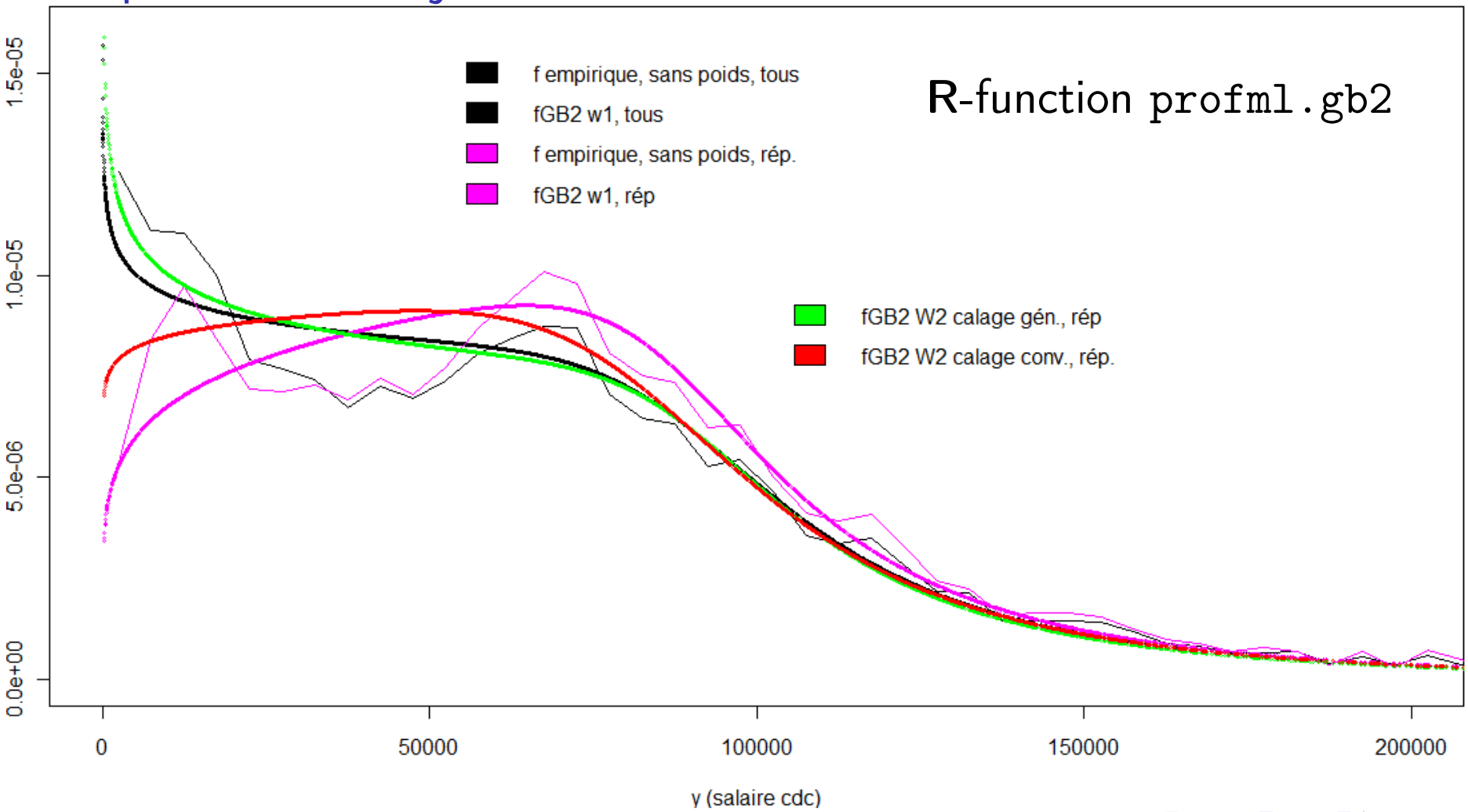
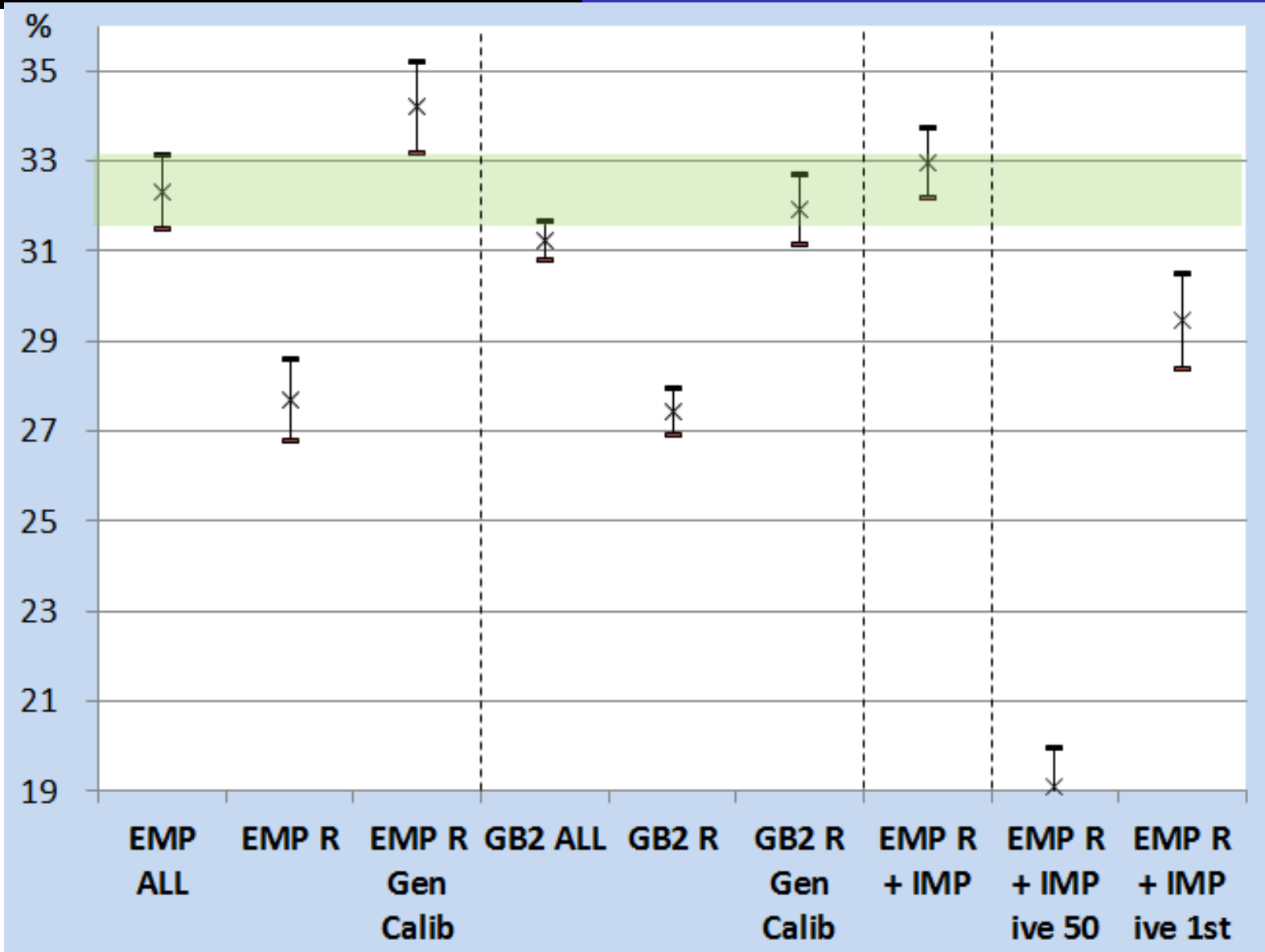


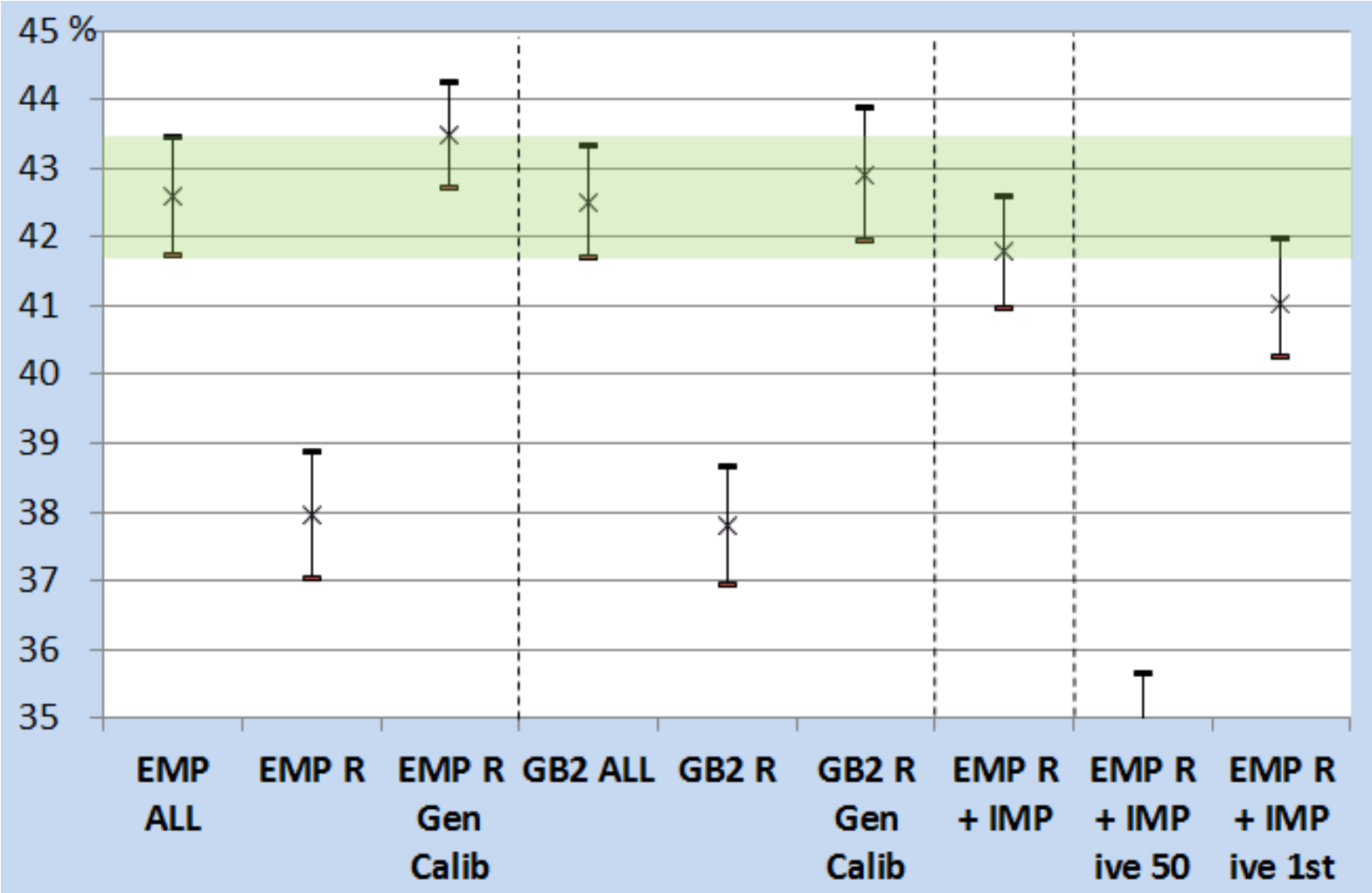
Results SILC09, salaried income

ARPR



Results SILC09, salaried income

GINI



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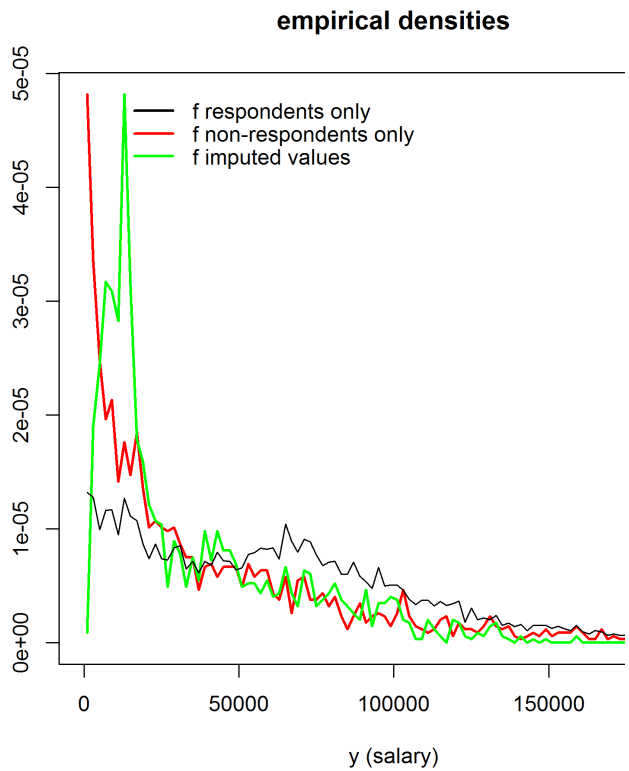
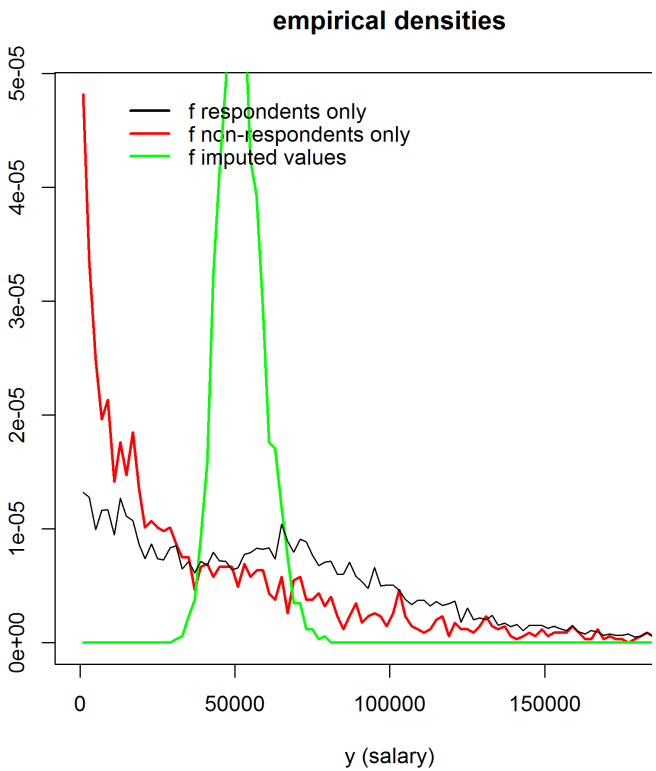
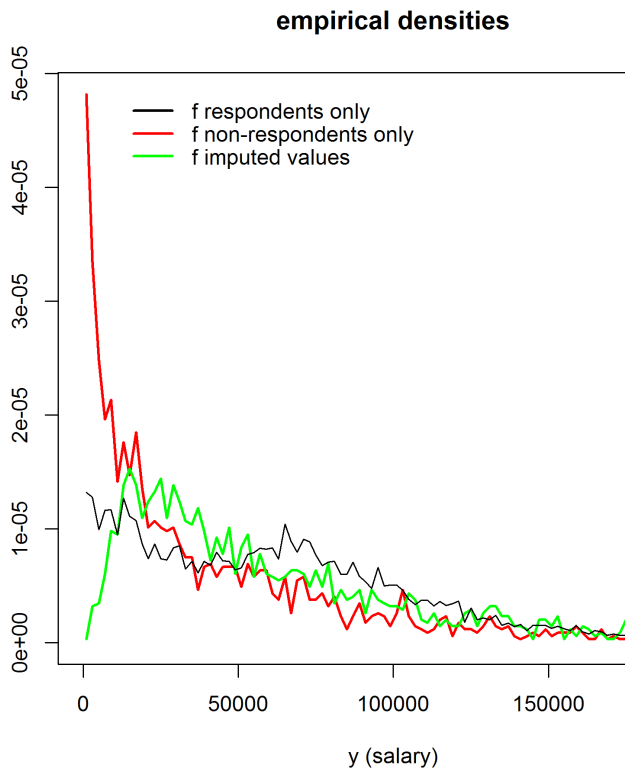


Our method.



IVEware, mean on 50 imp.



IVEware, 1 imp.

Conclusions - Imputation method

- We used the GB2 adjustment as a graphical test to identify the best combination of auxiliary/instrumental variables defining the gen. calib. $\Rightarrow$ weights compensating for NMAR NR.
- the choice of good combination of aux. and instr. variables is more important than the choice of the calibration function.
- Imputations fit and respect the natural distribution of income variables.
- Significatively better results than IVEware.

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Outline

Motivation & Introduction

Imputation of Income Data with Generalized Calibration and GB2 Distribution : Illustration with SILC Data

Variance Estimation Using Linearization for Laeken Indicators

- Generalized linearization
- Empirical influence functions for poverty indicators
- Estimation of income density function
- Results from simulations
- Conclusion for generalized linearization

Generalized linearization technique

- Usable for estimating the variance of sample-based non linear statistics (Deville, 1999 ; Demnati and Rao, 2004)
- Relies on the concept of influence functions proposed in robust statistic (Hampel, 1974)
- Applied by Osier (2009) for several EU-SILC indicators

Key result : Under asymptotic conditions, that are in principle satisfied if the sample is "large enough" : the variance of the estimated total of *estimated linearized variable* or *empirical influence function* $\hat{z}_k$ is an approximation of the variance of the (complex) statistic $\widehat{A}$.

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Influence functions for poverty and inequality measure

See Osier (2009) ; Langel and Tillé (2011, 2013) ; Graf and Tillé (2014)

Example : $A = \mathrm{ARPT}$ At risk of poverty threshold

$$\hat{z}_k^{\mathrm{ARPT}} = -\frac{0.6}{f(\hat{q}_{50})}\frac{1}{\hat{N}}[\mathbf{1}_{[y_k \leq \hat{q}_{\mathbf{50}}]} - 0.5]$$

$\hat{z}_k^{\mathrm{ARPR}}$, $\hat{z}_k^{m_p}$ and $\hat{z}_k^{\mathrm{RMPG}}$ also require to estimate $f(\cdot)$ in several points of the income distribution.

Estimation of income density function f

Deville (1999) ; Osier (2009) propose to proceed through Gaussian kernel density estimation

(1) $$\widehat{f_1}(x) = \frac{1}{h\sqrt{2\pi}} \frac{1}{\hat{N}} \sum_{k \in S} w_k \exp\left[-\frac{(x-y_k)^2}{2h^2}\right]$$

- h is the *bandwidth* that Osier estimates by $\hat{h} = \hat{\sigma}\hat{N}^{-0.2}$
- $\hat{\sigma}$ the empirical standard deviation of the income variable $\mathcal{Y}$.
- The estimation of σ is non robust (sensitive to extreme values)
- In surveys, one encounter often some concentration of observations around some values what may cause trouble with a fixed bandwidth.

(2) Estimate through the logarithm

$$\widehat{f}_2(x) = \frac{\widehat{f}_1(log(x))}{x}$$

Diminishes problems due to extreme values in the GK density estimation

(3) Nearest neighbours with minimal bandwidth $\widehat{f}_{NNMB}(x) = \frac{p(x)}{nh(x)}$

At each point of the distribution, use at least p neighbours and a minimal bandwidth $h(x) \geq h_{opt}$ where h_{opt} is the *rule of thumb* of Silverman (1986) to determine the bandwidth.
This solution is more robust and avoids problems arising from the concentration of observations around some values.

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Results from simulations

Relative bias of the variance

10,000 SRSWR from Swiss SILC09 data

y = salaried income, $N = 7,922$

Indicator	Sample size (sampling rate)								
	$n = 500(6.3\%)$			$n = 750(9.5\%)$			$n = 1000(12.6\%)$		
GINI	-0.03			-0.03			-0.02		
QSR	-0.00			0.00			0.00		
	$\hat{f}_1$	$\hat{f}_2$	$\hat{f}_3$	$\hat{f}_1$	$\hat{f}_2$	$\hat{f}_3$	$\hat{f}_1$	$\hat{f}_2$	$\hat{f}_3$
ARPT	0.07	0.05	**0.13**	0.06	0.04	**0.10**	0.06	0.03	0.08
ARPR	-0.05	-0.04	-0.02	-0.05	-0.04	-0.01	-0.06	-0.05	-0.02
RMPG	**0.61**	**0.12**	**0.15**	**0.60**	**0.11**	0.08	**0.59**	0.09	0.05
MEDP	**0.73**	**0.17**	**0.18**	**0.72**	**0.16**	**0.10**	**0.72**	**0.15**	0.07
MED	0.07	0.04	**0.13**	0.06	0.04	**0.10**	0.05	0.03	0.07

Conclusion for generalized linearization I

Generalized linearization permits for large samples ($\geq$ 1000 units) to estimate variances for widely used complex indicators, but one must be careful how the income density is estimated in such computations :

- The Gaussian kernel density estimation method currently implemented in most cases is not recommended without at least using the logarithm.

Conclusion for generalized linearization II

- The nearest neighbour method which also imposes a minimum bandwidth, may yield even better results, especially if there are agglomerations of observations with certain values in the given data. However, this method requires setting a minimum number p of neighbours.
- Attention : In presence of nonresponse, we have conducted other studies showing that ignoring the stage of imputation in the variance estimation may lead to severe underestimation of the total variance.

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- Prof. Yves Tillé, University of Neuchâtel,
- Dr. Philippe Eichenberger and Dr. Jean-Pierre Renfer from section METH at the Swiss Federal Statistical Office (SFSO),
- SFSO : this work was carried out within the framework of a cooperation agreement between the Institute of Statistics of the University of Neuchâtel and the SFSO.
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Thank you for your attention !

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