Original Article

Generalized marginal risk

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ABSTRACT An important aspect of portfolio risk management is the analysis of the overall risk with respect to the assets' allocations. Marginal risk is the traditional tool, however, this metric is only meaningful when a position is levered or when the proceeds from the sale of a position are put in the cash account. This article proposes an extension of the traditional marginal risk approach as a means of overcoming this deficiency. The new concept addresses situations where the change in a position results in changes to other positions as well. An illustration is provided for a real-world portfolio.

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INTRODUCTION

Portfolio risk management requires assessing the aggregated risk of a portfolio. Nowadays, the industry standards for such risk measures are the Value-at-risk (VaR) and the expected shortfall (ES). VaR is the value of the portfolio return such that lower returns will only occur with, at most, a preset probability level, which typically is between 1 and 5 per cent. ES is the expected return below the VaR. A large stream of research has been devoted to their unbiased and efficient estimation; see, for example, Duffie and Pan (1997), Jorion (2001) and Gourieroux and Jasiak (2009). However, as mentioned by Litterman (1996a), aggregated risk measures are useful for monitoring risk but they do not provide much guidance for practical risk management.

In order to manage the risks of a portfolio effectively, the risk impact of new trades and/or reallocations within the portfolio must be assessed. Moreover, the sources of risk in the current portfolio need to be identified. Generally speaking, the aim of the portfolio risk analysis is to gain insight through the sensitivity of the aggregated risk with respect to the portfolio holdings, as well as the attribution of the portfolio risk to the underlying components through decomposition. In the financial literature, these concepts are referred to as *marginal risk* and *component risk*, respectively. For an introduction, the reader is referred to Litterman (1996a, 1997a, b) and Jorion (2001).

The marginal risk aims at measuring how investment decisions affect the risk profile of the portfolio. Mathematically, this is simply the gradient of the portfolio risk measure with respect to the allocation weights. It is therefore defined as the linear approximation of the change in the portfolio risk when a position is altered while all other positions remain the same. This sensitivity measure is precise for infinitesimal changes; however these are rarely the case in practice. A portfolio manager would typically relate this marginal change with the expected return on the various assets in the portfolio in order to increase its risk-adjusted performance.

The risks of the portfolio holdings generally do not sum up to the overall portfolio risk. While this is desirable from a diversification viewpoint, this does not allow for a straightforward decomposition of risk in the portfolio. The component risk is an attempt at measuring the proportion of the portfolio risk that can be attributed to each of the individual positions. With this metric, a portfolio manager is able to target the most significant sources of risk; the so-called *hot spots* (Litterman, 1996a). The mathematical construction of component risk is based on the Euler decomposition of positive homogeneous functions and was first used by Litterman (1996a) for decomposing the standard deviation of a portfolio. Garman (1996, 1997) used this approach for decomposing the portfolio VaR. This mathematical decomposition expresses the portfolio risk as a sum of each position's weight times the marginal risk of the position. The marginal risk is therefore a building block of the component risk. While the component risk provides a way to decompose the portfolio risk, we stress that it needs to be interpreted carefully. Indeed, a pure mathematical decomposition of the portfolio risk measure does not, in general, guarantee that the results are meaningful in the financial sense; see Sharpe (2002) and Qian (2006) for further details.

While appealing by nature, the traditional definition of marginal risk (and by construction the component risk) faces a main drawback. The concept relies on the gradient, so that it measures the risk impact in the portfolio when altering a position while keeping the others constant. Therefore, it is applicable when a position is levered or when a position is reduced and the proceeds are put in the cash account of the portfolio. However, it leads to flawed results when the adjustments are carried out through capital in- or outflows in the portfolio, as well as reallocations within the portfolio, for instance. This is obviously caused by the change in all of the relative positions in the portfolio when there are capital adjustments. This important point is discussed by Sefton *et al* (2002), where the authors analyze the role of the cash-account in a multivariate Gaussian framework with volatility as risk measure. They investigate the so-called 'cash-puzzle', that is, why the marginal risk of the cash-account is always zero, and therefore does not have a marginal contribution to the (active) portfolio risk. They also introduce a new measure, which quantifies the sensitivity of the portfolio tracking error to the trade of marginally increasing the active position of a stock by selling the benchmark (market) portfolio.

Our article extends the study by Sefton *et al* (2002) to more generic risk metrics and non-specific distributional assumptions, by introducing a novel approach, which we name *generalized marginal risk*. As for the traditional marginal risk, the new concept allows a portfolio manager to measure the sensitivity of the portfolio to new marginal allocations. However, it ensures that potential effects on the other positions are correctly taken into account. This therefore helps analyze the risk impact under more general and realistic scenarios. Moreover, we show that the generalized marginal risk and its traditional counterpart are directly related. Therefore, once the marginal risks have been estimated, a portfolio manager can run a generalized sensitivity analysis in a straightforward manner. We illustrate the usefulness of the new metric with a real-world portfolio within the elliptical framework.

The remainder of this article is organized as follows. The next section briefly reviews the marginal and component risk, the subsequent section introduces the new concept of generalized marginal risk, the penultimate section illustrates the new metric and the final section concludes.

MARGINAL RISK

First, let us introduce some notation. We assume that the portfolio is composed of n assets whose arithmetic returns are given by the $(n \times 1)$ random vector $\boldsymbol{R} = (R_1, \ldots, R_n)'$ and whose allocation weights are collected into the $(n \times 1)$ vector $\boldsymbol{w} = (w_1, \ldots, w_n)'$. In addition, the portfolio consists of a cash account with weight $w_0 = 1 - \sum_{i=1}^{n} w_i$, which we assume as risk-free. The portfolio is levered when $w_0 < 0$. We denote the portfolio return by $P(\boldsymbol{w})$ to emphasize the fact that it is a function of $\boldsymbol{w}$. For underlying arithmetic returns, this function is linear, that is, $P(\boldsymbol{w}) = \boldsymbol{w}'\boldsymbol{R}$. Finally, we denote the risk measure of the portfolio return by $\rho(\boldsymbol{w}) = \rho\{P(\boldsymbol{w})\}$ where the notation again emphasizes the fact that it is a function of $\boldsymbol{w}$. We assume that $\rho(\boldsymbol{w})$ is at least once differentiable.

Definition (marginal risk)

For the risk measure ρ, the marginal risk of the ith asset in the portfolio, denoted by ρ_i^m, is defined as the change in the portfolio risk measure for an infinitesimal change in the allocation to the ith asset. Formally, this is the derivative of $\rho(\boldsymbol{w})$ with respect to w_i:

$$\rho_i^m(\boldsymbol{w}) = \frac{\partial}{\partial w_i}\rho(\boldsymbol{w}). \tag{1}$$

For convenience, the n marginal risks of the portfolio are collected into the $(n \times 1)$ vector $\boldsymbol{\rho}^m = (\rho_1^m, \ldots, \rho_n^m)'$; $\boldsymbol{\rho}^m$ is the gradient of $\rho(\boldsymbol{w})$.

In some cases, the marginal risks ρ_i^m can be computed explicitly (see the penultimate section). If a parametric model is available for the distribution of $P(\boldsymbol{w})$, the derivatives with respect to the holdings are either obtained in closed-form or can be computed efficiently by numerical methods. For Monte Carlo approaches (that is, when the portfolio return distribution is obtained by simulation), the so-called *brute force* or *before and after* approach described in Dowd (1998) can be used. In this case, a marginal Δw is added to each w_i iteratively, the risk of the new portfolio (that is, with the new allocation) is computed, and the derivative is approximated by first difference. However, this approach is time demanding in high-dimensions. Jorion (2001) and Hallerbach (2003) provide guidelines for the estimation of marginal VaR using analytical and simulation methods; see also Tasche (1999) and Gourieroux *et al* (2000).

The component risk is based on the Euler decomposition of positive homogeneous risk measures ρ, that is, when $\rho(\lambda\boldsymbol{w}) = \lambda\rho(\boldsymbol{w})$ for $\lambda > 0$. This is the case for common risk measures such as the VaR and ES; see, for example, McNeil *et al* (2005). Under this condition, the risk measure can be decomposed as

$$\begin{aligned}\rho(\boldsymbol{w}) &= \sum_{i=1}^{n} w_i \frac{\partial}{\partial w_i}\rho(\boldsymbol{w}) = \sum_{i=1}^{n} w_i \rho_i^m(\boldsymbol{w}) \\ &= \sum_{i=1}^{n} \rho_i^c(\boldsymbol{w}),\end{aligned} \tag{2}$$

where the term $\rho_i^c = w_i \rho_i^m(\boldsymbol{w})$ is defined as the component risk of the ith asset in the portfolio.

As we can see in (2), the marginal risk is a building block of the component risk. We can therefore interpret ρ_i^c as the linear approximation of the risk impact of the ith allocation if we remove the corresponding asset in the portfolio and put the proceeds in the cash account. On the other hand, if a new asset is included in the portfolio, the component risk approximates the risk impact of the new position in the augmented portfolio; this is known as the *incremental risk* of the new asset. In both cases, the larger the position size the worse the linear approximation.

It is important to emphasize two limitations of the marginal and component risk. First, both concepts are based on a marginal argument, and this must be kept in mind when interpreting the measures. Indeed, consider the case where a particular position accounts for half the risk according to the component risk. This implies that a small percentage increase in that position will increase the portfolio risk, as much as a combined similar percentage increase in all other positions. However, it does not imply that eliminating that position entirely will reduce risk by half. Indeed, as the size of the position of a contributor to risk is reduced, the marginal contribution of that position to risk will be reduced as well (Litterman, 1996b, p. 29).

Second, the marginal risk is the linear approximation of the risk impact of *leveraging* the corresponding position in the portfolio. Indeed, the gradient is the linear approximation of the change in the portfolio risk when *a position is altered while all others remain constant*. In order to illustrate this point, let us assume that $w_0 = 0$ (that is, $\sum_{i=1}^{n} w_i = 1$), which is the case for a fully funded portfolio with an empty cash account. The marginal risk does introduce leverage for this case since altering a position and leaving all others constant implies $w_0 \neq 0$. In the case where we are interested in the portfolio risk impact for an increase in size of a certain position we obviously have $w_0 < 0$, even for an infinitesimal increase of any portfolio weight. This point is often neglected in practice; this leads to false conclusions in a sensitivity analysis, where capital might be shifted in the portfolio but the sensitivity measure relies on the leveraged scenario. The differences can be substantial, as illustrated in our empirical analysis.

GENERALIZED MARGINAL RISK

In practice, it is also important to consider scenarios where the change in a position results in the change of other positions as well. This is typically the case when there are capital in- and outflows in the portfolio since all percentage allocations change in this setting. Another example is when the increase in a position is funded by the reduction of other positions. In this case, the weights of other components must be rescaled accordingly when computing the sensitivity of the portfolio risk. The concept of generalized marginal risk aims at dealing with these scenarios.

Definition (generalized marginal risk)

Let us denote by $\tilde{\boldsymbol{w}}_i(\delta) = \boldsymbol{w} + \delta \boldsymbol{a}_i(\boldsymbol{w})$ the new allocation vector of the portfolio after allocating an additional δ per cent of the investor's total wealth to the ith asset. The function $\boldsymbol{a}_i\colon \Re^n \rightarrow \Re^n$ describes how an additional δ per cent investment in the ith position affects the positions. It can be interpreted as an allocation scheme (examples are given below). The generalized marginal risk of the ith asset in the portfolio, denoted by ρ_i^{gm}, is defined as the derivative of $\rho(\tilde{\boldsymbol{w}}_i(\delta))$ with respect to δ, evaluated at $\delta = 0$:

$$\begin{aligned}\rho_i^{gm}(\boldsymbol{w}) &= \frac{\partial}{\partial \delta}\rho(\tilde{\boldsymbol{w}}_i(\delta))|_{\delta=0} \\ &= \left(\frac{\partial}{\partial \boldsymbol{w}}\rho(\tilde{\boldsymbol{w}}_i(\delta))\right)'\Bigg|_{\delta=0} \times \frac{\partial \tilde{\boldsymbol{w}}_i(\delta)}{\partial \delta}\Bigg|_{\delta=0} \\ &= \boldsymbol{\rho}^{m}(\boldsymbol{w})' \boldsymbol{a}_i(\boldsymbol{w}). \end{aligned} \tag{3}$$

The n generalized marginal risks of the portfolio are collected into the $(n \times 1)$ vector $\boldsymbol{\rho}^{gm} = (\rho_1^{gm}, \ldots, \rho_n^{gm})'$ for convenience.

Our definition of $\tilde{\boldsymbol{w}}_i(\delta)$ assumes that the allocation scheme is linear in δ, but this does not need to be the case. In many real situations, the linear setup is sufficient (see below). This also simplifies the presentation. Expression (3) shows the direct relationship between the marginal and the generalized marginal risk in the linear setting. Therefore, once the marginal risks of the positions have been computed, a portfolio manager can run a generalized sensitivity analysis in a straightforward manner.

In order to gain insight on this new concept, we assume that an investor has an additional δ to invest in the portfolio, expressed as a percentage of the total wealth (that is, the current wealth plus the additional wealth). If the investor adds this capacity to the ith asset, the new allocation vector reads $\tilde{\boldsymbol{w}}_i(\delta) = \boldsymbol{w}(1-\delta) + \delta\boldsymbol{e}_i$ where $\boldsymbol{e}_i$ denotes the ith $(n \times 1)$ basis vector. The term $\boldsymbol{w}(1-\delta)$ represents the effect of adding δ amount of new capital to the portfolio, while the term $\delta\boldsymbol{e}_i$ reflects the fact that the ith position is increased by δ. In this case, $\boldsymbol{a}_i(\boldsymbol{w}) = (\boldsymbol{e}_i - \boldsymbol{w})$ and using (3) yields $\boldsymbol{\rho}_i^{gm}(\boldsymbol{w}) = \boldsymbol{\rho}^m(\boldsymbol{w})'(\boldsymbol{e}_i - \boldsymbol{w})$. By stacking (column-wise) n times the weight vectors in a $(n \times n)$ matrix $\boldsymbol{W}$, we can express the $(n \times 1)$ vector of generalized marginal risks as

$$\boldsymbol{\rho}^{gm}(\boldsymbol{w}) = \boldsymbol{\rho}^m(\boldsymbol{w})'(\boldsymbol{I}_n - \boldsymbol{W}), \qquad (4)$$

where $\boldsymbol{I}_n$ denotes the $(n \times n)$ identity matrix.

Another example arises when a portfolio manager is interested in changes of the portfolio risk when reallocating capital within the portfolio. For instance, consider the case where the δ increase in the ith position is financed through an equal reduction of all other positions. After this adjustment, the allocation vector reads $\tilde{\boldsymbol{w}}_i(\delta) = \boldsymbol{w} + \delta\boldsymbol{\lambda}_i$ where $\boldsymbol{\lambda}_i$ denotes a $(n \times 1)$ vector whose components are all equal to $-1/(n-1)$ except the ith position which equals one. In this case, $\boldsymbol{a}_i(\boldsymbol{w}) = \boldsymbol{\lambda}_i$ and using (3) yields $\boldsymbol{\rho}_i^{gm}(\boldsymbol{w}) = \boldsymbol{\rho}^m(\boldsymbol{w})'\boldsymbol{\lambda}_i$. In vector form we obtain

$$\boldsymbol{\rho}^{gm}(\boldsymbol{w}) = \boldsymbol{\rho}^m(\boldsymbol{w})' \times \left(\frac{n}{n-1}\boldsymbol{I}_n - \frac{1}{n-1}\boldsymbol{J}_n\right), \qquad (5)$$

where $\boldsymbol{J}_n$ denotes the $(n \times n)$ matrix of ones. Obviously, we do not necessarily need to finance the reallocation by reducing all other positions proportionally. By modifying the vector $\boldsymbol{\lambda}_i$, a portfolio manager has full control on how assets are to be shifted within the portfolio. For instance, the investor could look for the direction $\boldsymbol{\lambda}_k$, which reduces risk the most in order to find the most suitable portfolio adjustments to increase the kth position.

As a last example, consider the increase in the ith position financed through leverage. The allocation vector reads $\tilde{\boldsymbol{w}}_i(\delta) = \boldsymbol{w} + \delta\boldsymbol{e}_i$. In this case, $\boldsymbol{a}_i(\boldsymbol{w}) = \boldsymbol{e}_i$ and using (3) we obtain $\boldsymbol{\rho}_i^{gm}(\boldsymbol{w}) = \boldsymbol{\rho}_i^m(\boldsymbol{w})$. Therefore, the generalized marginal risk for a scenario of leverage equals the traditional marginal risk.

Finally, note that if we multiply the generalized marginal risk with the corresponding asset weight, we obtain a linear approximation of the change in the portfolio risk if a position is closed and the proceeds are treated as defined through the function $\boldsymbol{a}_i(\boldsymbol{w})$. Contrary to the marginal risk, the decomposition of the portfolio risk in terms of generalized marginal risks is not possible since $\sum_{i=1}^n w_i\rho_i^{gm} \neq \rho$ in general. We could consider the portfolio risk ρ as a function of $\boldsymbol{w}$ and δ, and then perform the Euler decomposition. Since δ equals zero for the current portfolio we would simply obtain the traditional component risk. Alternatively, the products $w_i\rho_i^{gm}$ could be rescaled in order to sum up to the portfolio risk. However, we prefer to have a unique decomposition of risk with a meaningful financial interpretation.

ILLUSTRATION

We illustrate the concept of generalized marginal for a real-world equity portfolio whose asset returns are modeled by a multivariate elliptical distribution. An appealing property of elliptical distributions is that they are closed under affine transformations. Moreover, elliptical distributions are numerically tractable, even for very high dimensions. Finally, there is numerous empirical evidence that multivariate return data of similar types look roughly elliptical (see, for example, McNeil *et al*, 2005). Note that we could account for the co-skewness in this setting by using the Cornish-Fisher extension; see Boudt *et al* (2008).

A $(n \times 1)$ random vector $\boldsymbol{R}$ of asset returns, which follows an elliptical distribution is denoted by $\boldsymbol{R} \sim E_n(\boldsymbol{\mu},\boldsymbol{\Sigma},\psi)$ where $\boldsymbol{\mu}$ is a $(n \times 1)$ location vector, $\boldsymbol{\Sigma}$ is a $(n \times n)$ dispersion matrix and ψ is the characteristic generator. As the class is closed under affine transformations, the distribution of the linear portfolio $P(\boldsymbol{w}) = \boldsymbol{w}'\boldsymbol{R}$ is obtained in closed-form as $P(\boldsymbol{w}) \sim E_1(\boldsymbol{w}'\boldsymbol{\mu}, \boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w}, \psi)$.

Risk measures such as the VaR or the ES are obtained in a straightforward manner within the elliptical framework. Indeed, since $P(\boldsymbol{w}) \sim E_1(\boldsymbol{w}'\boldsymbol{\mu},\boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w},\psi)$ we have $P(\boldsymbol{w}) = (\boldsymbol{w}'\boldsymbol{\mu} + (\boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w})^{1/2}Z)$ where $Z \sim E_1(0,1,\psi)$. Using the latter expression, we obtain closed-form expressions for the VaR and ES as

$$\begin{aligned}\rho(\boldsymbol{w}) &= \rho\{\boldsymbol{w}'\boldsymbol{\mu} + (\boldsymbol{w}'\Sigma\boldsymbol{w})^{\frac{1}{2}}Z\} \\ &= \boldsymbol{w}'\boldsymbol{\mu} + \rho\{Z\}(\boldsymbol{w}'\Sigma\boldsymbol{w})^{\frac{1}{2}}, \qquad (6)\end{aligned}$$

where, for example, $\rho\{Z\} = -1.645$ for the VaR at the 95 per cent confidence level within the Gaussian framework; see Landsman and Valdez (2003) for other elliptical distributions and other risk measures. In the sequel, we will focus on the ES at the 95 per cent confidence level.

Using expression (6), it is straightforward to calculate the derivative in (1). This yields the following expression for the vector of marginal risks:

$$\boldsymbol{\rho}^m(\boldsymbol{w}) = \boldsymbol{\mu} + \rho\{Z\}(\boldsymbol{w}'\Sigma\boldsymbol{w})^{-1/2}\Sigma\boldsymbol{w}. \qquad (7)$$

The vector of component risks are easily obtained from (2) using (7). Finally, the vector of generalized marginal risk can be calculated in a straightforward manner for in- or outflows and reallocation scenarios trough the application of (7) in expressions (4) and (5).

In our illustration, we consider a portfolio of 20 equities, whose allocations are chosen to replicate the Swiss Market Index (SMI) as of 24 March 2010. We use 10 years of monthly closing prices for the SMI constituents ranging from January 2000 to December 2009. The monthly arithmetic asset returns are modeled by a multivariate Student-*t* distribution. The Student-*t*

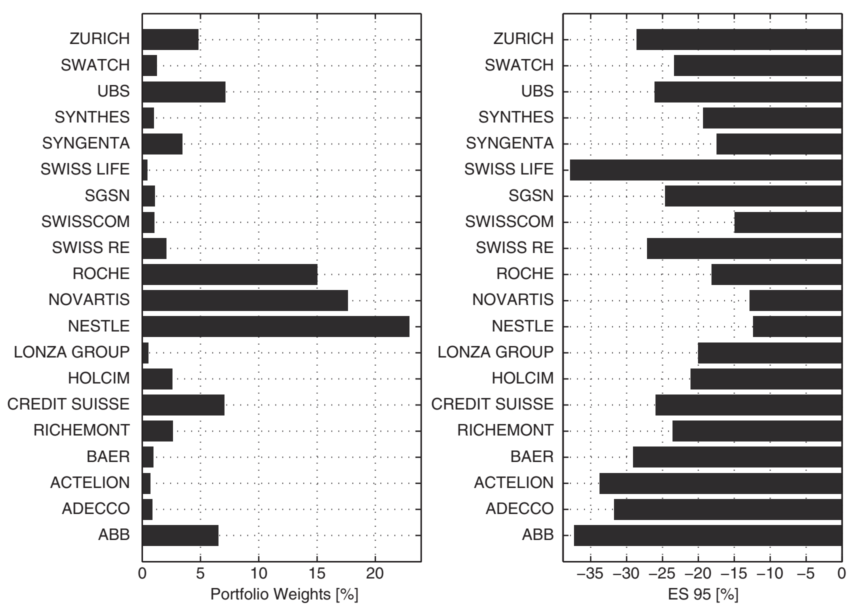

Figure 1: Left: SMI portfolio weights (in per cent). Right: individual monthly ES95.

belongs to the class of elliptical distributions, and allows us to capture the fat tail aspect of the behavior of equity returns. The parameters of the multivariate Student-*t* distribution, that is, the mean vector, the covariance matrix and the degrees of freedom parameter, are estimated by the Expectation Maximization algorithm (see McNeil *et al*, 2005, p. 81).

Figure 1 displays the SMI portfolio weights (left) together with the individual monthly ES at the 95 per cent confidence level (ES95) risk figures. The portfolio is concentrated in half a dozen positions. Individual monthly ES95 range from −12.4 per cent for Nestle to more than −37.8 per cent for Swiss Life. The overall portfolio ES95 is −12.2 per cent.

Figure 2 reports the (relative) marginal and component ES95 for the assets in the portfolio. Relative measures are obtained by dividing the sensitivity measures by the portfolio ES95. Therefore, a positive (negative) value of x per cent indicates an increase (decrease) of x per cent of the current portfolio risk after an additional 1 per cent allocation in the corresponding asset. From the marginal ES95 numbers, the portfolio manager can infer that the portfolio risk will increase if any position is levered. Conversely, if the portfolio manager divests from a position and puts the proceeds in the cash account, the portfolio risk is reduced. If the investor wants to decrease the portfolio ES95, the marginal risk suggests to reduce the allocations in Asea Brown Boveri (ABB) first. For instance, reducing (in the absolute sense) the position in ABB by 1 per cent (that is, from 6.5 to 5.5 per cent) would reduce (in the relative sense) the portfolio ES95 by 2.1 per cent (that is, from −12.2 to −11.9 per cent). The component risk analysis indicates that the portfolio risk is concentrated in around half a dozen positions. The hot spots in the portfolio happen to be the holdings with large weights.

Let us now consider the generalized marginal risk as an additional decision tool for the portfolio manager. We consider two scenarios: (i) there are capital inflows in the portfolio, which corresponds to expression (4); (ii) a position is increased by an equal decrease in all other positions, which corresponds to

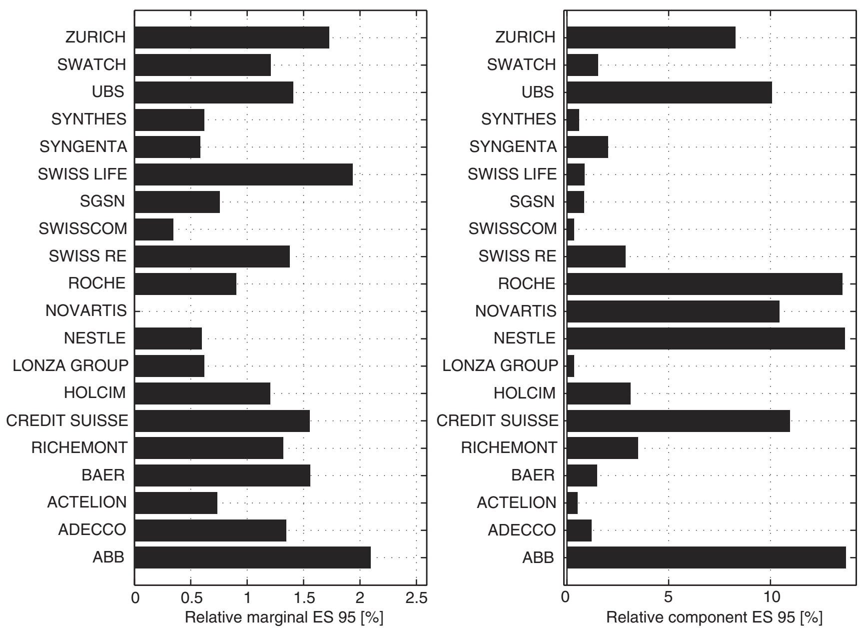

Figure 2: Left: (relative) marginal ES95 for the assets in the SMI portfolio. Right: (relative) component ES95. *Note*: Relative measures are obtained by dividing the sensitivity measures by the portfolio ES95.

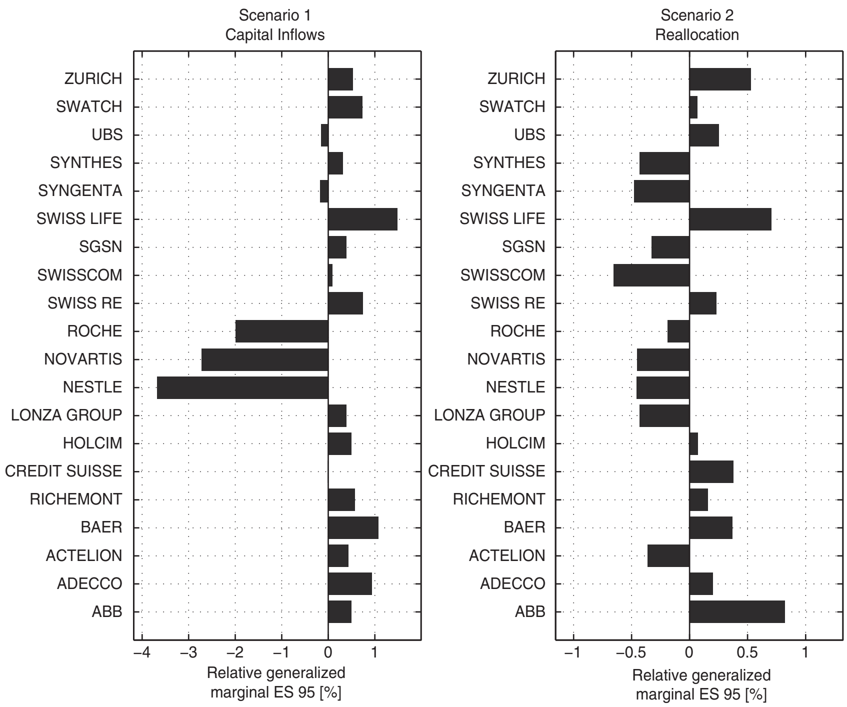

Figure 3: Left: (relative) generalized marginal ES95 for the assets in the SMI portfolio when additional capital is brought in the portfolio and invested in one position. Right: Case where the increase in one position is financed by an equal reduction in all other positions.
Note: Relative measures are obtained by dividing the sensitivity measures by portfolio ES95.

expression (5). Both situations are relevant for mutual fund managers and institutional investors, which cannot allocate more than a given percentage (often 5 per cent) of the portfolio value on the portfolio cash account. Figure 3 displays the results of the sensitivity analysis. The left-hand side reports the generalized marginal ES95 in the case of capital inflows in the portfolio. In this case, additional capital invested in Nestle will have the most effect on decreasing the risk in the new portfolio. For instance, an additional 1-per cent allocation in Nestle (that is, from 22.9 to 23.9 per cent) would reduce the portfolio risk by 4.56 per cent. On the right-hand side, the case where assets are shifted within the portfolio is displayed. Under this scenario, reallocating capital to Swisscom will decrease the overall risk the most. Note that under the reallocation scenario, the generalized marginal ES95 should be reflective of the return expectations of the portfolio manager. For instance, if the portfolio manager does not have a strong performance expectation on ABB, the position in ABB should be reduced and the proceeds invested equally in the other assets. This sensitivity analysis is especially helpful for an investor, who aims at implementing views if a benchmark is to be beaten on a risk-adjusted basis. Overall, we show that depending on the portfolio's adjustment scheme pursued by the portfolio manager, the traditional and the new sensitivity measures can vary substantially. This underlines the importance of accurately modeling the way the portfolio is adjusted and choosing adequately the sensitivity measure.

CONCLUSION

Assessing the sensitivity of the aggregated portfolio risk with respect to the underlying holdings is important for a portfolio manager to support the sizing of the portfolio positions. The traditional concept to measure the portfolio risk sensitivity is the marginal risk. Mathematically, this is simply the gradient of the portfolio risk measure with respect to the allocation weights. However, since this metric relies on the gradient, it is only meaningful when a position is levered or when the proceeds of the sale of a position are put in the cash account of the portfolio. This is certainly not always the case in practice. Counter examples are in- and outflows of capital in the portfolio, as well as reallocations within the portfolio. This article proposes a novel approach for measuring the risk sensitivity of a portfolio when the traditional marginal risk fails. The new sensitivity measure, referred to as generalized marginal risk, can deal with cases where the changes in the portfolio results in changes of other position as well. We illustrate the usefulness of the new approach with a real-world portfolio within the elliptical framework.

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