

## HOLOGRAPHIC OPTICAL ELEMENTS (HOE) FOR SEMICONDUCTOR LASERS

Hans Peter HERZIG

*Institut de Microtechnique de l'Université, CH-2000 Neuchâtel, Switzerland*

A two step method is described to produce off-axis holographic lenses with high diffraction efficiency and without astigmatism for semiconductor lasers. The hologram is recorded in the visible (514 nm) and reconstructed in the infrared (800 nm). The principal parameters (recording and reconstruction angles, astigmatic focal lengths) for each hologram and wavelength are calculated analytically using second order approximation. Numerical methods, based on classical ray-tracing applied to holographic diffraction, have been developed to calculate spot diagrams and to investigate the effects of higher order aberrations.

### 1. Introduction

Holographic optical elements (HOE) for semiconductor lasers cannot be realized directly, since there are no suitable holographic emulsions available for the infrared. Recording HOE in the visible and using them in the infrared requires careful control of all wavelength dependent parameters, such as Bragg angle, focal length and astigmatism.

The techniques to solve the problem of wavelength shift work usually with computer generated holograms [1], or with specially designed optical systems [2] to compensate the aberrations, due to the difference between recording and reconstruction. Another method, a simple copying process, was proposed by Lin [3] to realize HOE's for HeNe lasers in dichromated gelatin, a material which is not sensitive at 633 nm. A hologram is recorded with the red light of a HeNe-laser in a photographic emulsion and then copied with the blue light of an Ar-laser. For this method it is necessary to have an original hologram recorded at the final wavelength. In the case of HOE's for infrared light, the problem becomes more difficult, since there are no photographic films available to perform the original hologram.

The method presented in this paper may be considered as an extension of Lin's two step process for the infrared, where simple copying is no longer pos-

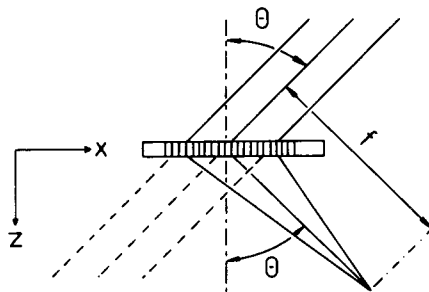


Fig. 1. Holographic optical element (HOE) producing a spherical wave of focal length  $f$  from an incident plane wave. The drawing shows symmetric deflection under Bragg-condition in a thick hologram.

sible. The realization of a HOE with the following properties (fig. 1) will be described in detail:

- Reproduction wavelength in the IR (800 nm),
- Symmetric deflection of the incident plane wave.
- Reproduction of a spherical wave front without aberrations.
- High diffraction efficiency of the hologram (Bragg).
- Recording wavelength in the visible (514 nm).

## 2. Theory

The principal parameters (recording and reconstructing angles, astigmatic focal lengths) for each hologram and wavelength can be calculated analytically using second order approximation [4]. In the case of hologram recording and reconstruction of point sources in a common plane of incidence, one gets

$$k_{\text{r}} \sin \theta_{\text{P}} = k_{\text{r}} \sin \theta_{\text{r}} + k_{\text{R}} \sin \theta_{\text{O}} - k_{\text{R}} \sin \theta_{\text{R}},$$

$$k = 2\pi/\lambda, \quad (1)$$

$$(k_{\text{r}}/f_{\text{P}}^{\parallel}) \cos^2 \theta_{\text{P}} = (k_{\text{r}}/f_{\text{r}}^{\parallel}) \cos^2 \theta_{\text{r}} + (k_{\text{R}}/f_{\text{O}}^{\parallel}) \cos^2 \theta_{\text{O}} - (k_{\text{R}}/f_{\text{R}}^{\parallel}) \cos^2 \theta_{\text{R}}, \quad (2)$$

$$(k_{\text{r}}/f_{\text{P}}^{\perp}) = (k_{\text{r}}/f_{\text{r}}^{\perp}) + (k_{\text{R}}/f_{\text{O}}^{\perp}) - (k_{\text{R}}/f_{\text{R}}^{\perp}), \quad (3)$$

where  $\theta_i$  are the angles of incidence and  $f_i$  the focal lengths measured from the hologram, as shown in fig. 1. The indices refer to the four waves involved, namely R to the recording reference, r to the reconstruction reference, O to the object point and P to the reconstructed astigmatic ray pencil with the focal lengths  $f_{\text{P}}^{\parallel}$  parallel and  $f_{\text{P}}^{\perp}$  perpendicular to the plane of incidence.

To get high diffraction efficiency from thick volume holograms, the Bragg-condition has to be satisfied. That means for the wave vectors

$$k_{\text{P}} - k_{\text{r}} = k_{\text{O}} - k_{\text{R}}. \quad (4)$$

For the components in the hologram plane (x-direction) eqs. (1) and (4) are identical. For the components normal to the hologram (z-direction) it follows additionally

$$k_{\text{r}} (\cos \theta_{\text{P}} - \cos \theta_{\text{r}}) = k_{\text{R}} (\cos \theta_{\text{O}} - \cos \theta_{\text{R}}). \quad (5)$$

If one works under symmetric conditions ( $|\theta_{\text{P}}| = |\theta_{\text{r}}| = \theta_{\text{m}}$ ,  $|\theta_{\text{O}}| = |\theta_{\text{R}}| = \theta_{\text{n}}$ ) eq. (5) is always satisfied. In this case the fringes in the holographic emulsion are perpendicular to the hologram plane and therefore emulsion shrinkage does not change the grating geometry.

If the recording and reconstructing references are plane waves, the terms  $1/f_{\text{r}}$  and  $1/f_{\text{R}}$  in eqs. (2) and (3) become zero. Now eqs. (1), (2) and (3) can be rewritten in a different way, also admitted astigmatism of the object wave during recording:

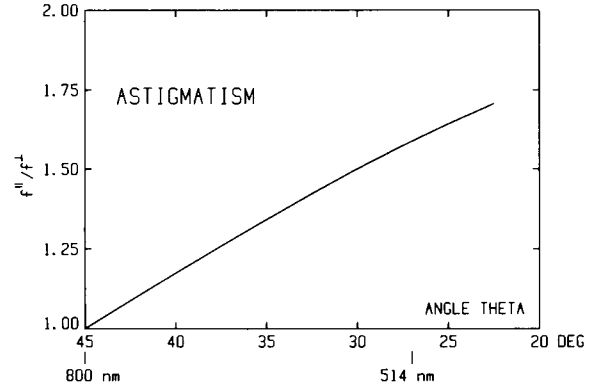


Fig. 2. Angular dependence of the astigmatism, represented by the ratio of the focal lengths  $f^{\parallel}$  and  $f^{\perp}$ . The angles corresponding to the Bragg-condition are 45 deg for 800 nm and 27.0 deg for 514 nm.

$$\sin \theta_{\text{m}} = (\lambda_{\text{m}}/\lambda_{\text{n}}) \sin \theta_{\text{n}}, \quad (6)$$

$$f_{\text{m}}^{\parallel} = (\lambda_{\text{n}}/\lambda_{\text{m}}) (\cos^2 \theta_{\text{m}}/\cos^2 \theta_{\text{n}}) f_{\text{n}}^{\parallel}, \quad (7)$$

$$f_{\text{m}}^{\perp} = (\lambda_{\text{n}}/\lambda_{\text{m}}) f_{\text{n}}^{\perp}. \quad (8)$$

The equations are now symmetric. The indices  $m$  refer to the recording parameters and  $n$  to the reconstruction parameters, respectively.

A change of wavelength between recording and reconstruction requires also a change of angle (eq. (6)). Eqs. (7) and (8) describe the curvatures of the wavefront parallel ( $f^{\parallel}$ ) and perpendicular ( $f^{\perp}$ ) to the plane of incidence. Even in the case that one of the waves is spherical (e.g.  $f_{\text{n}}^{\parallel} = f_{\text{n}}^{\perp}$ ), the other one is necessarily astigmatic ( $f_{\text{m}}^{\parallel} \neq f_{\text{m}}^{\perp}$ ), when the wavelength is changed ( $\lambda_{\text{m}} \neq \lambda_{\text{n}}$ ,  $\theta_{\text{m}} \neq \theta_{\text{n}}$ ). The dependence of this astigmatism on the change of wavelength is presented in fig. 2. It is assumed that the wave is spherical at  $\lambda_{\text{n}} = 800$  nm and  $\theta_{\text{n}} = 45$  deg. For a wavelength of  $\lambda_{\text{m}} = 514$  one gets then  $\theta_{\text{m}} = 27.0$  deg and  $f_{\text{m}}^{\parallel}/f_{\text{m}}^{\perp} = 1.6$ , which is a very important astigmatism. To get a perfectly focussing hologram at 800 nm, this astigmatism has to be provided at 514 nm, where the hologram is recorded for sensitivity reasons.

## 3. Realization

What one needs is an astigmatic wavefront as object wave at 514 nm. This astigmatism can be created

by using another hologram recorded at 633 nm and reconstructed at 514 nm. This hologram is not necessarily of the same high efficiency material as the final one. The correct astigmatism is obtained by choosing the angles of recording and reconstruction appropriately. It is possible to satisfy the Bragg-condition (symmetric reference and reconstruction) at any time to work always with maximum efficiency.

The steps are the following (fig. 3):

(i) Recording of a first hologram H1 with a HeNe-laser at  $\lambda_1 = 633$  nm. The reference is a plane wave ( $\theta_1$ ), the object is a spherical wave ( $f_1^{\parallel}, f_1^{\perp}$ ).

(ii) Reconstruction of H1 with a plane wave ( $\theta_2$ ) from an Ar-laser at  $\lambda_2 = 514$  nm respecting eq. (6). The hologram produces an astigmatic wave ( $\theta_3, f_2^{\parallel}, f_2^{\perp}$ ).

(iii) Recording of a second hologram H2 with the astigmatic wave ( $f_3^{\parallel}, f_3^{\perp}$ ) is provided by the hologram and a plane reference wave at  $\lambda_3 = 514$  nm.

(iv) Reconstruction of H2 with a plane wave ( $\theta_4$ ) from a GaAs-laser at  $\lambda_4 = 800$  nm produces now the desired spherical wave ( $\theta_4, f_4^{\parallel} = f_4^{\perp}$ ).

The parameters  $\theta_3, f_3^{\parallel}, f_3^{\perp}$  for the recording of the hologram H2 are obtained from eqs. (6), (7) and (8)

with  $m = 3, n = 4, \lambda_3 = 514$  nm,  $\lambda_4 = 800$  nm,  $\theta_4 = 45$  deg and  $f_4^{\parallel} = f_4^{\perp} = f_4$ , where  $f_4$  is the desired focal length of the final holographic lens. The divergent astigmatic wave ( $f_3^{\parallel}, f_3^{\perp}$ ) is provided by the hologram H1, which is placed at a distance  $\lambda$  from the hologram H2. The focal lengths with respect to H1 ( $f_2^{\parallel}, f_2^{\perp}$ ) are therefore shorter than those of H2, namely

$$f_2^{\parallel} = f_3^{\parallel} - d, \quad (9)$$

$$f_2^{\perp} = f_3^{\perp} - d. \quad (10)$$

Now the focal lengths  $f_2^{\parallel}, f_2^{\perp}$  of H1 at  $\lambda_2 = 514$  nm are known. This hologram will be recorded at  $\lambda_1 = 633$  nm with a spherical object wave ( $f_1^{\parallel} = f_1^{\perp} = f_1$ ). The unknown parameters  $\theta_1, \theta_2$  and  $f_1$ , can be calculated from eqs. (6), (7) and (8) with  $m = 1, n = 2, \lambda_1 = 633$  nm,  $\lambda_2 = 514$  nm and  $f_1^{\parallel} = f_1^{\perp} = f_1$ . This yields the relations

$$f_1 = (\lambda_2/\lambda_1)f_2^{\perp}, \quad (11)$$

$$\sin^2\theta_2 = [1 - (f_2^{\perp}/f_2^{\parallel})] / [(\lambda_1/\lambda_2)^2 - (f_2^{\perp}/f_2^{\parallel})], \quad (12)$$

$$\sin\theta_1 = (\lambda_1/\lambda_2) \sin\theta_2. \quad (13)$$

The hologram H1, recorded at  $\lambda_1 = 633$  nm with a plane reference and a spherical object wave ( $f_1, \theta_1$ ) and reconstructed with a plane wave ( $\theta_2$ ) at  $\lambda_2 = 514$  nm, creates the astigmatic wave necessary to record the hologram H2.

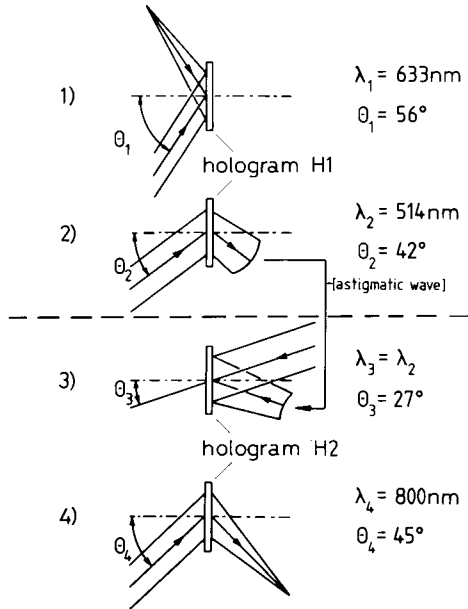


Fig. 3. Hologram recording process using two holograms and three different wavelengths.

#### 4. Experimental results

A holographic lens with  $f_4 = 300$  mm at  $\lambda_4 = 800$  nm and  $\theta_4 = 45$  deg (fig. 1) has been realized by the described method in a  $15 \mu\text{m}$  thick emulsion (dichromated gelatin based on Kodak 649 F photographic plates). The parameters of the required astigmatic wave for the recording at  $\lambda_3 = 514$  nm were calculated using eqs. (6), (7) and (8) to be  $\theta_3 = 27.0$  deg,  $f_3^{\parallel} = 741$  mm,  $f_3^{\perp} = 467$  mm. With a distance  $d = 100$  mm between hologram H2, the astigmatic wave reconstructed from H1 at  $\lambda_2 = 514$  nm has to have the focal lengths  $f_2^{\parallel} = 641$  mm and  $f_2^{\perp} = 367$  mm, following eqs. (9) and (10). From eqs. (11), (12) and (13) one gets finally the parameters  $f_1 = 298$  mm,  $\theta_2 = 42.3$  deg and  $\theta_1 = 56.0$  deg for recording ( $\lambda_1 = 633$  nm) and reconstruction ( $\lambda_2 = 514$  nm) of hologram H1.

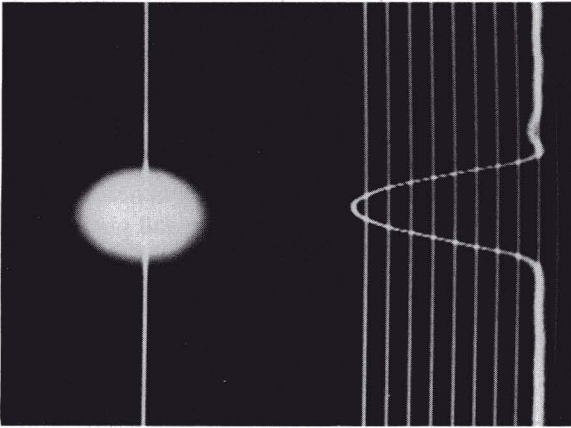


Fig. 4. Point spread function obtained from a holographic lens, when illuminated with a single-frequency laser-diode (Hitachi HL7801) at 800 nm.

Fig. 4 shows the focussed spot obtained from the holographic lens ( $f = 300$  mm), when illuminated with a parallel beam (diameter  $D = 5$  mm) of a single-frequency diode-laser (Hitachi HL7801 E) at 800 nm. The light distribution was observed and scanned by a camera. The measured spot has a diameter of about  $100 \mu\text{m}$ , which corresponds quite well to the diameter of the Airy disc expected from diffraction. The oval shape is due to the characteristics of the laser source. The influence of aberrations seems therefore to be smaller than the limits of diffraction.

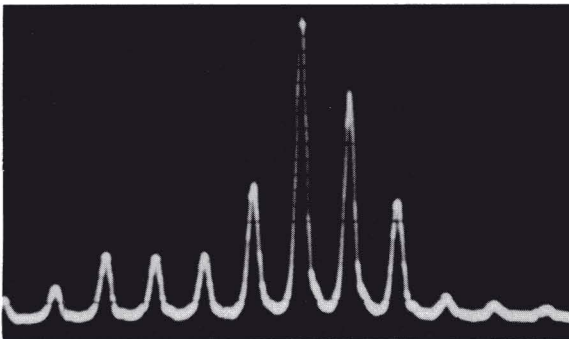


Fig. 5. Spectrum of a multi-frequency diode-laser obtained by a linear CCD array in the total plane of the holographic lens. Each peak corresponds to one longitudinal mode of the laser.

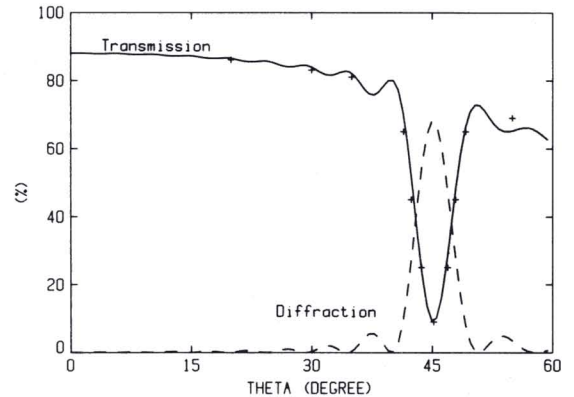


Fig. 6. Measured diffraction efficiency of the holographic lens at 800 nm versus angle of incidence.

With a multi-frequency diode-laser as light source one gets multiple spots reconstructed in the focal plane, one for each longitudinal mode. Fig. 5 was obtained with a linear photodiode-array replacing the TV camera in the focal plane, and it shows the spectrum of the laser source. In this case the setup works as a simple and compact grating spectrometer for diode-lasers.

The diffraction efficiency of this  $15 \mu\text{m}$  thick hologram depend strongly on the angle of incidence of the reconstructing wave (Bragg). Some measured values (crosses) of the transmitted light (zero order) versus angle of incidence are shown in fig. 6. The transmission and diffraction of a real hologram has been calculated based on the coupled wave theory for volume holograms [5] and taking account of the reflection and transmission losses. Fitting the transmission curve to the measured points yields a peak diffraction efficiency of about 70% at the desired angle of 45 deg, as shown in fig. 6.

## 5. Higher order aberrations

The theory which has been used for the calculations is based on second order approximations. Higher order aberrations are therefore not considered. Expansion of the theory to third order is not useful, the resulting relations are too cumbersome. Geometrical ray-tracing, a powerful method in classical optics, can also be applied to analyse the performance of holograms. A

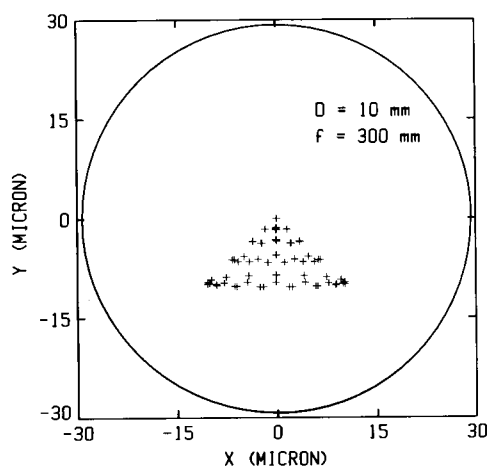


Fig. 7. Spot diagram of a hologram recorded at 514 nm and reconstructed at 800 nm:  $D$  is the diameter of the reconstructing reference,  $f$  is the focal length. The circle shows the size of the Airy disc.

computer program based on ref. [6] has been developed for that purpose.

To quantify the aberrations of the final hologram H2, the two step process of hologram recording and reconstruction (fig. 3) has been simulated by ray-tracing. The aberrations of the wave front, reconstructed by hologram H2 at 800 nm, are presented as spot diagram in the focal plane (fig. 7.). As predicted, the image is free of astigmatism. The remaining aberrations are of higher order. Choosing an aperture of  $D = 10$  mm for  $f = 300$  mm, which is twice as wide as in the experiment, the residual errors are still small compared to the diffraction effect, given by an Airy disc or radius  $r = 1.22 \lambda f/D = 29 \mu\text{m}$  for that case.

## 6. Conclusions

A simple method to produce highly efficient off-

axis holographic elements (HOE) for the near infrared (IR) is reported. It has been successfully applied to fabricate aberration free holographic lenses for semiconductor lasers at 800 nm. However, by changing the recording parameters, aberration free lenses can be produced up to 1300 nm.

The described method uses only spherical waves and plane waves, generated by lenses and pinholes, to record the intermediate and the final holograms. Aberrations of second order (astigmatism) are corrected exactly. Ray-tracing has been used to check for higher order aberrations. It shows, in good agreement with the experimental results, that the described lens of  $f = 300$  mm produces up to an aperture of at least  $D = 15$  mm a diffraction limited spot at 800 nm, which is in this case about  $40 \mu\text{m}$  in diameter. Interesting examples of applications are focussing deflectors for laser-printers and focussing gratings for diode-laser spectrum analyzers.

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## References

- [1] K. Winick, J. Opt. Soc. Am., 143.
- [2] M. Malin and H.E. Morrow, Opt. Eng. 20 (1981) 756.
- [3] L.H. Lin and E.T. Doherty, Appl. Opt. 10 (1971) 1314.
- [4] R. Dändliker, K. Hess and Th. Sidler, Isr. J. Techn. 18 (1980) 240.
- [5] H. Kogelnik, Bell. Syst. Techn. J. 48 (1969) 2909.
- [6] W.T. Welford, Optics Comm. 14 (1975) 322.