

Reviews – Etudes critiques – Buchbesprechung

Stephen Neale: *Facing Facts*.

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Suppose a one-place sentential connective \odot has the following two features:

1. It is *closed under logical equivalence* (CLE), i.e. if ϕ and ψ are logically equivalent sentences, then from $\odot\phi$ one can infer $\odot\psi$;
2. It is *closed under substitution of co-referential definite descriptions* (CSCD), i.e. if ϕ contains occurrences of a definite description $\iota x\alpha$ which are not within the scope of any non-extensional connective, and ψ is ϕ with one or more of these occurrences replaced by the definite description $\iota x\beta$, then from $\iota x\alpha = \iota x\beta$ and $\odot\phi$ one can infer $\odot\psi$.

Then, assuming Russell's Theory of Definite Descriptions, given any two sentences ϕ and ψ , from $\phi \equiv \psi$ and $\odot\phi$, one can infer $\odot\psi$. That is to say, \odot is *closed under material equivalence*, i.e. \odot is *truth-functional*. Call the proposition that any connective which is both CLE and CSCD is truth-functional FLATTENING.

FLATTENING, if true, imposes serious constraints on various theories of great philosophical interest, e.g. theories of modality, belief, and knowledge. This is quite obvious. The operators 'it is necessary that', 'it is possible that', 'Sam believes that' and 'Sam knows that' are certainly not truth-functional – at least one should recognise it if one takes them seriously. Thus by FLATTENING, any serious theory of necessity in which the notion is expressed by means of a sentential operator must give up Russell's Theory of Descriptions, or deny that the operator is CLE, or deny that it is CSCD – and the same holds *mutatis mutandis* of belief and knowledge.

FLATTENING also imposes constraints on various other theories, like for instance theories of facts, propositions, state of affairs or situations – though in these cases the connection is not so straightforward. Consider facts. Suppose there are such entities, and that some expressions of type 'the fact that ϕ ' (where ϕ is a true sentence) are terms which denote (facts). Let then ϕ be a true sentence such that 'the fact that ϕ ' is denoting, and let \odot be the "fact-operator" 'the fact that $\phi =$ the fact that ...'. Then $\odot\phi$ is true. Assuming that \odot is truth-functional, we must conclude that given any true sentence ψ such that 'the fact that ψ ' is denoting, 'the fact that $\phi =$ the fact that ψ ' is true. That is to say, we must conclude that all facts which can be denoted by prefixing 'the fact that'

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to a true sentence are identical, that there is only one such fact; and making the further assumption that all facts can be denoted that way in some language, the conclusion is that there is only one fact. These are conclusions any regular friend of facts is likely to reject. And by FLATTENING, rejecting the conclusions means giving up Russell's Theory of Descriptions, or denying that fact-operators are both CLE and CSCD.

FLATTENING is a consequence of a result I shall call CHURCH, which Neale extracts from a formal proof he takes to be an abstract version of "slingshot arguments" of a series which originates in Church's (1943) objection to Carnap's view that sentences denote propositions, and includes Davidson's (1967, 1969) celebrated slingshots. The work is done in chapter 8, and in the following chapter Neale establishes another general result I shall call GÖDEL, on the basis of another formal proof he takes to be an abstract version of a slingshot suggested by Gödel (1944).

Each result states that a one-place connective is truth-functional provided it satisfies certain conditions, some of them involving definite descriptions. Yet, on Neale's view, GÖDEL is superior to CHURCH insofar as the conditions which appear in GÖDEL are weaker than those which appear in CHURCH. Neale also takes it that the proof which delivers GÖDEL is superior to the one which delivers CHURCH, insofar as, in order to be correct, the latter, but not the former, has to be supplemented by substantial views about definite descriptions.

The main philosophical interest of these general results, as Neale rightly stresses, lies not so much in the fact that they may be used to undermine e.g. non-extensional discourse or talk of facts, but rather in the fact that they provide one with precise constraints any viable theory of facts or non-extensional connectives should satisfy.

Chapter 8 and 9 are the core of the book. In these chapters Neale gives the formal proofs and extracts CHURCH and GÖDEL. Chapters 6 and 7 introduce some formal prerequisites, the notions of scope and extensionality and the inference principles which will be involved in the following chapters. Both CHURCH and GÖDEL are extracted from proofs which make use of some assumptions about definite descriptions, which are indeed satisfied by Russell's Theory. Chapter 10 deals with the question as to how rival theories of descriptions behave with respect to these assumptions, and what results can be established if these theories are countenanced. In chapter 11 the consequences of CHURCH and GÖDEL for theories of facts and theories of causation are carefully examined. Chapters 2 to 5 are historical in character. Chapter 2 is devoted to Davidson. His semantic programme is examined, his slingshot argument against facts is exposed and its use against the correspondence theory of truth and the notion of representation is explained. Chapter 3 deals with the Fregean roots of slingshot arguments, and chapter 4 with Russell's own views about facts and definite descriptions. Finally the discussion about Russell's views on descriptions is expanded in the book. The book is rich in content, clearly written and precise, and its topic exciting. The next part of the review will be critical.

The central chapters 8 and 9 – in which Neale gives his abstract versions of Church's (1943) slingshot and of Gödel's (1944) slingshot, and extracts from these arguments the general results CHURCH and GÖDEL, respectively – contain some important mistakes and lacunae I shall try to repair or fill in here. (I shall follow Neale's technical vocabulary and notational conventions without explanation.)

CHURCH is the following proposition:

(CHURCH) If a one-place connective is both +PSLE and + ι -SUBS, then it is +PSME.

Neale holds that this result can be extracted from a formal proof he gives on page 173, assuming Russell's Theory of Descriptions. Unfortunately the proof is faulty under the Russellian treatment of descriptions. Neale assumes that by Russell's Theory, given any sentence ϕ and any singular terms a and b , ϕ is logically equivalent to $a = \iota x((x=a \bullet \phi) \vee (x=b \bullet \sim\phi))$, but this is not the case. In fact some models with only *one* individual make some sentences false, while given any sentence ϕ and any singular terms a and b , all such models make the Russellian rendering of $a = \iota x((x=a \bullet \phi) \vee (x=b \bullet \sim\phi))$ true.

But a correct proof can be constructed. It is in two parts. The first is given by Neale himself on page 170, and it establishes that if a one-place connective is both +PSLE and + ι -SUBS, then it permits the substitution *salva veritate* of truths for truths (we are still assuming Russell's Theory of Descriptions). The second part establishes that if a one-place connective is both +PSLE and + ι -SUBS, then it permits the substitution *salva veritate* of falsehoods for falsehoods:

1	[1] $\sim\phi$	premiss
2	[2] $\sim\psi$	premiss
3	[3] $\odot\phi$	premiss
3	[4] $\odot\sim(a = \iota x(x=a \bullet \sim\phi))$	3, \odot +PSLE
1,2	[5] $\iota x(x=a \bullet \sim\phi) = \iota x(x=a \bullet \sim\psi)$	1,2, def. of ' ιx '
1,2,3	[6] $\odot\sim(a = \iota x(x=a \bullet \sim\psi))$	4,5, \odot + ι -SUBS
1,2,3	[7] $\odot\psi$	6, \odot +PSLE

In both parts of the proof, Russell's Theory of Descriptions justifies the moves to lines [4], [5] and [7].

It is clear that less than Russell's Theory of Descriptions is required for the proof to be correct. Say that definite descriptions are *nice* iff the following two conditions are satisfied:

- (1) ϕ and $a = \iota x(x=a \bullet \phi)$ are logically equivalent;
- (2) $\iota x(x=a \bullet \phi) = \iota x(x=a \bullet \psi)$ is a logical consequence of ϕ, ψ .

Our proof establishes CHURCH on the sole assumption that definite descriptions are nice. The view that definite descriptions are nice is by no means a trivial one. Under Russell's Theory of Descriptions the view is correct, but as Neale stresses, under some alternative theories it is not (see chapter 10). Thus the proof from which CHURCH has been extracted, in order to be correct, must be supplemented with some substantial principles about definite descriptions. Neale rightly makes the same point about his (faulty) proof.

Let me now turn to GÖDEL:

(GÖDEL) If a one-place connective is both + ι -CONV and + ι -SUBS, then it is +PSME.

Neale extracts GÖDEL from a four-part proof given on pages 183-186. He claims that his proof for GÖDEL is superior to his proof for CHURCH because the latter, but not the former, involves some moves whose justification requires some substantial, semantical assumptions about definite descriptions. And he also claims that GÖDEL is in a certain sense more worrying than CHURCH.

I do not think Neale's first claim is correct. In his proof for CHURCH (page 173), line [4] is supposed to be justified by line [1] and the definition of ' ιx ', so there is the background assumption that given the definition of ' ιx ', line [4] is a logical consequence of line [1]. There is also a background assumption underlying the moves from line [2] to line [3] and from line [5] from line [6], to the effect that certain formulas containing the ι -operator are logically equivalent to other formulas. The truth-value of these assumptions turns on which particular semantic view about definite descriptions is countenanced, as Neale rightly points out. And of course, similar considerations apply to my two part proof for CHURCH. Now I claim that Neale's proof for GÖDEL is on a par with both proofs for CHURCH in this respect. The proof for GÖDEL makes use of two rules of inference which concern definite descriptions, ι -CONV and ι -SUBS, which are applied to extensional contexts. There is thus a background assumption governing the proof, namely that both rules apply to such contexts *salva veritate*. And the truth-value of such this assumption also turns on which particular semantic view about definite descriptions is countenanced.

Neale might reply that the background assumptions used in my proof for CHURCH (let me put aside Neale's faulty proof here) are substantial while those used in the proof for GÖDEL are not, i.e. that any theory of descriptions, in order to be taken seriously, must in any event make the latter assumptions true, while some such theories may be acceptable and yet inconsistent with the former assumptions. (On page 180 Neale notices that some views about descriptions do not licence ι -CONV, but he just boldly says: "any adequate theory of descriptions, it seems to me, must be compatible with [the fact that ι -CONV is valid in extensional contexts]".) But this should be carefully argued for, and in any event I am not sure Neale would be willing to defend that view. For consider the rule ι -CONV*, which licences the replacement of a sentence ϕ in a context by the sentence $a = \iota x(x=a \bullet \phi)$ and *vice versa*. The view that both ι -CONV* and ι -SUBS apply to extensional contexts entails that definite descriptions are nice. Now ι -CONV* is very similar to ι -CONV. Would Neale claim that there is a deep asymmetry between ι -CONV* and ι -CONV, in that the view that ι -CONV* applies to extensional contexts is a substantial one, while the view that ι -CONV applies to such contexts is not?

I do not think Neale's second claim is correct either. On page 201, he says that GÖDEL is more worrying than CHURCH "on the obvious assumption that every "Gödelian equivalence", as given by ι -CONV, is also a logical equivalence, but not vice versa". The idea seems to be the following. Let \mathbf{C} be the class of +PSLE, + ι -SUBS connectives, and let \mathbf{G} be the class of + ι -CONV, + ι -SUBS connectives. CHURCH states that every connective in \mathbf{C} is truth-functional, and GÖDEL that every connective in \mathbf{G} is truth-functional. Neale seems to think is that \mathbf{C} is a proper part of \mathbf{G} , and that it is for that reason that GÖDEL is more worrying than CHURCH. We may agree that if \mathbf{C} is a proper part of \mathbf{G} , then GÖDEL is indeed more worrying than CHURCH. But on the assumption that GÖDEL is true, \mathbf{C} is not a proper part of \mathbf{G} . In fact if a connective is in \mathbf{G} , then by GÖDEL it is +PSME, and so it is +PSLE (two logically equivalent sentences are materially equivalent), and so it is in \mathbf{C} . That is to say, if GÖDEL is true, \mathbf{G} is a (proper or improper) part of \mathbf{C} . Now the assumption Neale finds "obvious" entails that \mathbf{C} is a (proper or improper) part of \mathbf{G} . Thus as a conclusion, if both GÖDEL and Neale's assumption are true, then $\mathbf{C} = \mathbf{G}$, and GÖDEL and CHURCH have the same worrying character.

CHURCH logically follows from the two-part proof given above and the assumption that definite descriptions are nice. GÖDEL, on the other hand, does not logically follow from Neale's four-part proof and the assumption that ι -CONV and ι -SUBS apply to

extensional contexts. One reason is that some parts of Neale's proof (pages 185-186) make use of the assumption that the connective is +DNN. So at best the proof establishes that, on the assumption that i-CONV and i-SUBS apply to extensional contexts, if a one-place connective is +i-CONV, +i-SUBS and +DNN, then it is +PSME. Neale cannot but agree with that point. Now the proof does not even establish that weaker result. It establishes less, namely that:

(GÖDEL') If a one-place connective is + ι -CONV, + ι -SUBS and +DNN, then it is +PSME *with respect to subject-predicate sentences.*

Neale is aware of that fact, but he claims, following a suggestion of Gödel's (1944), that a certain assumption yields the more general result – to wit the assumption that “any sentence can be put into subject-predicate form” (see pages 130 and 186). It is not clear what this means. I suggest, following a hint given by Neale (page 130), that the principle is that in any context whatsoever, any occurrence of a sentence ϕ can be replaced *salva veritate* by the sentence $|\phi|(a)$ (read: ‘a is such that ϕ ’) for any (referring) singular term a. Call this *Gödel's principle*. Using GÖDEL' and Gödel's principle, we can indeed prove what we wanted, namely that:

(GÖDEL'') If a one-place connective is + ι -CONV, + ι -SUBS and +DNN, then it is +PSME

on the assumption that ι -CONV and ι -SUBS apply to extensional contexts. The proof is fairly simple:

1	[1] $\phi \equiv \psi$	premiss
2	[2] $\odot\phi$	premiss
1	[3] $ \phi (a) \equiv \psi (b)$	1, Gödel's principle
2	[4] $\odot \phi (a)$	2, Gödel's principle
1,2	[5] $\odot \psi (b)$	3,4, GÖDEL'
1,2	[6] $\odot\psi$	5, Gödel's principle

I do not know whether Neale would be prepared to accept my formulation of Gödel's suggestion. But whether or not he would, I think he would agree that GÖDEL cannot be extracted from his formal proof, even with the help of the principle suggested by Gödel (whatever its formulation), while GÖDEL'' can. (Note that if GÖDEL'' is true, then the class G'' of connectives satisfying the conditions in GÖDEL'' is a (proper or improper) part of the class C of connectives satisfying the conditions in Church, so that GÖDEL'', just like Gödel, is not “more worrying” than Church.)

Thus Neale's abstract version of Gödel's slingshot establishes that GÖDEL' is true if ι -CONV and ι -SUBS apply to extensional contexts, and from this result and Gödel's principle we get the conclusion that GÖDEL'' is true if ι -CONV and ι -SUBS apply to such contexts. Now I wish to show that if Gödel's principle is accepted, then the very same conclusion can be drawn directly from a fairly simple formal proof, a proof which is actually simpler than Neale's. The proof is the following:

1	[1] ϕ	premiss
2	[2] ψ	premiss
3	[3] $\odot\phi$	premiss
1	[4] $ \phi (a)$	1, Gödel's principle
2	[5] $ \psi (a)$	2, Gödel's principle
3	[6] $\odot \phi (a)$	3, Gödel's principle

1	[7]	$a = \text{tX}(x=a \bullet \phi (x))$	4, t-CONV
2	[8]	$a = \text{tX}(x=a \bullet \psi (x))$	5, t-CONV
3	[9]	$\text{C}(a = \text{tX}(x=a \bullet \phi (x)))$	6, C+1-CONV
1,3	[10]	$\text{C}(a = a)$	7,9, C+1-SUBS
1,2,3	[11]	$\text{C}(a = \text{tX}(x=a \bullet \psi (x)))$	8,10, C+1-SUBS
1,2,3	[12]	$\text{C} \psi (a)$	11, C+1-CONV
1,2,3	[13]	$\text{C}\psi$	12, Gödel's principle
1	[1]	$\sim\phi$	premiss
2	[2]	$\sim\psi$	premiss
3	[3]	$\text{C}\phi$	premiss
1	[4]	$\sim \phi (a)$	1, Gödel's principle
2	[5]	$\sim \psi (a)$	2, Gödel's principle
3	[6]	$\text{C} \phi (a)$	3, Gödel's principle
3	[7]	$\text{C}\sim\sim \phi (a)$	6, C+DNN
1	[8]	$a = \text{tX}(x=a \bullet \sim \phi (x))$	4, t-CONV
2	[9]	$a = \text{tX}(x=a \bullet \sim \psi (x))$	5, t-CONV
3	[10]	$\text{C}\sim(a = \text{tX}(x=a \bullet \sim \phi (x)))$	6, C+1-CONV
1,3	[11]	$\text{C}\sim(a = a)$	8,10, C+1-SUBS
1,2,3	[12]	$\text{C}\sim(a = \text{tX}(x=a \bullet \sim \psi (x)))$	9,11, C+1-SUBS
1,2,3	[13]	$\text{C}\sim\sim \psi (a)$	12, C+1-CONV
1,2,3	[14]	$\text{C} \psi (a)$	13, C+DNN
1,2,3	[15]	$\text{C}\psi$	14, Gödel's principle

This proof actually deliver a stronger result, namely that GÖDEL' is true if t-CONV alone applies to extensional contexts. (It is also possible to modify Neale's proof so as to establish that GÖDEL' is true if t-CONV alone applies to extensional contexts.)

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