Stress-Testing With Parametric Models and Fully Flexible Probabilities

David Ardia
Institute of Financial Analysis, University of Neuchâtel, Neuchâtel, Switzerland and Department of Finance, Insurance and Real Estate, Laval University, Québec City, Canada
Keven Bluteau
Institute of Financial Analysis, University of Neuchâtel, Neuchâtel, Switzerland,
e-mail: keven.bluteau@unine.ch

Abstract
We propose a simple methodology to simulate scenarios from a parametric risk model while accounting for stress-test views via fully flexible probabilities.

Keywords
fully flexible probabilities, GARCH, stress testing

1 Introduction

Stress testing is the process of determining the reaction of a portfolio profit and loss (P&L) distribution in some (observed or unobserved) specific (extreme) market conditions. To perform stress testing in a nonparametric risk modeling framework, Meucci (2010, 2013) proposes a simple approach referred to as fully flexible probabilities (FFP). This method consists of modifying the probabilities associated with historical scenarios (realizations) to account for a user-defined stress-test view (on endogenous or exogenous risk factors). The modified probabilities are then used to generate new scenarios (or to weight historical scenarios), leading to a P&L distribution reflecting the stress-test view.

In a parametric framework, it is more difficult to specify or incorporate views, especially on exogenous risk factors, into the model parameters. In this paper, we propose a simple solution to stress-test a portfolio with a parametric model using the FFP framework. Typically, a model fitted to financial data is estimated on rolling windows. Using FFP on exogenous risk factors (e.g., macroeconomic variables) and assigning the resulting probabilities to the time window on which the parametric model parameters are estimated, we generate simulations that account for both the parametric specification and the stress-test view. From the simulated P&L distribution, we can easily compute relevant risk indicators such as the value-at-risk or the expected shortfall.

2 Fully flexible probabilities

Assume that we have T historical (observed) realizations $\{y_{1,t}, \dots, y_{N,t}\}_{t=1}^{T}$ on the N-dimensional variable $Y \equiv (Y_1, \dots, Y_N)$ and the associated set of probabilities $\{p_t\}_{t=1}^{T}$, $p_t \equiv 1/T$, where T is the sample size. The variable Y encompasses a set of risk factors or macroeconomic variables of interest (e.g., stock prices, implied volatilities, etc.).

In order to stress-test the historical distribution of the portfolio's P&L, $\{P\&L_t\}_{t=1}^{T}$, where the P&L at time t is a simple mapping of (a subset of) Y_t [i.e., $P\&L_t \equiv F(Y_t)$], the FFP approach consists of modifying the set of *prior* probabilities $\{p_t\}_{t=1}^{T}$ to a set of *posterior* probabilities $\{p_t^\star\}_{t=1}^{T}$ that suits a given stress-test condition, and recomputing the portfolio P&L with this new set of probabilities. Below, we discuss how this set of probabilities can be obtained.

2.1 Time-window conditioning

The simplest way to modify the prior is by setting the probabilities constant over a time window of interest and zero otherwise:

$$p_t^\star \equiv \begin{cases} 1 & \text{if} \quad t \in [t_0, t_1] \\ 0 & \text{else}, \end{cases}$$

where $1 \leq t_0 < t_1 \leq T$; then, normalize the probabilities such that they sum to one. This simple method can be used to assess the risk of a portfolio in a crisis period (e.g., by setting the time window from January 2008 to January 2011).

2.2 Market conditioning

The second approach defines the probabilities according to some risk factor or macroeconomic conditions. Let $Y_{\bullet,t}$ be the value of (a single or several) relevant macroeconomic indicator(s) at time t and set a condition C on this (these) variable(s) that we want to fulfill – i.e., the stress-test scenario. The market-conditioning approach defines the new probabilities as follows:

$$p_t^\star \equiv \begin{cases} 1 & \text{if} \quad y_{\bullet,t} \in C \\ 0 & \text{else}, \end{cases}$$

then, normalize the probabilities such that they sum to one. For example, we might focus on the (forward-looking) volatility consensus of the market and rely on a volatility index such as the VIX. The portfolio's P&L can be stress tested by emphasizing days for which the VIX is higher than 30 percent – i.e., $\mathcal{C} \equiv [0.3, \infty]$.

2.3 Partial information market conditioning

In the third approach, we modify the set of probabilities to fit a stress-test view on a feature of the risk factor(s) or macroeconomic variable(s) $Y_\bullet$ – e.g., expected value(s) or variance(s). We *optimally* distort the prior to reflect this view. Meucci (2010, 2013) proposes relying on the *relative entropy* to solve this problem.

Formally, the relative entropy measure ε between the prior probabilities $\{p_t\}_{t=1}^T$ and a set of candidate probabilities $\{q_t\}_{t=1}^T$ is given by

$$\varepsilon\left(\{q_t\}_{t=1}^T ; \{p_t\}_{t=1}^T\right) \equiv \sum_{t=1}^T q_t \ln\left(\frac{q_t}{p_t}\right)$$

If the variable on which we have a view is $Y_\bullet$, and our view is that its expected value is $\mathbb{E}[Y_\bullet] \gtreqless Y_\bullet^\star$, with $Y_\bullet^\star$ a value set by the user, then the view under the scenario framework writes as

$$\sum_{t=1}^T y_{\bullet,t} q_t \gtreqless Y_\bullet^\star .$$

In this case, the optimal set of *posterior* probabilities is found by solving

$$\{p_t^\star\}_{t=1}^T \equiv \underset{\{q_t\}_{t=1}^T}{argmin} \;\; \varepsilon\left(\{q_t\}_{t=1}^T ; \{p_t\}_{t=1}^T\right)$$

$$\text{subject to } \sum_{t=1}^T y_{\bullet,t} q_t \gtreqless Y_\bullet^\star . \qquad (1)$$

Meucci (2008) shows how the optimization problem (1) can be solved efficiently, and provides illustrations of (in-)equality views on other quantities of interest such as standard deviation or quantiles.

3 Modeling portfolio returns

We consider a simplistic example with a universe of two assets. We model the assets' log-returns in two steps. First, we model each individual asset with the asymmetric GARCH model of Glosten *et al.* (1993) with Student-*t* innovations. Second, we model the dependency between the models' innovations with a Gaussian copula. This constitutes our parametric risk model.

The model parameters θ are estimated by maximum likelihood. The estimators $\hat\theta$ depend on the sample to which the model is fitted (i.e., the sample's time period in our time-series setup). Given this, we design a methodology that allows us to combine FFP with the parametric fits.

Suppose we have a time series that begins at time $t = 1$ and ends at time $t = T$. We denote by $\hat\theta_{t,\Delta+t-1}$ the parameter estimates of the model fitted with the sample corresponding to a time window of size $\Delta (1 \leq \Delta \leq T)$, starting at time t. For simplicity, we assume that the models are re-estimated on rolling windows at a daily frequency. The set of rolling parameter estimates is $\hat\Theta \equiv \{\hat\theta_{1,\Delta}, \hat\theta_{2,\Delta+1}, \ldots, \hat\theta_{T-\Delta+1,T}\}$. Once the parameters are estimated on the rolling windows, we assign probabilities for each element in the set. This is done with the FFP approach.

Specifically, we first use one of the methods described in Section 2 to obtain a set of probabilities $\{p_t^\star\}_{t=1}^T$ fulfilling the stress-test view. In a second step, we define the probability of $\hat\theta_{t,\Delta+t-1}$ as

$$p_{\hat\theta_{t,\Delta+t-1}} \equiv \sum_{\tau=t}^{\Delta+t-1} p_\tau^\star ,$$

where $p_\tau^\star$ is the posterior probability at time τ. The probabilities are then normalized to sum to one. This leads us to the set $\mathcal{P} \equiv \{p_{\hat\theta_{1,\Delta}}, \ldots, p_{\hat\theta_{T-\Delta+1,T}}\}$. Figure 1 illustrates the process.

4 Illustration

Using an estimation (rolling) window of $\Delta = 500$ days and a daily recalibration frequency, we estimate the model in Section 3 on an equally weighted portfolio composed of the S&P500 index and IBM stock. The time frame ranges from January 2002 to December 2014. As a macroeconomic variable to express various stress-test views, we consider the VIX index.

For each stress-test, we simulate 100,000 portfolio returns. This is achieved by generating parameter draws in the set $\hat\Theta$ with probabilities $\mathcal{P}$ and simulating returns according to the Glosten–Jagannathan–Runkle model with Student- innovations and the Gaussian copula.

Stress-test results are presented in Table 1, where we report the value-at-risk (VaR) and the expected shortfall (ES) of the simulated unconditional P&L distribution. In addition to the stress-test scenarios presented in Section 2, we consider the

Figure 1. Converting p_t to $P_{\hat\theta t,\Delta+t-1}$. The figure illustrates how we convert the posterior probabilities into probabilities for the parameter estimates.

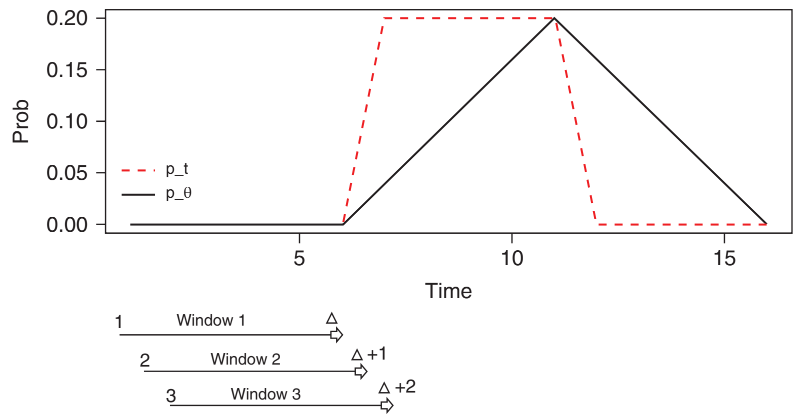

Table 1: Stress-test results. Relevant risk measures for each stress-test case. VaR reports the value-at-risk measures and ES reports the expected shortfall measures for 95%, 99% and 99.9% risk levels. All numbers are in percentages and computed from the P&L distribution obtained with 100,000 simulations.

	Stress-test case			
Risk measure	**Historical**	**Crisis**	**VIX ≥ 30%**	**$\mathbb{E}$[VIX] = 30%**
VaR 95%	−1.9	−3.0	−2.9	−2.5
VaR 99%	−3.9	−5.7	−5.3	−4.7
VaR 99.9%	−6.1	−10.9	−9.6	−9.5
ES 95%	−3.0	−4.7	−4.4	−4.0
ES 99%	−4.8	−7.9	−7.2	−6.7
ES 99.9%	−6.8	−13.7	−12.6	−11.8

case of no stress-test (i.e., historical figures for the P&L). We notice an increase in risk measures for the various stress-test scenarios.

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David Ardia is professor of finance at the University of Neuchâtel, Switzerland, and visiting professor of finance at Laval University, Canada. Dave spent 4+ years in the financial industry. He was senior analyst at aeris CAPITAL AG and head of research at Tolomeo Capital AG, two Swiss-based asset managers. In 2008, he received the Chorafas prize for his book "Financial Risk Management with Bayesian Estimation of GARCH Models" published by Springer. He is the author of several scientific articles and statistical packages. He holds an MSc in applied mathematics, a MAS in finance and a PhD in Bayesian econometrics.

Keven Bluteau is PhD student in finance at the University of Neuchâtel, Switzerland. He obtained his BSc and MBA from Laval University. His research is centered on quantitative methods for finance and risk management.

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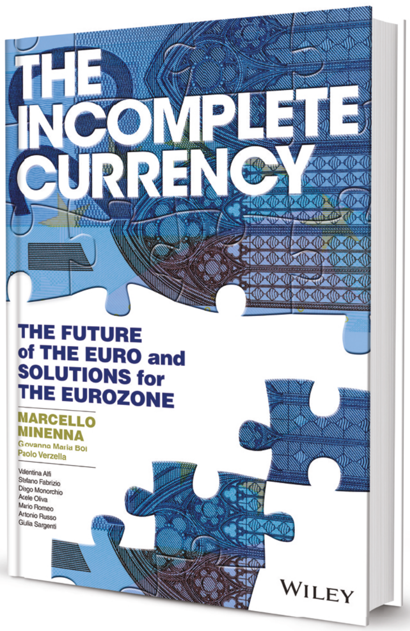

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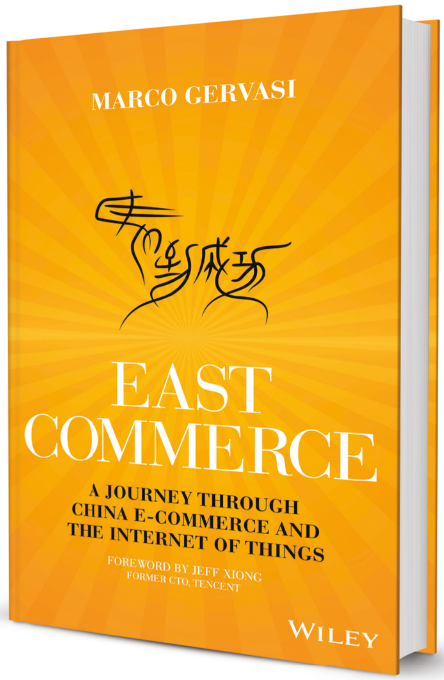

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