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On some Consequences of the Definitional Unprovability of Hume's Principle

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1. Neo-logicism, Hero and the bad company objection

In *Grundlagen* Frege proposes[1] to define the concept of number by laying down what, following a suggestion of Boolos'[2], is nowadays known as *Hume's Principle*:

(HP) The number of *F*s = The number of *G*s if and only if there is a one-one correlation between the *F*s and the *G*s.

However, he rejects[3] the proposal because of the so-called *Caesar problem*, that is, the problem that (HP) succeeds in settling the truth-value of statements of the form 'The number of *F*s = The number of *G*s', but not that of statements of the form 'The number of *F*s = *q*', where '*q*' is an arbitrary singular term. Thus, Frege shifts to an explicit definition in terms of extensions of concepts and later, in *Grundgesetze*, supplies a theory which is intended to show the purely logical nature of the concept of extension. This theory is embodied in the well-known and unfortunate *Axiom V*:

(V) The extension of *F*s = The extension of *G*s if and only if the *F*s and the *G*s are coextensive.

[1] Frege 1884, § 63.
[2] Boolos 1986-87, 171.
[3] Frege 1884, § 66.

Then, he proceeds with the proof of the basic laws of arithmetic from the definition of numbers in terms of extensions through second-order logic. He is therefore seen as having subscribed to *logicism*, the thesis that arithmetic is derivable from logic and suitable definitions. This thesis, on Frege's definition of analyticity, amounts to the thesis that arithmetical truths are analytic. Unfortunately, Russell's Paradox is derivable from (V) through simple logical rules of inference: (V), far from being a logical truth, turns out to be inconsistent.

Crispin Wright in his *Frege's Conception of Numbers as Objects*[4] claims that Frege's choice is premature: it is possible to solve the Caesar problem without turning to the definition of numbers in terms of extensions and therefore without making use of Axiom V. For Frege, while sketching in *Grundlagen* the derivation of the Peano Axioms from logic and definitions, makes use of the notion of extension only when he identifies numbers with extensions. This allows him to prove the truth of (HP)[5]. The notion of extension does not play any further role in the derivation of the basic laws of arithmetic: the Peano Axioms are theorems of *Frege Arithmetic*[6] (henceforward FA), the system resulting from the adjoining of (HP) to a suitable axiomatization of second-order logic. This result is now known as *Frege's Theorem*[7]. Moreover, FA is equiconsistent with mathematical analysis and, therefore, presumably consistent[8].

Wright ascribes to Frege's Theorem a deep philosophical significance, claiming that it can be the basis for the recovery of the logicist program: *neo-logicism*[9] is the thesis that arithmetical truths are

[4] Wright 1983.

[5] Frege 1884, § 73.

[6] Boolos so labels the system in Boolos 1987, 183-201.

[7] Boolos 1990b, 209.

[8] See Boolos 1987, 187-191.

[9] The term 'neo-logicism' is sometimes used to describe the entire program in the philosophy of mathematics. The program is also called 'neo-Fregean'. On the one hand, this term strikes me as more apt to refer to the two main theses (logicism and platonism) which distinguish the program and which Frege may be reasonably taken as having held. On the other hand, it raises some exegetical worries, since it may be doubted that the spirit of the program is completely Fregean (see Dummett 1991; Ruffino 2003). In this paper I will feel free to use the two terms interchangeably, since I will be mainly concerned with the epistemological component of the program.

analytic, the Peano Axioms being logical consequences of an analytic truth, namely (HP). Although Frege was wrong in thinking that arithmetic is reducible to logic, the philosophical essence of logicism, Wright argues, can be saved: arithmetical truths are epistemically innocent, since they inherit this status from (HP) through logical consequence.

In recent writings[10], Wright seeks to clarify the epistemological value of Frege's Theorem by showing how Hero, a subject endowed with mastery of second-order logic, could come to acquire knowledge of arithmetical truths on receipt of Hume's Principle. The idea is that Hero stipulates Hume's Principle and, in so doing, comes to know that it is true. Then, he can gain knowledge of arithmetic simply by reflecting upon its logical consequences.

Since the publication of Wright's book, many objections have been raised to the analyticity of (HP). Let me focus on one of them, the well-known *bad company* objection. (HP) is an instance of a kind of principles, known as *abstractions principles*, which give necessary and sufficient conditions for the identity of objects of some kind in terms of the obtaining of an equivalence relation between entities of another kind. The idea is to introduce a concept by giving identity criteria for the objects which fall under it. Call this method of concept-introduction the *method of abstraction*. The troubles begin as soon as we realize that (V), as well as (HP), has the form of an abstraction principle. We introduce the concept of extension by stipulating that the truth-conditions of identity statements between extensions are to be given in terms of coextensivity. But how can Hume's Principle be thought of as analytic if it belongs to a genre of principles which has inconsistent instances? However, this line of thought is, according to Wright, too simple-minded. Single failures of a method of concept-introduction do not show that the whole method is defective. What they show, rather, is that the method needs further refinement. Just lay down consistency as a constraint on the acceptability of an abstraction

[10] See Wright 1998b, 247-255 and Wright 1998c, 263-271.

principle and you will get what you need. But, as Boolos has shown[11], this reply will not do, since there are consistent abstraction principles which are inconsistent with (HP). In order to overcome this new difficulty, Wright has proposed further restrictions, such as conservativeness[12] and modesty[13]. On the other hand, further counterexamples have been put forward by the critics of neo-logicism[14]. The debate is still open and alive. However, I will not enter into it here. What I want to do, rather, is to explore some consequences of the introduction of restrictions.

2. The definitional unprovability of Hume's Principle and Success by Default

First of all, let us inquire after the epistemological status of restrictions. Consider the most intuitive one: consistency. Let Con(FA) be a sentence in the language of FA which expresses FA's consistency. Gödel's second incompleteness theorem tells us that, if FA is consistent, then Con(FA) is not provable in FA. We need other resources, resources which cannot be formalized in FA. Can we attribute such resources to Hero? *Ex hypothesi*, the only resources at his disposal are second-order logic and (HP). So there seems to be no way for him to prove the consistency of FA, even in principle. At this stage someone might point out that Boolos' proof of the equiconsistency of FA and analysis gives us strong reasons to believe that FA is consistent. Indeed, from our point of view, this is a very important result. Hero, however, cannot rely on Boolos' proof for two reasons. First, as Boolos himself emphasized, doubts may be raised about its actually delivering knowledge of consistency, since it leaves open the possibility that FA and analysis are both inconsistent[15]. Secondly and

[11] Boolos 1990b, 214-215.

[12] Wright 1997, 295-297.

[13] Wright 1998a, 324-330.

[14] Shapiro & Weir 1999 and Weir, forthcoming.

[15] Boolos 1997, 313.

crucially, Hero cannot be credited with knowledge of analysis, on pain of giving up the foundational significance of Frege's Theorem. So there can be no proof through which Hero can go in order to make sure of the consistency of Hume's Principle. What about the consistency of others abstraction principles? As Heck has shown[16], the consistency of second-order abstraction principles (that is, those abstraction principles whose equivalence relation is a relation on concepts) is in general undecidable.

The introduction of restrictions, then, forces the neo-Fregean to endorse the following thesis (**Definitional Unprovability**):

> (**DU**) For most abstraction principles AP (including (HP)), Hero cannot prove that AP is good.

Since Hero's story is intended to show the foundational significance of Frege's Theorem, endorsement of **DU** amounts to accepting that, when inquiring after the epistemological status of (HP), we cannot presuppose a demonstrative warrant for its truth. Nonetheless, the neo-Fregean claims that if (HP) is true, then Hero knows that it is true. On this view, Hero seems to be credited with something like a default justification for the truth of (HP). This point, already noted by Fraser MacBride[17], has been recently emphasized in a somehow different way by Augustín Rayo[18]. He claims that the neo-Fregean program rests on a hidden assumption, which he labels *Success by Default*. According to *Success by Default*, we are justified in thinking that a stipulation is successful (i.e. bestows meaning on the expressions which introduces in such a way that the stipulated sentence turns out to be true) unless there are reasons to believe otherwise. Rayo goes on to argue that since *Success by Default* has not yet received a proper justification, there is a serious gap in the neo-logicist program. What I will argue in the next section is that, given **DU**, even if we grant the correctness of *Success by Default*, it does not follow that Hero knows that (HP) is true.

[16] Heck 1992, 492-493.

[17] MacBride 2003.

[18] Rayo 2003.

3. Definitional unprovability and knowledge

In order to fully appreciate the situation, let us approach the matter by recalling a well-known example in the theory of knowledge. The example is Goldman's, on behalf of the theory of relevant alternatives[19]. You are asked to imagine a man, Henry, who, while driving a car in the countryside, identifies various objects on the landscape for his son: cows, tractors, and the like. Pointing at the objects, he says: 'That's a cow', 'That's a tractor', and so on. After a while, a barn comes into his view. As usual, he says: 'That's a barn'. Almost everyone would agree that Henry *knows* that the object he indicated is a barn. Now imagine that, unknown to Henry, the district across which he is driving is full of papier-mâché facsimiles of barns. However, the barn to which Henry has drawn his son's attention is an actual one. Even though Henry truly believes that there is a barn and seems to be perfectly justified in holding this belief, we would not ascribe knowledge to him in such situation. The reason, it is often said, is that the truth of what he said largely depends on lucky circumstances. But I think we can distinguish two ingredients which, taken together, lead our intuitions not to count the example as knowledge: (i) Henry cannot distinguish between the actual barn and the fake barns; (ii) the number of fake barns, compared to the number of actual barns, is remarkable. A single fake barn in a district full of actual barns would not deprive his belief of the status of knowledge.

Now consider Hero. He lays down Hume's Principle and believes it to be true. Moreover, Hume's Principle is successful and therefore true. Finally, he is justified in holding it to be so, provided *Success by Default* is correct. However, this is not enough for his belief to count as knowledge. In fact, there seems to be striking similarities between Henry's and Hero's epistemic situations. **DU** assures us that (i*) Hero cannot distinguish between (HP) and the inconsistent examples. When deciding whether to accept (HP), all he can do is check whether it has the form of an abstraction principle. Besides being satisfied by (HP),

[19] Goldman 1976.

however, this requirement is also satisfied by the inconsistent examples of the method of abstraction. To be sure, had he accepted an inconsistent principle, he could have scrutinized whether the usual lines to contradiction succeed in that case. But this would have only worked for few cases. So we have a perfect analogy with point (i) in the barn example. I guess it is (ii) which, to neo-Fregean eyes, reveals the real disanalogy between the two cases. While trying to show that failures of a method of concept-introduction are not usually taken as questioning the possibility of using such a method, Wright sometimes considers the method of fixing the sense of a sentence by stipulating its truth-conditions. Since no one would think that such method is mistaken simply on the basis of its having troublesome instances, why should we have a different attitude towards the method of abstraction? So Wright makes it sound as if he thinks that this method and the method of abstraction are perfectly analogous. The *rationale* seems to be that in both cases we are concerned with forms of definitions and introduction of concepts and expressions in our language. This analogy, however, is of no help here. What matters in the present situation is not the aim of the method, but whether its failures are isolated cases. But even if this were the case as regards the abovementioned method of sense-fixing, it would not follow that it is the case as far as the method of abstraction is concerned. At this stage, someone might object that this kind of worry is ill-founded, pointing out that the method of abstraction is widely entrenched in mathematical practice. But this does not really tell us very much. For it still remains to be ascertained whether over the years mathematics has been emended in such a way as to remove the bad companions from it, thanks to considerations which Hero cannot take into account. In any event, the method does fail quite often, and we do register frequent historical failures. This sort of concern can get the support of some passages taken from some articles of Boolos. He expresses the following misgiving:

> It is clear that an account of logical truth that attempts to distinguish Hume's Principle as a logical truth will have the hard task of explaining why Hume's Principle is a logical truth even though two

> other similar-looking principles are not. [...] They read: Extensions of concepts are identical if and only if those concepts are coextensive; and: Relation numbers of relations are identical if and only if those relations are isomorphic. Russell showed the former inconsistent; Harold Hodes has astutely observed that the latter leads to the Burali-Forti paradox.[20]

Returning to the bad company objection at a distance of years, he writes more explicitly:

> What I think I was doing was illustrating that what is called [...] "contextual definition" is not, *in general*, a permissible way of introducing a concept. [...] I cited Hodes' splendid observation that the relation-number principle [...] leads to the Burali-Forti paradox in order to point out that Basic Law V was not an isolated case.[21]

I guess what Boolos wanted to stress is that failures of a method of concept-introduction throw an unfavourable light on the legitimacy of the whole method. As we have seen, Wright is not sympathetic at all towards this kind of reasoning. In Boolos' passage, however, there is another important point, which might be taken as showing that a perfect analogy with (ii) applies to Hero's situation: (ii*) the number of inconsistent examples, compared to the number of consistent examples, is remarkable.

Hero's story astonishingly resembles Henry's. You have a subject who masters second-order logic. This subject stipulates (HP) and comes to believe it. If *Success by Default* holds, his belief is justified, since there are no reasons to think that such stipulation is not successful. Since this stipulation *is* successful, it is true. However, there are many other inconsistent principles which he could have stipulated and which he has no means to distinguish from (HP). How can we ascribe knowledge to him? To stress: I am not claiming that the analogy conclusively establishes that Hero and Henry are on the same epistemic foot. What I am claiming, rather, is that even granted the correctness of *Success by Default*, the neo-Fregean still owes us an argument which shows that the bad companions, despite their frequent

[20] Boolos 1990b, 214.

[21] Boolos 1997, 311.

occurrence, really are isolated cases, if she wants to claim that the stipulation of (HP) results in knowledge useful for foundational purposes.

4. Definitional unprovability, bad definitions and use

In this section, I will argue that, given **DU**, the neo-Fregean seems to have troubles in explaining our practices and our behaviour towards definitions. In doing this, I will be outlining a different conception of how I think we should regard abstraction principles.

The neo-Fregean completely subscribes to what she calls the *traditional connection*, i.e. the thesis that some important kinds of a priori knowledge, in particular logical and mathematical knowledge, are grounded in the stipulation of implicit definitions. On this conception, the path from implicit definitions to a priori knowledge proceeds as follows. We stipulate the truth of a certain sentence '#*f*' – where '#_' is a matrix whose content is known and '*f*' is a previously contentless expression whose syntactic category is such as to make it suitable to fill the gap in '#_'. The stipulation, then, confers meaning on '*f*' in such a way that (i) '#*f*' can be understood and (ii) the meaning of '*f*' is such as to render '#*f*' true. This puts us in a position to recognize that the sentence is true, since stipulated to be so. This, in turn, results in a priori knowledge of that very sentence.

Prima facie, what appears surprising is the idea that the truth of a sentence depends upon a decision of ours. To be plausible, this idea cannot be held unrestrictedly, without further qualification. To this end, the neo-Fregean specifies two conditions which, when satisfied, guarantee that we can indeed stipulate the truth of '#*f*' and that the meaning so bestowed on '*f*' will be such as to make '#*f*' true.

The first condition requires that some stipulations should not be *arrogant*, where a stipulation is arrogant when its truth "cannot justifiably be affirmed without collateral (a posteriori) epistemic work"[22]. This condition is needed to characterize a class of sentences

[22] Hale & Wright 2000, 128.

whose truth can be legitimately stipulated without going outside of the domain of the a priori. Sometimes, to say the least, a justification for the truth of a sentence demands cooperation from the world.

The second condition requires that the stipulation of '#f' determine a meaning for 'f'. This condition is needed to deal with the obvious objection that there are cases in which the stipulated sentence, although not arrogant, far from being true, proves to be inconsistent. For someone might say that these cases show that when we stipulate a sentence we do not establish its truth. The neo-Fregean replies that when we stipulate a sentence we do establish its truth, but that there are cases in which the stipulation fails to determine a genuine meaning for the *definiendum*. The distinction between these cases and the good ones can be drawn thanks to some constraints. The neo-Fregean is therefore committed to the following thesis:

> (T) The stipulation of an abstraction principle which does not meet the admissibility constraints fails to provide the *definiendum* with a genuine meaning.

(T), however, is very problematic once **DU** is accepted. For **DU** opens up the possibility of a scenario where Hero comes to accept an abstraction principle which is, by neo-Fregean criteria, bad. It seems that Hero can go on to use the concept that the bad definition serves to introduce. What account are we to give of Hero's use? The only viable strategy for the neo-Fregean is to claim that Hero's use does not count as a *proper* use. However, this seems to clash with the way we conceive of our practices. We usually think that good definitions and bad definitions, although differing as regards features such as consistency, deliver concepts which can be perfectly understood and used. None of us would say that the concept of phlogiston does not have a genuine meaning. The neo-Fregean, for her part, would stress the difference between scientific theoretical terms and logical and mathematical expression. But are we really willing to say that, for example, when using naïve set theory, mathematicians did not genuinely mean what they were talking about?

Moreover, what account are we to give of Hero's attitude towards an abstraction principle if he cannot have a demonstrative warrant for its truth? From Hero's point of view the stipulation of an abstraction principle seems to be only an *attempt*. Hence, it cannot be right to describe him as establishing the *truth* of the abstraction principle. When he stipulates an abstraction principle, his attitude seems rather to be that of *trying* to capture a truth. If we adopt this stance, we also become free to drop (**T**). On this view, in fact, the meaning-constituting role is not played by the *truth* of the principle, but by our *regarding* it *as* true. The stipulation bestows genuine meaning on the *definiendum* independently of its success.

To be sure, there is a familiar move which would put us in a position to deal with the two above-mentioned problems while saving the standard account that a stipulation consists in establishing the truth of a sentence. On a long-standing tradition about the definition of scientific theoretical terms[23], we cannot stipulate the truth of the theory itself, since we have to allow for the possibility that the theory could be false. Nevertheless, we can stipulate the truth of some other sentence that suffices to bestow the intended meaning on the *definiendum* while remaining true if the theory gets disconfirmed by experience. On this conception, the theory is factorized into two components:

(R) $\exists x\,(\#x)$

and

(C) $\exists x\,(\#x) \rightarrow \#f.$

The first component, known as the *Ramsey sentence*, captures the empirical content of the theory, whereas the second component, usually called the *Carnap conditional*, serves to confer meaning on the *definiendum*. Empirical disconfirmation of the theory affects the truth of (R) but not that of (C), whose truth we are therefore free to stipulate. Analogously, we would be free to stipulate the truth of the Carnap conditional of an abstraction principle, even though this might be, for all that Hero knows, inconsistent. This strategy would also enable us to drop (**T**), since on this conception the meaning-constituting role is not played by the theory but by its Carnap conditional.

[23] See, for example, Carnap 1928 and Ramsey 1931.

Wright and Hale claim that even if this kind of account is correct as far as scientific theoretical terms are concerned, it does not follow that the same account must be given as regards the introduction of mathematical and logical expression. However, **DU** implies that Hero cannot rule out the possibility that the theory which (HP) embodies is false. So if we really want to hold on the idea that implicit definitions consist in the stipulation of the truth of certain sentences, what should be taken as being stipulated is not (HP) but the following principle:

(HP*) If there is a function % from properties to objects such that %Fs = %Gs if and only if there is a one-one correlation between the *F*s and the *G*s, then The number of *F*s = The number of *G*s if and only if there is a one-one correlation between the *F*s and the *G*s.

What I am suggesting, then, is that, given **DU**, we have two options. Either we hold on the thesis that what is stipulated is the truth of a sentence, in which case we have to allow for the possibility that the theory is false, falling back on its Carnap conditional, or we accept that a stipulation is only an attempt, in which case we have to give up the idea that what we do when we stipulate a sentence is to establish its truth. Either way, Hume's Principle loses its special role in the neo-Fregean program. The former option is useless for the neo-Fregean's purposes, since (HP*) is not strong enough to provide for the derivation of the basic laws of arithmetic in second-order logic. The latter option deprives (HP) of its special epistemological value, since an attempt, even if successful, can be hardly said to result by itself in knowledge.

5. Extending our results and a bit of holism

I now want to show how the considerations of the previous section could have a wider range of application and extend from Hero's situation to ours. Let S be the set of constraints on the acceptability of an abstraction principle which have been laid down so far (consistency, conservativeness, etc.) and let Q be a complete set of constraints suitable to separate the good companions from the bad ones. Even if we consider ourselves able to know that (HP) meets the constraints in S, we know that it is successful only if:

(i) $Q \subseteq S$.

I said at the outset that I would not enter into the debate concerning the bad company objection. Let me say, however, that there are serious reasons to doubt that all the constraints in Q have been found[24]: Q seem to be a proper extension of S, contrary to (i). Thus, we seem to be in the same situation as Hero: our stipulation is an attempt because an abstraction principle might be bad owing to a constraint which is in Q but not in S. So if we want our stipulation to be something more than an attempt, at least two further conditions need to be satisfied:

(ii) we have succeeded in giving a complete list of the members of Q

(iii) we know that (HP) meets all the constraints in Q.

(ii), however, can be satisfied only if there is a set such as Q. But what guarantee do we have that there is such a set? While working on some analogies between the method of abstraction and other methods of concept-formation, Wright writes:

> [W]e should not, presumably, conclude that the whole idea that properties can be defined by the stipulation of satisfaction condition is misbegotten. Rather, some kind of restriction is wanted. Nor, crucially, pending such a restriction, should we suspend judgement about what appear to be perfectly innocent examples of such procedure. The sensible response is rather that there is a distinction to be

[24] See Weir, forthcoming.

> drawn *which we are, perhaps, not clear how exactly to draw* but which, once drawn, will safeguard the vast majority of cases where we fix a property by stipulating satisfaction conditions [*my emphasis*].[25]

In this passage, Wright himself seems to acknowledge that it is not completely clear how exactly to draw the distinction between the good and the bad instances of a method of concept introduction. But, then, why should we take it for granted that such distinction can be drawn? In a later article, Wright and Hale say, more cautiously, that they see "no reason for pessimism that such a complete set of constraints can be given"[26]. But the growing number of counterexamples might just be taken as casting doubts on the possibility of characterizing the required set. Moreover, any kind of consideration seems to be potentially relevant in this context. We could be led to modify or reject a concept for many different reasons. So many that it can be doubted that a complete list of constraints can actually be given. So our being in the same situation as Hero might be something more than a temporary situation. Relatedly, in the absence of a set such as Q, how can we say that the considerations we will have to call into account in order to separate the good companions from the bad ones will not jeopardize its analyticity? A posteriori threatens.

The picture which has emerged, then, is the following. We introduce a concept. This introduction, however, is only an attempt to capture a truth. We do not establish that a principle is true even when the principle *is* true. The discovery of inconsistencies or other problems will lead us to modify the concept. If the concept cannot be modified in such a way as to make it acceptable from the new perspective, the concept will be dropped. Let me end, in this connection, with an historical remark which seems to shed light on how this picture seems to fit well with our practices. In 1893, Frege introduced the concept of extension by laying down Axiom V in the first volume of *Grundgesetze*. Russell's discovery of the paradox in 1902 forced him to try to emend the concept of extension in such a way as to make it

[25] Wright 1997, 288.
[26] Wright & Hale 2000, 137.

consistent but powerful enough for his purposes. In 1906, he realized that this could not be done and decided to drop the concept. His behaviour closely resembles that of a scientist who, when presented with contrary empirical evidence, decides to modify and eventually comes to reject the theory. The difference between scientific and mathematical enterprises, though important, should not be exaggerated.

References

BOGHOSSIAN P. & PEACOCKE C. (eds). (2000). *New Essays on the A Priori*. Oxford: Clarendon.

BOOLOS G. (1986-87). Saving Frege from Contradiction. *Proceedings of the Aristotelian Society* 87. 137-151. [Boolos 1998, 171-182].

BOOLOS G. (1987) The Consistency of Frege's *Foundations of Arithmetic*. In Thomson 1987, 3-20. [Boolos 1998, 183-201].

BOOLOS G. (ed). (1990a). *Meaning and Method: Essays in Honor of Hilary Putnam*. Cambridge University Press.

BOOLOS G. (1990b). The Standard of Equality of Numbers. In Boolos, 1990a, 261-278. [Boolos 1998, 202-219].

BOOLOS G. (1997). Is Hume's Principle Analytic? In Heck 1997, 245-262. [Boolos 1998, 301-314].

BOOLOS G. (1998). *Logic, Logic, and Logic*. Cambridge Mass.: Harvard University Press.

CARNAP R. (1928). *Der Logische Aufbau der Welt*. Berlin: Schlachtensee Weltkreis-Verlag.

DUMMETT M. (1991). *Frege. Philosophy of Mathematics*. London: Duckworth.

FREGE G. (1884). *Die Grundlagen der Arithmetik*. Breslau: Koebner.

FREGE G. (1893, 1903). *Grundgesetze der Arithmetik, begriffsschriftlich abgeleitet*, 2 vol., Jena: Pohle.

GOLDMAN A. I. (1976). Discrimination and Perceptual Knowledge. *Journal of Philosophy* 73. 771-91.

HALE B. & WRIGHT C. (2000). Implicit Definition and the A Priori. In Boghossian & Peacocke 2000, 286-319. [Hale & Wright 2001, 117-150].

HALE B. & WRIGHT C. (2001). *The Reason's Proper Study. Essays Towards a Neo-Fregean Philosophy of Mathematics*. Oxford: Clarendon.

HECK JR. R. (1992). On the Consistency of Second-Order Contextual Definitions. *Noûs* 64. 491-494.

HECK JR. R. (ed). (1997). *Language, Thought and Logic: Essays in Honour of Michael Dummett*. Oxford University Press.

MACBRIDE F. (2003). Speaking with Shadows: A Study of Neo-Logicism. *British Journal for the Philosophy of Science* 54/1. 103-163.

RAMSEY F. P. (1931). Theories. In his *The Foundations of Mathematics and Other Logical Essays*. London: Routledge & Kegan Paul. 212-237.

RAYO A. (2003). Success by Default? *Philosophia Mathematica* 11/3. 305-322.

RUFFINO M. (2003). Why Frege Would Not Be a Neo-Fregean. *Mind* 112. 51-78.

SHAPIRO S. & WEIR A. (1999). New V, ZF and Abstraction. *Philosophia Mathematica* 7/3. 293-321.

SCHIRN M. (ed). (1998). *The Philosophy of Mathematics Today*. Oxford: Clarendon.

THOMSON J. (ed). (1987). *On Being and Saying: Essays in Honor of Richard Cartwright*. Cambridge Mass.: MIT Press.

WEIR A. (Forthcoming). Neo-Fregeanism: An Embarassment of Riches. *Notre Dame Journal of Formal Logic*.

WRIGHT C. (1983). *Frege's Conception of Numbers as Objects*. Aberdeen University Press.

WRIGHT C. (1997). On the Philosophical Significance of Frege's Theorem. In Heck 1997, 201-244. [Hale & Wright 2001, 272-306].

WRIGHT C. (1998a). Is Hume's Principle Analytic? *Notre Dame Journal of Formal Logic* 40/1. 307-332. [Hale & Wright 2001, 307-32].

WRIGHT C. (1998b). On the (Harmless) Impredicativity of Hume's Principle. In Schirn 1998. 339-68. [Hale & Wright 2001. 229-255].

WRIGHT C. (1998c). Response to Dummett. In Schirn 1998. 389-405. [Hale & Wright 2001. 256-271].