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ASYMPTOTICAL ANALYSIS OF TIMING IMPERFECTIONS IN UWB RECEIVERS

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***Abstract*— In this paper, a model is proposed to analyze the influence of timing imperfections on ultra-wide bandwidth (UWB) time-hopping spread-spectrum (TH-SS) signaling schemes, where N pulses are coherently added to increase the received signal-to-noise ratio (SNR). Based on this model, we derive asymptotical (valid for large N) expressions for the received signal after pulse summation and matched filtering.**

As an application example, we study the influence of timing jitter and frequency offsets on the maximum SNR, assuming a 2nd derivative Gaussian pulse shape. The analytical results we obtain are verified with numerical simulations.

***Index Terms*— Ultra-wide bandwidth (UWB), impulse radio, jitter, frequency offset.**

I. Introduction

UWB impulse radio (IR) spreads the energy of the radio signal over a wide bandwidth by transmitting train of pulses with very short duration [1]. This allows high data rates. However, due to the short duration of the pulses (sub-nanoseconds), an accurate time domain analysis is mandatory.

In [2], [3], the effect of timing jitter on the spectral density has been studied. A comparison between bit error rates (BER) of different UWB modulation schemes has been performed in the presence of timing jitter in static and Rayleigh fading channels in [4]. BER performance has been studied in correlated random timing jitter in [5]. Jitter models in delay locked loop (DLL) and phase locked loop (PLL) have been used to evaluate the sensitivity of sampling and correlation in a UWB digital receiver in [6]. BERs of different modulation schemes under different conditions (multipath, multiple-access interference, narrowband interference, and timing jitter) have been evaluated in [7]. The influence of timing jitter on BER in the case of orthogonal Hermite pulse shapes has been studied in [8], [9]. Finally, using computer simulations, the performance of UWB-IR in terms of throughput has been shown sensitive to timing jitter and tracking in [10], [11].

In contrast to the above literature, we develop in this paper a model to investigate the influence of any timing imperfection in the transmitter or the receiver on the pulse combining gain, during the transmission of a known training sequence. This case arises in transmission systems during synchronization or in positioning systems. In the latter, a high gain in SNR (large N) is mandatory for accurate time of arrival estimation.

This paper has been organized as follows. Section II and III contains the general UWB system model and analytical analysis, respectively. In section IV the theory is applied to assess the influence of timing jitter and frequency offsets on the achievable SNR for a matched filter receiver. Finally, Section V contains the conclusions.

II. System Model

In the presence of one transmitter, a pulse position modulated (PPM) UWB sequence is defined by the received pulse shape $p(t)$, the average time between two consecutive pulses or pulse repetition time T_f, a pseudorandom sequence h_n used to distinguish between many simultaneous users and to spread the energy over the bandwidth, the duration of addressable time delay bins T_h, the data sequence d_n, the modulation index δ, and a phase error term $\epsilon(n)$, as

$$s(t) = \sum_n p(t - nT_f - h_n T_h - d_n\delta + \epsilon(n)). \qquad (1)$$

The error term $\epsilon(n)$ can be used to model different imperfections. In this paper, we use it to model the timing jitter of the transmitter and/or the receiver, and to model a frequency offset between the received sequence and the receiver.

For additive noise channels, the received sequence $r(t)$ will be the sum of the signal $s(t)$ containing the pulses and a noise term $n(t)$, i.e., $r(t) = s(t) + n(t)$.

To coherently sum the pulses, the receiver shifts the nth sequence by[1] $nT'_f + h_n T'_n + d_n\delta'$. Assuming that N pulse sequences are summed and each sequence of length T_f is sufficiently long so that even in the presence of timing errors the shifted pulses within the sequences are not truncated, we can express the sequence after summation as

$$r_{\text{sum}}(t) = \sum_{n=1}^{N} p(\tau) + \underbrace{\sum_{n=1}^{N} n(\tau)}_{n_{\text{sum}}(t)}\ , \ -\frac{T_f}{2} < t < \frac{T_f}{2} \qquad (2)$$

where $\tau = t - n(T_f - T'_f) - h_n(T_h - T'_h) - d_n(\delta - \delta') + \epsilon(n)$.

Assuming that $n(t)$ is a white Gaussian noise process of zero mean and variance $\mathcal{N}_o/2$, $n_{\text{sum}}(t)$ will also be a white Gaussian noise process of zero mean and variance $N\mathcal{N}_o/2$.

III. Time Domain Analysis

By combining any of the above timing errors in τ into a common error term $\epsilon'(n)$, we obtain

$$r_{\text{sum}}(t) = \underbrace{\sum_{n=1}^{N} p(t + \epsilon'(n))}_{s_{\text{sum}}(t)} + n_{\text{sum}}(t) \qquad (3)$$

[1]The prime is used to separate the respective time intervals at the receiver with the time intervals in the received sequence in the presence of timing errors.

Noting that we can write $p(t+\epsilon'(n))$ using the convolution integral as[2]

$$\begin{aligned} p(t+\epsilon'(n)) &= \int_{-\infty}^{\infty} p(\tau)\delta(t+\epsilon'(n)-\tau)\,d\tau \\ &= p(t)\circ\delta(t+\epsilon'(n)) \end{aligned} \quad (4)$$

and using the linearity properties of the convolution, we can write the normalized summed signal $s_{\text{sum}}(t)/N$ as

$$\begin{aligned} \frac{s_{\text{sum}}(t)}{N} &= \frac{1}{N}\sum_{n=1}^{N}\left[p(t)\circ\delta(t+\epsilon'(n))\right] \\ &= p(t)\circ\frac{1}{N}\sum_{n=1}^{N}\delta(t+\epsilon'(n)). \end{aligned} \quad (5)$$

At the limit $N\to\infty$, and assuming that $\epsilon'(n)$ is mean ergodic[3], we thus have

$$\begin{aligned} \chi(t) &\stackrel{\text{def}}{=} \lim_{N\to\infty} E\left[\frac{s_{\text{sum}}(t)}{N}\right] = p(t)\circ E\left[\delta(t+\epsilon'(n))\right] \\ &= p(t)\circ\left[\int_{-\infty}^{\infty}\delta(t+\epsilon'(n))f_{\epsilon'}(t)\,dt\right] \\ &= p(t)\circ f_{\epsilon'}(t) \end{aligned} \quad (6)$$

where $E[\cdot]$ denotes statistical expectation and $f_{\epsilon'}(t)$ is the probability density function (pdf) for $\epsilon'(n)$.

For sufficiently large N, we can thus approximate $s_{\text{sum}}(t)$ as

$$s_{\text{sum}}(t) \approx N\chi(t) = Np(t)\circ f_{\epsilon'}(t). \quad (7)$$

From (7), we can also approximate the energy $\mathcal{E}_{\text{sum}}^{N}$ of the time averaged signal $s_{\text{sum}}(t)$ with the energy of the statistically averaged normalized signal $\chi(t)$ as

$$\begin{aligned} \mathcal{E}_{\text{sum}}^{N} &= \int_{-\infty}^{\infty}\left(s_{\text{sum}}(t)\right)^2 dt \\ &\approx N^2\int_{-\infty}^{\infty}(\underbrace{p(t)\circ f_{\epsilon'}(t)}_{\chi(t)})^2 dt \stackrel{\text{def}}{=} \mathcal{E}. \end{aligned} \quad (8)$$

Assuming a matched filter matched to $s_{\text{sum}}(t)$ and recalling that $n_{\text{sum}}(t)$ has variance $N\mathcal{N}_o/2$, the maximum SNR at the output of the filter will thus be [13]

$$\text{SNR}_{\text{opt}} = 2\mathcal{E}_{\text{sum}}^{N}/(N\mathcal{N}_o). \quad (9)$$

It is also interesting to consider the gain in SNR G due to the acquisition of N pulses:

$$G = \frac{\text{SNR}_{\text{opt}}}{\text{SNR}_o} = \frac{\mathcal{E}_{\text{sum}}^{N}}{N} \approx \frac{\mathcal{E}}{N} \quad (10)$$

where $\text{SNR}_o = 2/\mathcal{N}_o$ corresponds to the acquisition of one pulse with normalized unit energy.

In the next section, analytical results for $\chi(t)$ and G are given and compared with numerical results for finite N.

[2] $\circ$ denotes the convolution operator.

[3] Note that assuming $\epsilon'(n)$ to be mean ergodic is not a very restrictive assumption, as it can be shown that a sufficient condition for a discrete-time white-sense stationary (WSS) random process $\epsilon'(n)$ to be mean ergodic is $C_{\epsilon'}(0) < \infty$ and $\lim_{n\to\infty} C_{\epsilon'}(n) = 0$ where $C_{\epsilon'}(n)$ is the covariance of $\epsilon'(n)$ [12].

IV. Application Example

A. Assumptions

Due to the differentiating effect of the antennas, the received pulse is modelled as the derivative of a Gaussian monopulse [4], i.e., as

$$p(t) = \frac{\partial^2}{\partial t^2} A\exp\left(-\frac{t^2}{2t_n^2}\right) \quad (11)$$

where t_n defines the width of the pulse.

Assuming a unit energy pulse, i.e., $\int_{-\infty}^{\infty} p^2(t)\,dt = 1$, we have $A = -\sqrt{\frac{4t_n^3}{3\sqrt{\pi}}}$, and the normalized pulse is thus

$$p(t) = \frac{2(t_n^2 - t^2)}{\sqrt{3\pi^{1/2}t_n^5}}\exp\left(-\frac{t^2}{2t_n^2}\right). \quad (12)$$

Pulse shapes for different t_n are plotted in Fig. 1. We assume furthermore an additive white Gaussian noise (AWGN) transmission channel.

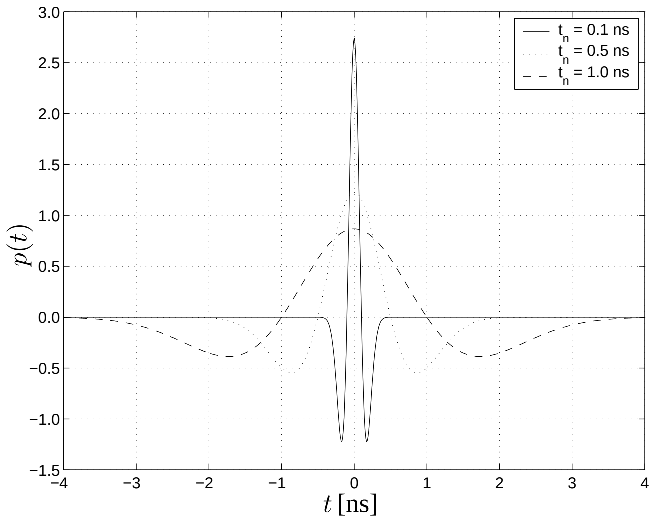

Fig. 1: Normalized 2nd derivative Gaussian monopulse for $t_n \in \{0.1, 0.5, 1.0\}$ ns.

B. Gaussian distributed jitter

We assume a zero mean Gaussian distributed jitter with variance σ^2, i.e,

$$\epsilon'(n) \sim f_{\epsilon'}^{(\text{G})}(t) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{t^2}{2\sigma^2}\right). \quad (13)$$

Substituting (12) and (13) in (6), we obtain

$$\chi^{(\text{G})}(t) = \frac{2t_n^{5/2}(\sigma^2 + t_n^2 - t^2)}{\sqrt{3\pi^{1/2}(\sigma^2+t_n^2)^5}}\exp\left(-\frac{1}{2}\frac{t^2}{\sigma^2+t_n^2}\right). \quad (14)$$

From (8), the normalized energy is

$$\mathcal{E}^{(\text{G})}/N^2 = \int_{-\infty}^{\infty}(\chi^{(\text{G})}(t))^2\,dt = \left(\frac{t_n}{\sqrt{\sigma^2+t_n^2}}\right)^5. \quad (15)$$

Using (10) we can thus express the gain $G^{(\text{G})}$ as

$$G^{(\text{G})} = \left(\frac{t_n}{\sqrt{\sigma^2+t_n^2}}\right)^5 N. \quad (16)$$

As expected, the SNR gain is proportional to N and depends on the pulse width t_n and the jitter variance σ^2. The slope is maximal and equals 1 in the absence of Gaussian distributed jitter ($\sigma = 0$). We note that the gain continues to increase with increasing N.

Fig. 2 represents the gain for a 2nd derivative Gaussian monopulse and a Gaussian distributed timing jitter of variance σ^2. As expected, the degradation due to Gaussian distributed jitter is more severe for narrow pulses.

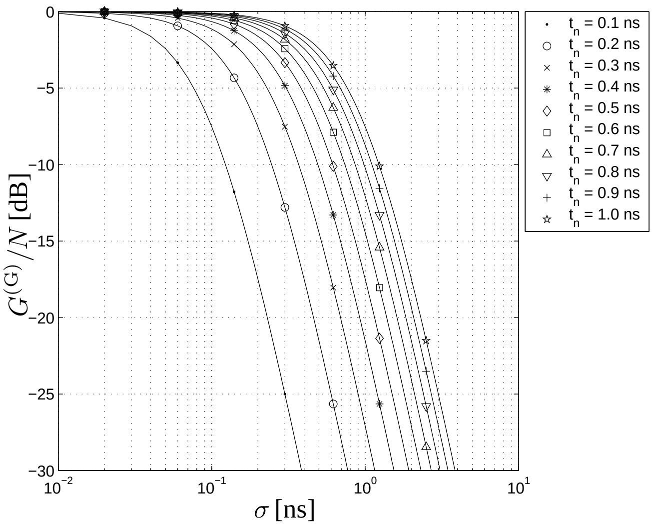

Fig. 2: Theoretical gain for a 2nd derivative Gaussian monopulse and a Gaussian distributed timing jitter of variance σ^2.

In Fig. 3, we plotted the outcomes of Monte-Carlo trials where we computed experimentally the gain for a number of summed pulses $N \in \{1, \ldots, 200\}$, assuming a pulse's width of $t_n = 0.5$ ns and a jitter's standard deviation of $\sigma = 0.2$ ns. We note that the convergence occurs effectively around the predicted value (calculated using (15)) of -1.6 dB.

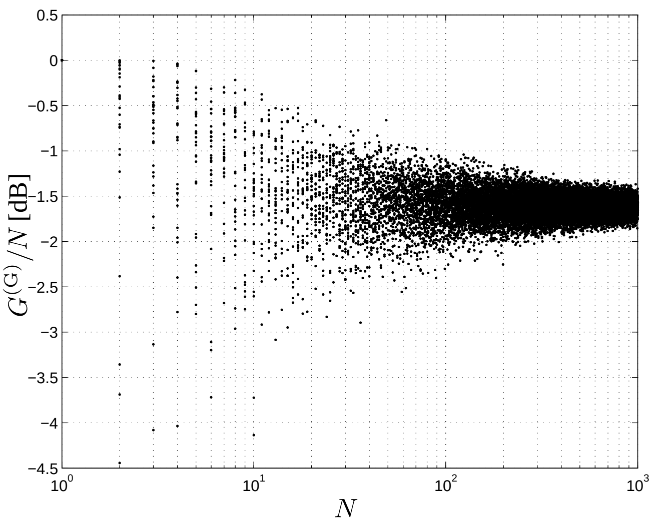

Fig. 3: Gain obtained by Monte Carlo simulations for a pulse's width of $t_n = 0.5$ ns and a Gaussian distributed timing jitter with standard deviation $\sigma = 0.2$ ns, as a function of the number of summed sequences N.

C. Frequency offset

A receiver which applies the correct spreading sequence h_n, but is not synchronized to the sender can be modelled with a linear error term $\epsilon'(n) = (T_f' - T_f)n$. Letting the time offset after N summations to be Δt, we obtain

$$\Delta t = N\alpha = N(T_f' - T_f) \tag{17}$$

where α is the time offset between two consecutive pulse acquisitions. Note that the constraints imposed on the receiver's clock will increase for increasing N.

It can be shown that an asymmetry in the timing jitter pdf does not influence the power spectral density of the signal [3]. According to Parseval's theorem the energy is also independant of an asymmetry. We can thus consider the following symmetric uniform pdf for the error term $\epsilon'(n)$

$$\epsilon'(n) \sim f_{\epsilon'}^{(\mathrm{U})}(t) = \begin{cases} 1/\Delta t & , -\Delta t/2 < t < \Delta t/2 \\ 0 & , \text{otherwise} \end{cases} \tag{18}$$

Proceeding as previously, the statistical expectation of the normalized summed signal as $N \to \infty$ becomes

$$\begin{aligned} \chi^{(\mathrm{U})}(t) \;&= \frac{\Delta t + 2t}{\sqrt{3t_n\sqrt{\pi}}\Delta t} \exp\left(-\frac{1}{8}\left(\frac{\Delta t + 2t}{t_n}\right)^2\right) \\ &\quad + \frac{\Delta t - 2t}{\sqrt{3t_n\sqrt{\pi}}\Delta t} \exp\left(-\frac{1}{8}\left(\frac{\Delta t - 2t}{t_n}\right)^2\right) \end{aligned} \tag{19}$$

which results in the following normalized energy

$$\mathcal{E}^{(\mathrm{U})}/N^2 = \frac{4t_n^2}{3\Delta t^2} + \left(\frac{2}{3} - \frac{4t_n^2}{3\Delta t^2}\right) \exp\left(-\frac{1}{4}\frac{\Delta t^2}{t_n^2}\right). \tag{20}$$

In Fig. 4, the gain in a matched filter receiver computed using (8)-(10) is plotted as a function of the time offset Δt for various pulse widths t_n.

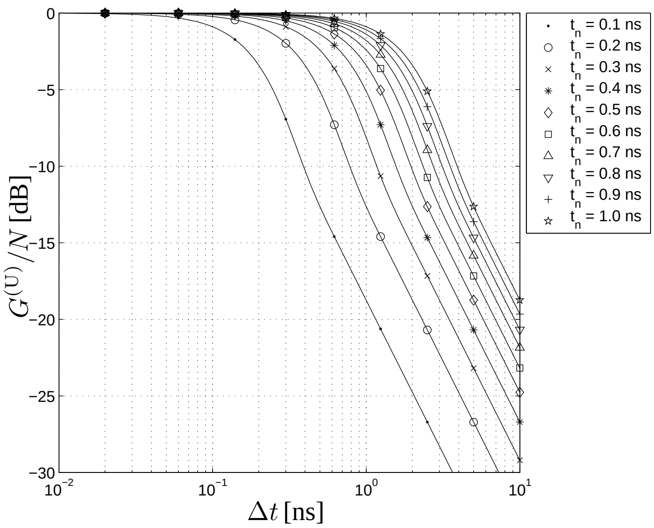

Fig. 4: Theoretical gain for a 2nd derivative Gaussian pulse shape and a frequency offset $\Delta t = N(T_f' - T_f)$.

Inserting (17) in (20), we can express the gain $G^{(\mathrm{U})}$ as a function of N and a constant time offset α

$$G^{(\mathrm{U})} = \frac{4t_n^2}{3N\alpha^2} + \left(\frac{2N}{3} - \frac{4t_n^2}{3N\alpha^2}\right) \exp\left(-\frac{1}{4}\frac{N^2\alpha^2}{t_n^2}\right). \tag{21}$$

The maximum achievable gain $G_{\max}^{(\mathrm{U})}$ can now be found by solving $\partial G^{(\mathrm{U})}/\partial N = 0$.

$$G_{\max}^{(\mathrm{U})} \approx 0.9564\frac{t_n}{\alpha} \tag{22}$$

$$N_{\max}^{(\mathrm{U})} \approx 1.6102\frac{t_n}{\alpha} \tag{23}$$

Fig. 5 shows the maximum achievable gain $G^{(U)}_{\max}$ as a function of the time offset α. Note that (22) can also be used to compute the maximum time offset $\alpha_{\max}$ that can be tolerated for a given desired gain $G^{(U)}_{\max}$, as illustrated in the following example.

Example: Assuming that $t_n = 100\,\text{ps}$ and the desired gain is $20\,\text{dB}$, (22) yields a maximum value for α to achieve the required gain of $\alpha \leq \alpha_{\max} \approx 1\,\text{ps}$. This corresponds to a timing accuracy of $100\,\text{ppm}$ for a pulse repetition time $T_f = 10\,\text{ns}$. From (23), a summation over 161 individual pulses is required to achieve this specified gain.

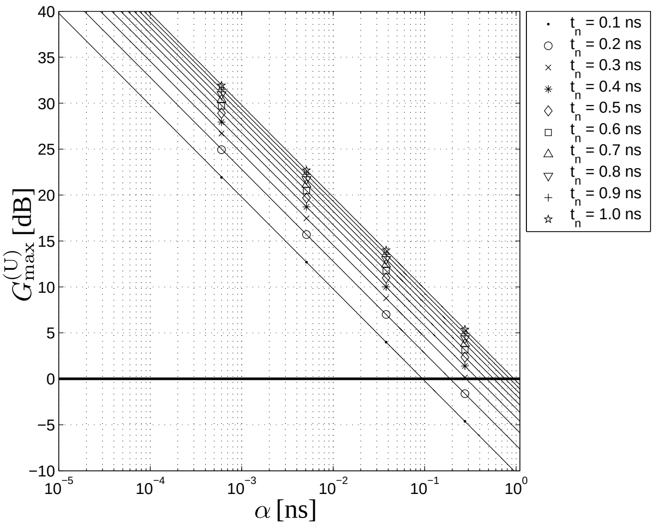

Fig. 5: Maximum achievable gain for a 2nd derivative Gaussian pulse shape as a function of the time offset α.

In Fig. 6, we verified experimentally the convergence of the gain for a finite number of pulses, assuming a pulse's width of $t_n = 0.5\,\text{ns}$, and a time offset after N summations of $\Delta t \in \{1, 2, \ldots, 9\}$ ns. As expected, the asymptotical values in Fig. 6 agree with the theoretical values in Fig. 4.

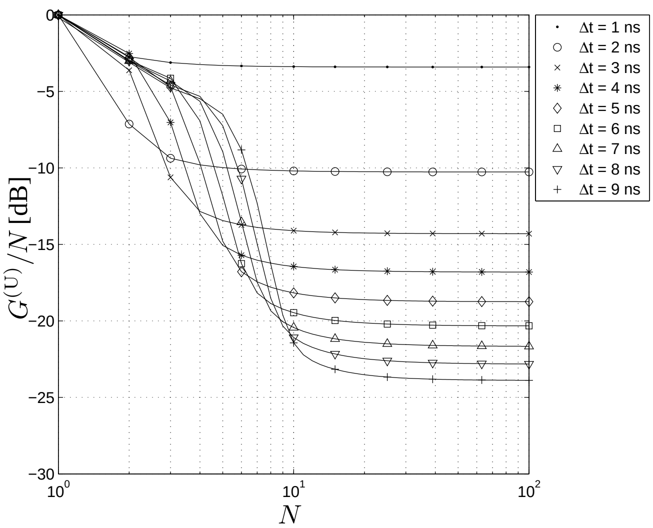

Fig. 6: Numerically evaluated gain for a pulse shape's width of $t_n = 0.5\,\text{ns}$ and a time offset after N summations of $\Delta t \in \{1, 2, \ldots, 9\}$ ns, as a function of the number of summed sequences N.

V. Conclusion

Our asymptotical analysis provides valuable information about the nuisance effects of timing imperfections on the achievable gain after pulse summation and matched filtering. The analytical results can be applied to any pulse shape and different timing jitter assumptions.

Closed form solutions are provided for a Gaussian distributed jitter and a frequency offset assuming a 2nd derivative Gaussian monopulse. As expected, the numerical simulations converge to the analytical results for moderate N. The maximum achievable gain for a given frequency offset is also computed.

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