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High-resolution measurement of phase singularities produced by computer-generated holograms

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Abstract

We present measurements of the intensity as well as the phase distribution in the various diffraction orders of computer-generated holograms designed to generate a higher order Gauss–Laguerre beam. For the direct measurement of the phase distribution in the diffraction orders a high-resolution interferometer is used, which allows access to a lateral length scale for the localization of phase singularities below the wavelength. It is experimentally shown that in beams that carry multiple singularities, the dislocations do not degenerate. This effect cannot be seen by analyzing only the intensity distribution of the laser beam.

Keywords: Computer-generated holograms; Phase singularities; High-resolution microscopy

1. Introduction

Phase singularities are points in space where the real and imaginary parts of a scalar component of the electromagnetic field vanishes: the amplitude is zero, the phase is not determined and its gradient becomes infinite [1]. Basically, one distinguishes between edge dislocations and screw dislocations.

Screw dislocations are line singularities parallel to the propagation direction [2], which appears as a point if the phase is measured in a plane perpendicular to the propagation direction. The phase on a closed circle around this point will change by a multiple m of 2π, with m being the strength of

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the dislocation, also called the charge [3]. In nature they appear in speckle fields for example [4]. Laser beams that carry screw dislocations can be generated in a controlled manner within laser resonators [5], by appropriate phase plates [6,7], by combining the two method [8], or by computer-generated holograms (CGHs) [9], amongst other methods. Beams with this kind of singularity find manifold applications, e.g. for the trapping of particles [10] or atoms [11].

The measurement of the generated phase distribution is normally deduced from intensity recordings of the interference patterns between the beam that carries the dislocations and a tilted reference plane wave, a technique called fork interferometry [3]. The straight lines of constructive and destructive interference are disordered at the point of the singularity and m lines either appear or vanish. The position of the singularities can be found with a resolution that depends basically on the angle between object and reference beam. However, the angular difference between the two beams is fairly usually low and a resolution in the order of tens of micrometers is typically achieved. More recently, Leach et al. [12] investigated in depth the phase distribution in beams with a non-integer dislocation using an interferometric technique based on a Fourier-transformation of a larger number of superpositions between the object and the reference beam.

We use a high-resolution interference microscope to measure the phase distribution in the different diffraction orders of a CGH designed to generate Gauss–Laguerre beams in the first order. Such a type of interferometer has been used in the past to analyze with high precision the amplitude and phase distribution around single micro-structures [13,14], periodic objects [15] and in the focal region of microlenses [16]. We will understand the resolution of the microscope to be its capability to determine the position of the phase singularities. Such a definition of the resolution will not violate that classical resolution limit (e.g. the two-point resolution criteria), as we intend to analyze the phase distribution in a wave-field that has no physical counterpart in the object plane. This resolution is limited by technological constrains that correspond in the present case to a few pixels of the CCD-Camera. A pixel corresponds approximately to 100 nm in the object plane [17].

2. Experimental set-up

The experimental set-up of the Mach-Zehnder interferometer is shown in Fig. 1. As a coherent light source for the microscope we have used an Argon laser that emits at $\lambda = 488$ nm. For practical reasons the beam is coupled into a mono-mode fiber. The object and reference beam are split at an integrated beam splitter by a ratio of 90/10 and the polarization of both beams can be adjusted at the exit of the fiber for optimizing the contrast of the resulting interference fringes. The sample is placed on a piezo-stage that permits the local positioning of the CGH in all three spatial directions.

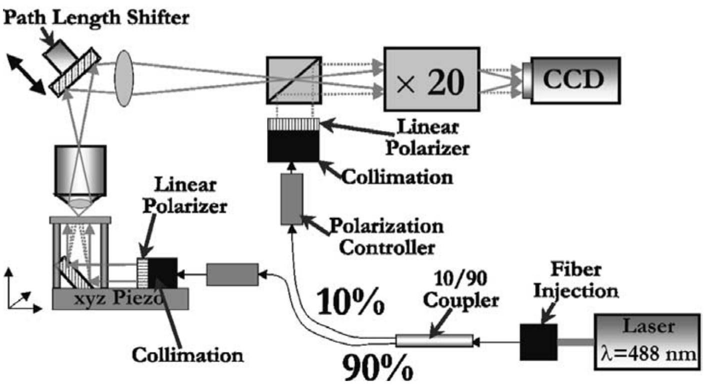

Fig. 1. Experimental set-up of the Mach-Zehnder interferometer.

Collimating optics are placed at the exit of the fibers in the object and in the reference arm. In the object arm, the beam is focused on the sample such that ideally the waist of the Gaussian beam is in the object plane. The optics in the reference arm can be used to minimize the difference in the curvature between the reference and object wave. The microscope consists of two stages: the first is a telescopic system with a 10× objective and a 250 mm lens. At an intermediate plane an aperture is positioned that allows selection of a single diffraction order for analysis. We have chosen such a weak objective in order to avoid interference between neighboring diffraction orders that would appear if the first magnification were stronger. After superposition with the reference wave, a similar optical system with a 20× objective magnifies the intermediate image in a second step. The measured interference images in the waist of the magnified laser beam are recorded with an 8-bit CCD camera and the phase distributions are calculated using a five-step phase algorithm [18]. The two combined telescopic systems allow a magnification such that a CCD-pixel corresponds to 100 nm in the object plane.

We have investigated two different CGHs that have been designed to produce as off-axis holograms Gauss–Laguerre beams with the orders ($p = 0$, $m = 1$) and ($p = 0$, $m = 2$), respectively [19], p being the radial mode parameter and m the azimuthal mode parameter. The second parameter m corresponds to the strength of the singularity [20].

The patterns of the binary computer-generated hologram have been deduced from the calculated lines of constructive interference between the laser modes to be generated and the plane reference wave propagating under a design angle relative to the laser mode of $\tan\theta = \lambda/\Lambda$ [3]. The period of the resulting grating is Λ and the first diffraction order will propagate at an angle of θ with respect to the optical axis. The holograms were printed using a photocomposer and subsequently reduced in a photolithographic process by a factor of ten and then imaged on a substrate covered with photoresist. After developing the samples, the CGHs can be used as phase-only objects in transmission. The period of the samples used for the measurement was 10 μm and the depth was approximately 600 nm, as deduced by measurements with an atomic force microscope (AFM).

3. Results

3.1. Gauss–Laguerre mode 01

Fig. 2(a) shows the measured intensity distribution in the +1 diffraction order of the hologram

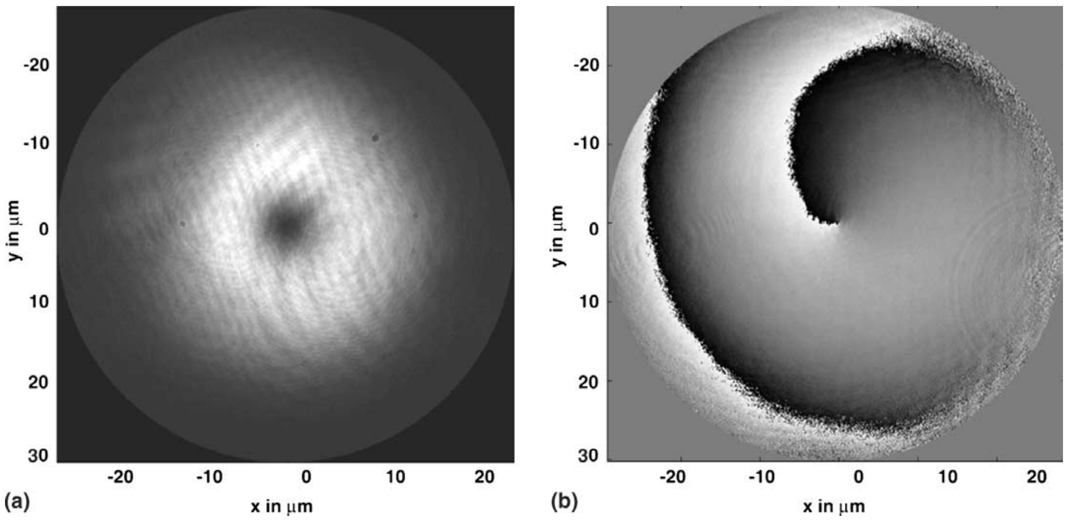

Fig. 2. Measured (a) intensity and (b) phase distribution in the +1 diffraction order of the GL01-CGH.

that generates a Gauss–Laguerre 01 beam. The ring-shaped intensity distribution of the doughnut laser mode can be seen, with an intensity zero in the center of the beam. The measured corresponding phase distribution is shown in Fig. 2(b). In all the figures, which show a phase distribution, black corresponds to a phase value of $-\pi$ and white to π. A difference in the radius of curvature between the wave in the object and in the reference arm will cause the slightly spiral-like distortion of the phase distribution. Without such a misalignment, the isophases would be straight radial lines, originating from the center of the beam, where the dislocation is situated. The strength of the dislocation for the generated laser mode is -1. We use a positive sign for the dislocations if the singularity produces a right screw, and a negative sign for the opposite case. However, due to conservation of the sum of charges in a wave-field, in the negative diffraction order a similar beam is generated that has for a perfect binary grating the same intensity distribution but the strength of the singularity in the beam center has the opposite sign. This will be shown in the following measurements. The position of the origin for the dislocation can be determined with a precision of a few pixels of the CCD-camera (<3 pixel) and is limited by the optical noise in the measurements and the number of discretization levels of the CCD-camera. In the case where we want to record the entire intensity distribution of the beam, saturation of the CCD-camera is avoided by the adjustment of the gain factor. This limits the resolution, because all intensity values below a half discretization step are clipped to zero intensity. The size of the corresponding region can be regarded as the resolution limit for the position of the singularity. The advantage of measuring the phase distribution, instead of only the intensity distribution, is the elimination of incoherent background. This contribution smears out the otherwise well-defined minimum intensity and an exact determination of the position of the singularity is not possible. If one limits the observation to the singularity and its neighborhood, the gain of the CCD-camera and the power of the laser illumination can be increased, such that the ultimate limit for the resolution is given by a single pixel of the CCD-camera.

3.2. *Gauss–Laguerre mode 02*

Fig. 3(a) shows the measured intensity distribution in the -2 order of the CGH, and Fig. 3(b) shows the corresponding measured phase distribution. The intensity distribution has again a doughnut-like shape. By comparing the intensity

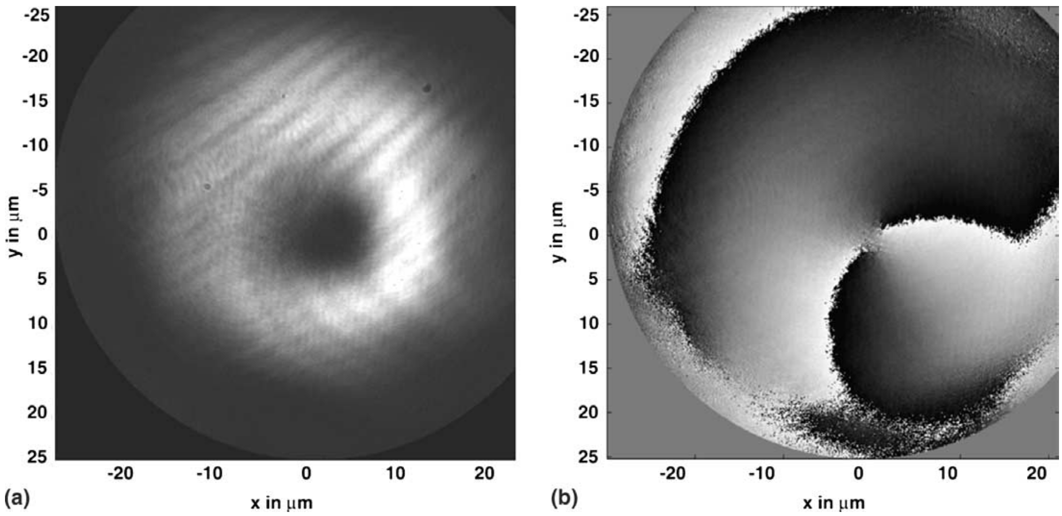

Fig. 3. Measured (a) intensity and (b) phase distribution in the -2 diffraction order of the GL01-CGH.

distribution in the second order with the intensity from the first order (Fig. 2(a)), it can be seen that the dark core is somewhat broader and the bright ring is thinner, which corresponds to the intensity signature of a Gauss–Laguerre mode of a higher azimuthal mode parameter. In the phase distribution, two easily distinguishable phase singularities appear with the same strength, which proves the excitation of a GL02 mode. The strength for each of the two dislocations is +1 and they are separated by approximately 2.5 μm.

The appearance of such a beam having two dislocations can be explained as follows. The phase difference between two subsequent diffraction orders is exactly equal to one wavelength. This means that the screw-like phase distribution produced by the CGH has an additional 2π phase shift. This will lead to a beam in the second order which carries a singularity that has twice the strength of the singularity generated in the first diffraction order. The generated beam can be regarded principally as a superposition of propagation invariant modes. Assuming Gauss–Laguerre modes as the basis for the mode expansion, the mode which corresponds to the induced phase distribution in the present case is a GL02 mode(the second diffraction order of a hologram designed to produce a GL01 mode in the first order). Correspondingly it will be the laser mode with the strongest fractional power. But the mode conversion is incomplete, for two reasons. The first is the non-optimized height of the structure, which will not induce a perfect screw into the phase distribution. Additional modes are excited and interfere with the dominant mode. The second reason for a non-perfect generation of the GL02 mode is the mismatch between the amplitude distribution of the illuminating mode and the GL02 mode. This will lead additionally to the excitation of other modes. Both effects will cause the GL02 to be perturbed and the dislocation, which originally has a charge of +2, will be split into the observed two dislocations. Both singularities have a charge of +1.

By plotting the logarithm of the intensity and the iso-phases on the same figure, as shown in Fig. 4, it can be seen that the positions of the singularity and the minimum of the beam can be located at the same position. The thick line corresponds to the change of 2π in the phase. It must been mentioned that the region with the intensity minimum is smeared out and the absolute minimum cannot be determined. In the phase distribution, however, the position of the two singularities can be distinguished clearly, even for a spacing as small as 2.5 μm.

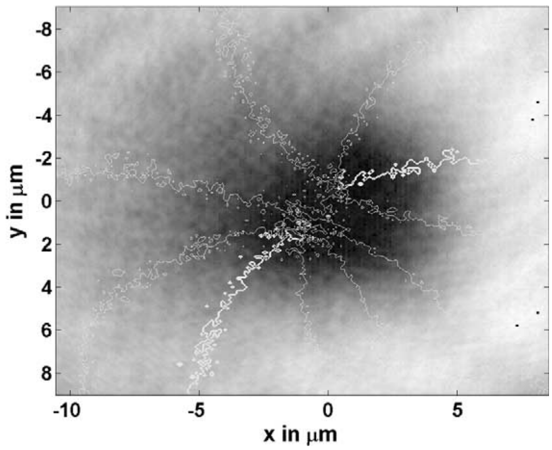

Fig. 4. Logarithm of the intensity and iso-phases for the measured field distribution in the −2 diffraction order of the GL01-CGH.

3.3. Gauss–Laguerre mode 03

The intensity and phase distribution in the −3 diffraction order is shown in Fig. 5. The generated beam contains three singularities with a charge of +1 each. The separation of the dislocations is increased compared with the measurements in the second order and is between 6 and 8 μm. The beam is dominated by the contribution from a GL03 mode, but the distribution is perturbed by significant contributions from other modes. This can be also seen in the intensity, where the ring-shaped distribution is broken and the dark core of the beam shows three well-defined local minima. The points with the lowest intensity coincide perfectly with the position of the dislocations, determined more precisely from the phase distribution. The distribution can be regarded as a superposition of a GL03 mode with a GL00 mode as explained before. The coherent superposition of the laser

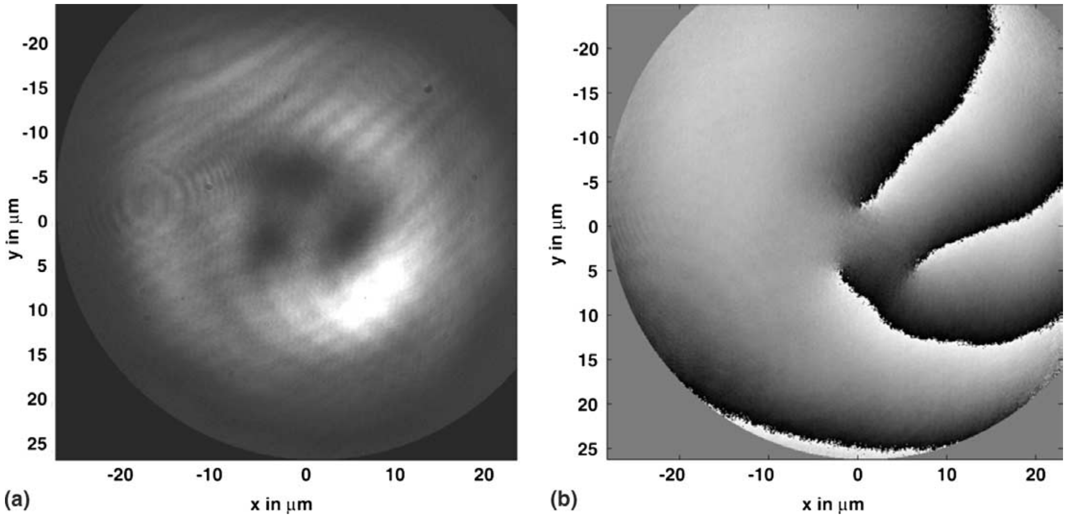

Fig. 5. Measured (a) intensity and (b) phase distribution in the −3 diffraction order of the GL01-CGH.

beams will cause a deviation from the radially symmetric intensity distribution due to constructive and destructive interference.

3.4. Gauss–Laguerre mode 04

Modes with a still higher number of dislocations have been generated by using a CGH that was designed to transform a Gaussian beam into a Gauss–Laguerre 02 mode for the first diffraction order. Measuring the field-distribution in the second diffraction order, a dominant GL04 mode is seen. The mechanism for the excitation is comparable to the described mode excitation in the second order of the previous hologram (GL01-CGH). The induced phase delay for the second order is twice that of the first order. Therefore, the screw-like phase delay transferred to this beam is $2 \times 4\pi$. Correspondingly, such a mode carries four dislocations. In Fig. 6, the measured intensity

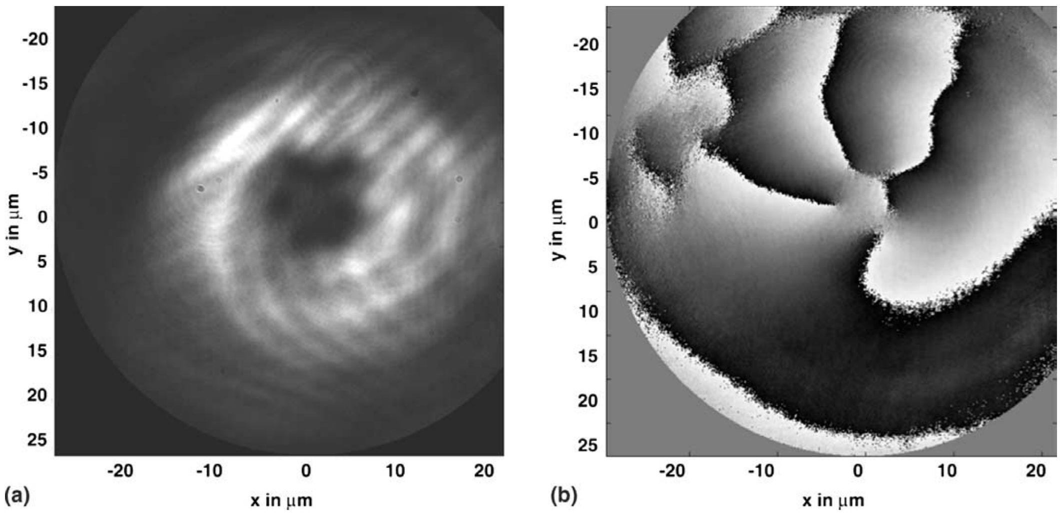

Fig. 6. Measured (a) intensity and (b) phase distribution in the +2 diffraction order of the GL02-CGH.

and phase distribution of such a beam are shown. The low contrast fringes in the intensity distribution originate from interference of the primary beam with scattered light of other diffraction orders. The origin of the scattered light is insufficient filtering of the diffraction order by the aperture. In the experiments, the higher diffraction orders are not clearly distinguishable and rather elongated. If the aperture is adjusted to a diameter that is too small, this unwanted scattered light is blocked but a significant portion of the laser beam is also lost.

In the phase distribution, the four singularities can be distinguished clearly and correspond to local intensity minima in Fig. 6(a).

4. Conclusions

In this paper, we have presented direct measurements of the intensity and phase distribution in the different diffraction orders of computer-generated holograms. They have been intentionally designed to work off-axis as mode transformers in first diffraction. It has been shown that in the higher diffraction orders, beams can be observed with an integer multiple of the singularity of the first diffraction order beam. The sign of the strength in the negative diffraction orders is the opposite of the strength from the singularities in the positive diffraction orders. The phase singularities of higher orders are not observed clearly. Due to excitation of other beam modes in the system, the singularity is split into a number of closely spaced singularities, each with a strength of unity. Using a high-resolution interference microscope, they can be well discriminated.

These findings helps to understand the limitations for practical applications of higher-order singularities. Such a higher order Gauss–Laguerre beam is for example applied as an optical tweezer for the trapping of particles with an index lower than the refractive index of the surrounding. These particles can be stably trapped in the dark core of the laser beam. For higher order modes the trapping efficiency is better because the gradient of the absolute value of the electric field is higher. However, if the singularities do not coincide, different points of stable trapping might exist in the beam. This will cause an instability in the particle position, which is undesired in high-precision applications.

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