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Efficiency limits for single-junction and tandem solar cells

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Abstract

Basic limitations of single-junction and tandem p–n and p–i–n diodes are established from thermodynamical considerations on radiative recombination and semi-empirical considerations on the classical diode equations. These limits are compared to actual values of short-circuit current, open-circuit voltage, fill factor and efficiency for amorphous (a-Si:H) and microcrystalline (μc-Si:H) silicon solar cells. For single-junction cells, major efficiency gains should be achievable by increasing the short-circuit current density by better light trapping. The limitations of p–i–n junctions are estimated from recombination effects in the intrinsic layer. The efficiency of double-junction cells is presented as a function of the energy gap of top and bottom cells, confirming the 'micromorph' tandem (a-Si:H/μc-Si:H) as an optimum combination of tandem solar cells.

Keywords: Amorphous and microcrystalline silicon; Single p–n and p–i–n junction solar cells; Tandem solar cells; Efficiency limits

1. Introduction

Thin-film silicon solar cells are regularly presented as a one of the main future options for cost-effective solar cells [1–3]. But, parallel to a price reduction, it is imperative to improve the efficiency of current solar cells. It is therefore useful to look in detail at the existing efficiency limitations, introducing estimated values for the dark current in the

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classical diode model for the solar cell and comparing them with values predicted by Ref. [4], as well as ultimate values predicted by Ref. [5].

The aim of this paper is, thus, to re-examine the fundamental limits for short-circuit current density (J_{sc}), open-circuit voltage (V_{oc}), fill factor (FF) and efficiency (η) as a function of the energy gap (E_g) for single-junction and tandem solar cells, in the context of thin-film silicon solar cells.

Additional losses in V_{oc} and FF due to the p–i–n configuration generally used in thin-film silicon solar cells will be estimated considering two different effects: (a) the diminution of the photogenerated current in p–i–n solar cells due to additional recombination in the intrinsic layer and (b) the increased dark (reverse) current of thin-film silicon p–i–n diodes, as can be attributed to thermal generation current from mid-gap defect states (dangling bonds).

The efficiency limitations for tandem (double-junction) solar cells will then be considered; it will be shown that tandem ‘micromorph’ (a-Si:H/μc-Si:H) solar cells correspond to an optimum combination of band gap values.

2. Single-junction solar cells

2.1. *Theoretical limit for short-circuit current, J_{sc}*

An upper limit for the J_{sc} can be computed by considering the normalized AM 1. 5 spectrum (IEC 904-3) and assuming that all photons with $h\nu > E_g$ (where h is Planck’s constant, $\nu = c/\lambda$ with c the speed of light and λ the wavelength, and E_g is the energy gap of the semiconductor material considered) are absorbed and converted into electron–hole pairs that can, in principle, be collected at short circuit conditions.

This results in J_{sc} (E_g) as presented in Fig. 1: microcrystalline (μc-Si:H) solar cells have an E_g of around 1.1 eV which corresponds to the limit $J_{sc} < 43.6\,\text{mA/cm}^2$, whereas amorphous (a-Si:H) solar cells have an E_g of 1.75 eV, corresponding to $J_{sc} < 21.1\,\text{mA/cm}^2$. At the moment, values of $J_{sc} \approx 20$–$30\,\text{mA/cm}^2$ for μc-Si:H solar cells and values of $J_{sc} \approx 12$–$17\,\text{mA/cm}^2$ for a-Si:H solar cells are obtained by most laboratories. A substantial gain in short-circuit current can therefore still be obtained, mainly by improving light trapping techniques [3].

2.2. *Limits for single-junction cells: V_{oc}, FF and η*

2.2.1. *Open-circuit voltage, V_{oc}*

According to the standard diode equation, the $J(V)$ characteristic of a single-junction solar cell under illumination can be written as the linear superposition of the dark characteristics of the cell and the photogenerated current

$$J = J_L - J_0\left[\exp\left(\frac{qV}{nkT}\right) - 1\right], \tag{1}$$

where J_L is the photogenerated current, J_0 is the reverse saturation current, q is the elementary charge, k the Boltzmann constant, T the absolute temperature and n the ideality factor with $n = 1$ standing for ideal p–n junctions, whereas $n \approx 2$ stands for p–i–n junctions.

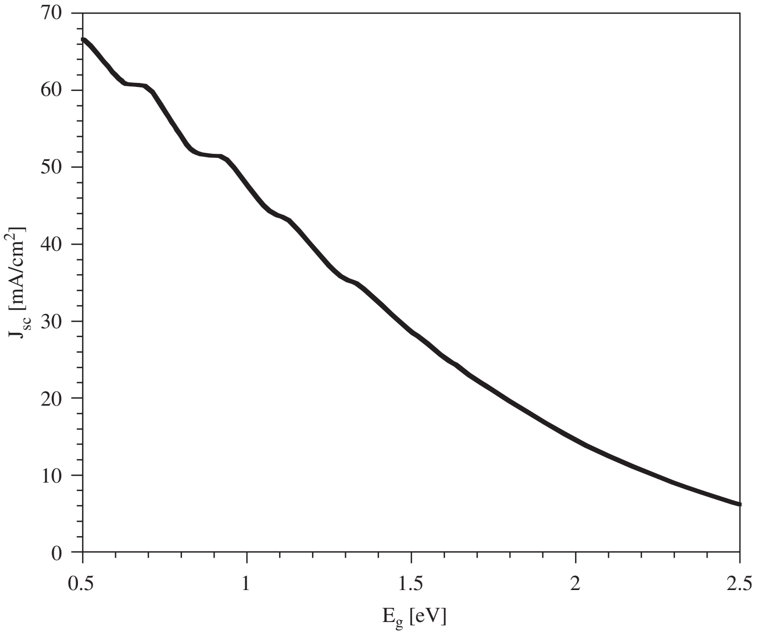

Fig. 1. Upper limit of the J_{sc} as a function of E_g of the solar cell material, under AM1.5 illumination.

The V_{oc} can be deduced from Eq. (1) with the condition $J = 0$

$$qV_{oc} = nkT \ln\left(\frac{J_L}{J_0} + 1\right). \tag{2}$$

The lower limit of the reverse saturation current J_0 is a function of the gap energy E_g

$$J_0 = J_{00} \exp(-E_g/nkT), \tag{3}$$

and Eq. (2) can therefore be rewritten as

$$qV_{oc} = E_g + nkT \ln(J_L/J_{00}) \quad \text{with} \quad \left(\frac{J_L}{J_{00}} + 1 \approx \frac{J_L}{J_{00}}\right). \tag{4}$$

Green [4] gives a lower limit for J_0 in a p–n junction, that corresponds to:

$$J_{00} = 1.5 \times 10^8\ [\text{mA/cm}^2], \tag{5}$$

whereas, based on thermodynamical considerations, Kiess and Rehwald [5] give the following limit for V_{oc}:

$$qV_{oc} = E_g + kT \ln\left[\left(\frac{h^3 c^2}{2\pi kT}\right)\left(\frac{\dot{N}_{\text{incident}}}{E_g^2}\right)\right] \quad \text{with} \quad \dot{N}_{\text{incident}} = J_{sc}/q. \tag{6}$$

For $T = 25\,^\circ\text{C}$ ($kT = 25\,\text{meV}$), Eq. (6) corresponds to setting

$$J_{00} = 4 \times 10^5 E_g^2\ [\text{mA/cm}^2] \quad \text{with} \quad E_g \quad \text{in} \quad [\text{eV}]. \tag{7}$$

Additional losses in V_{oc} (and in FF) due to the p–i–n configuration can be assumed to be the consequence of two separate effects: (a) additional recombination in the intrinsic layer and (b) the generation/recombination contribution to the dark current due to mid-gap defects (dangling bonds).

The recombination in the intrinsic layer of a p–i–n diode can be considered at a first approximation as a recombination source added to the electrical equivalent circuit for photovoltaic solar cells [6]. The linear superposition of the dark current and the photogenerated current (see Eq. (1) for ideal p–n junction) is, thus, not valid anymore: the illuminated $I(V)$ characteristic for p–i–n solar cells should now be written as

$$J = J_{\mathrm{L}} - J_{\mathrm{rec}} - J_0\left[\exp\left(\frac{qV}{nkT}\right) - 1\right], \tag{8}$$

with J_{rec} a function of J_{L}. Based on Ref. [7] and on experimental results, we may assume, as coarse approximation that $J_{\mathrm{rec}} \approx 0.8 J_{\mathrm{L}}$ for $V \approx V_{\mathrm{oc}}$.

A lower limit for the reverse saturation current in p–i–n diodes can be estimated considering that the dark current is thermally generated from mid-gap defects. This current may then basically be written as

$$J_{00} = (1/\tau_{\mathrm{gen}}) d_{\mathrm{i}} q N_{\mathrm{db}}, \tag{9}$$

with τ_{gen} the time constant of thermal generation in the intrinsic layer, d_{i} the intrinsic layer thickness and N_{db} the dangling bond density.

Note that this is an additional contribution to the reverse saturation current, in supplement to the current from Eq. (5) or (7). As a coarse approximation, only this additional contribution was taken into account for calculation of p–i–n diode limits presented in Figs. 2 and 3, as well as Fig. 5 hereafter.

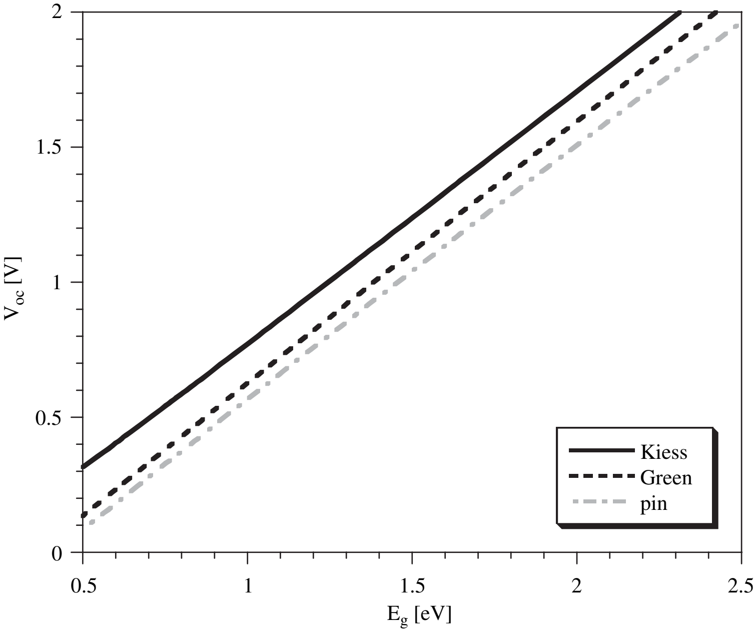

Fig. 2. Upper limits of the V_{oc} as a function of the gap energy of the solar cell material: limit based on Kiess model, limit based on Green's model and coarse approximation for p–i–n diode junction limit.

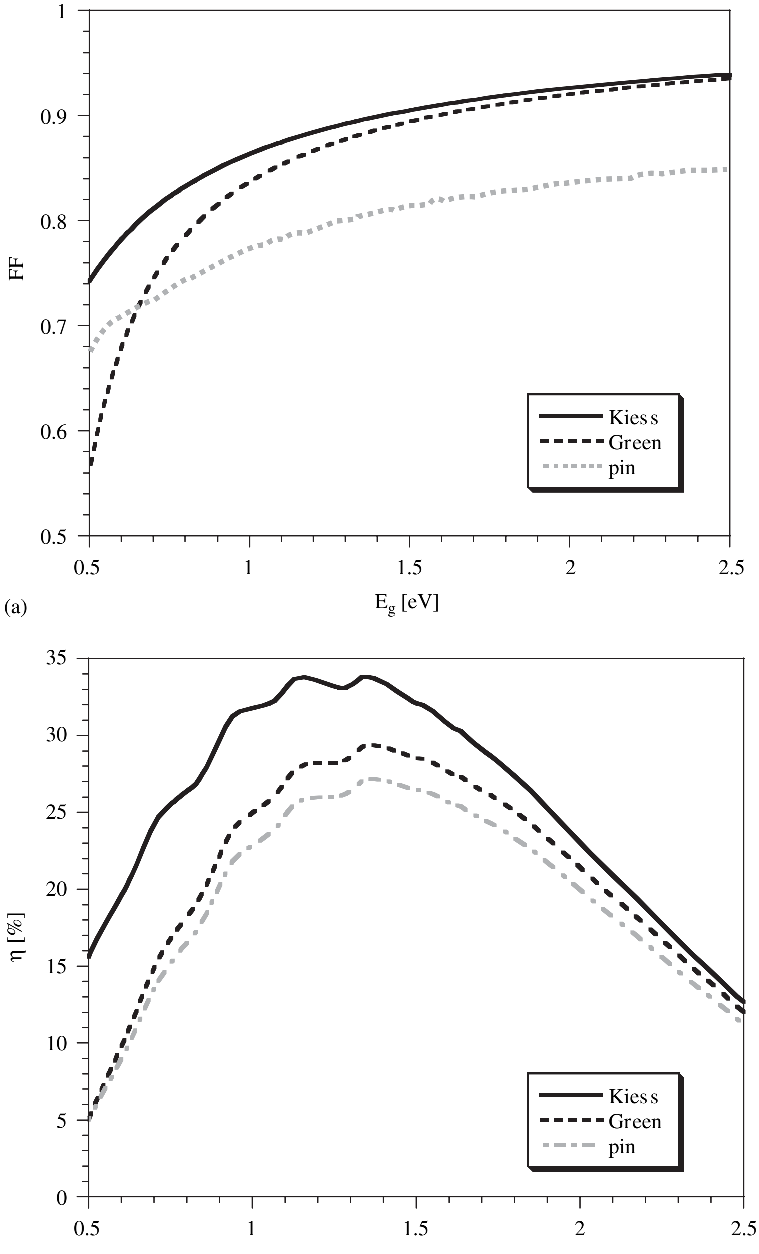

Fig. 3. Upper limits for solar cells, as a function of the gap energy E_g: thermodynamical limit [5], semi-empirical limit [4] and p–i–n diode junction limit; (a) FF and (b) η.

According to the model proposed by Street [8], and dark current measurements by our own group [9] and Roca et al. [10], which have yielded reverse saturation currents in the order of $I_{th} \approx 10^{-12}\,A/cm^2$, we may assume that J_{00} is in the order of

$$J_{00,\mathrm{pin}} = 5 \times 10^4\ [\mathrm{mA/cm^2}], \tag{10}$$

for state–of-the-art a-Si:H p–i–n diodes.

This value can be taken as a first approximation also for μc-Si:H diodes, as indeed defect densities are similar here [11].

The limits for V_{oc} (E_g) are presented in Fig. 2 for p–n and p–i–n diodes.

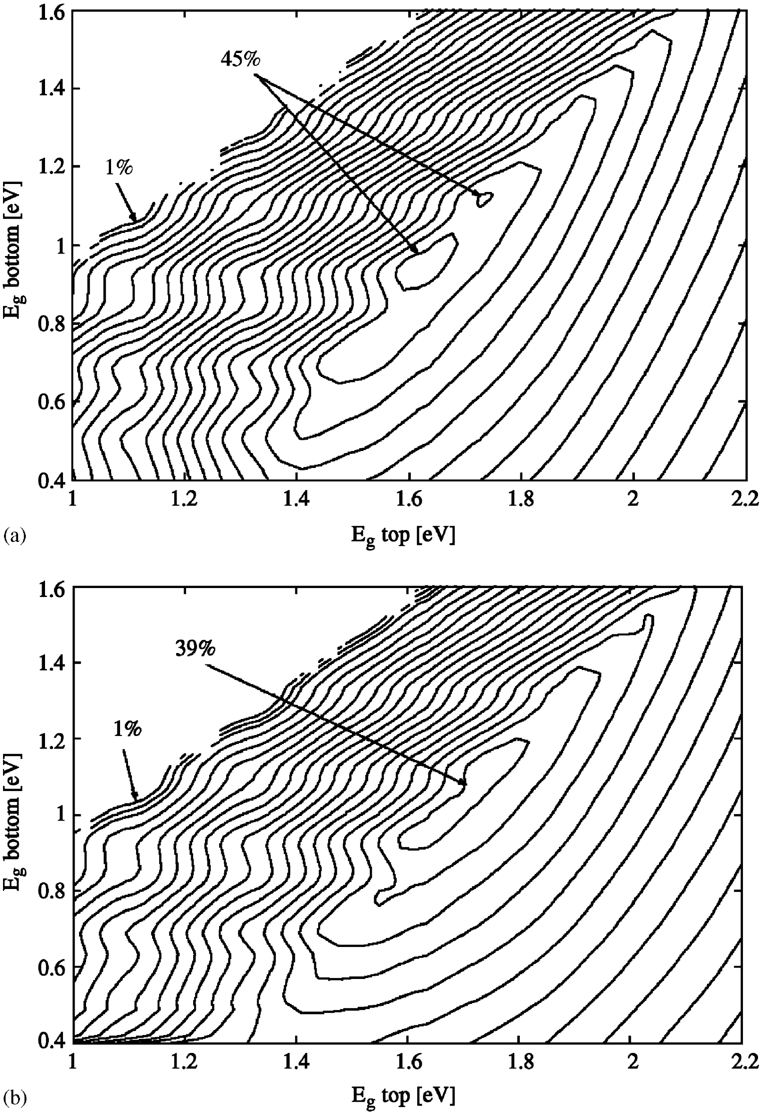

Fig. 4. Upper limits of the efficiency of p–n/p–n tandem solar cell as a function of the energy gaps $E_{g,\mathrm{top}}$ and $E_{g,\mathrm{bottom}}$, based on (a) Kiess and Rehwald thermodynamical upper limit and (b) on Green [4] upper limit.

The p–i–n limit is almost achieved by present μc-Si:H solar cells that present an $V_{oc} \approx 0.55$–$0.6\,V$.

2.2.2. *Fill factor, FF and efficiency, η*

The FF can be expressed [4] by

$$FF = (J_M V_M)/(J_{sc} V_{oc}), \tag{11}$$

where J_M and V_M are the current density and the voltage at the maximum-power point. V_M and J_M are obtained by maximising the (JV) product by differentiation, considering the diode equation with $n = 1$ (p–n) or $n = 2$ (p–i–n). Furthermore, the recombination current of Eq. (8) is taken into account for the calculation of FF in the p–i–n case. The curves FF (E_g) are presented in Fig. 3(a).

Finally, the limit efficiency of the solar cell can be computed by multiplying $J_{sc}V_{oc}$FF and by dividing by incident light energy. This yields the curves $\eta(E_g)$ of Fig. 3(b).

3. Tandem solar cells

Tandem solar cells are particularly interesting for the thin-film silicon technology, as they allow one to overcome practical efficiency limitations as well as to reduce degradation effects [12]. Based on the above, efficiency limits for tandem solar cells were also computed considering: (a) a perfect balance between top and bottom cell current densities and (b) the idealized case, where the bottom cell absorbs all the light transmitted by the top cell. Limits for tandem solar cells are shown in Figs. 4 and 5 as a function of the E_g of the bottom and

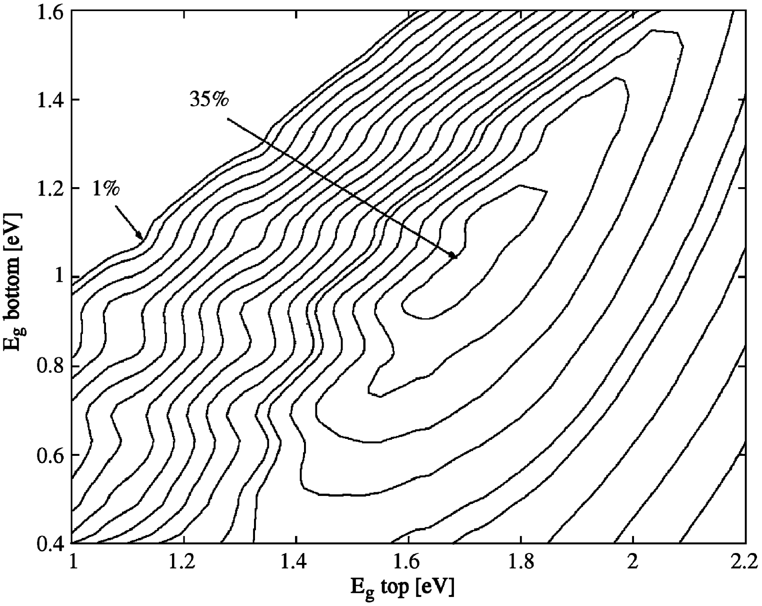

Fig. 5. Upper limit of the efficiency of p–i–n/p–i–n tandem solar cell as a function of the energy gaps $E_{g,top}$ and $E_{g,bottom}$.

top cells. Fig. 5 shows clearly that the ‘micromorph’ tandem (a-Si:H/μc-Si:H) [12] possesses an optimal band gaps combination for tandem solar cells.

4. Conclusions

Fundamental limits for p–n and p–i–n single and double-junction solar cells were established from well-known theoretical and semi-empirical considerations. These limits are, especially for the J_{sc} very much higher than present values, and indicate scope and the direction of further improvements that may still be obtained, e.g., by light trapping and materials research.

Losses in V_{oc} (and in FF) due to the p–i–n configuration have here been assumed to be the consequence of both an additional recombination in the intrinsic layer, and a generation/recombination contribution to the dark current due to mid-gap defects (dangling bonds). This part clearly needs to be further refined, by future experimental and theoretical works.

The ‘micromorph’ tandem (a-Si:H/μc-Si:H) is confirmed to correspond to the optimum combination for double-junction solar cells with an efficiency limit of 35%.

Acknowledgments

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