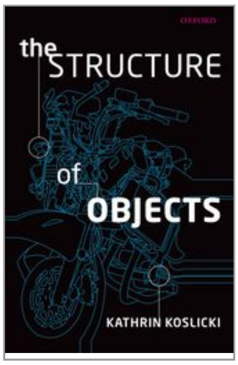

The Structure of Objects
Kathrin Koslicki

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The Standard Conception of Composition

Kathrin Koslicki (Contributor Webpage)

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Abstract and Keywords

This chapter is devoted to an exposition of the main concepts and principles of standard mereology, the system originally developed by Stanislaw Leśniewski and introduced into the English-speaking world in the guise of Henry Leonard and Nelson Goodman's ‘Calculus of Individuals’. The main source for this chapter is Peter Simons', *Parts: A Study in Ontology*, in particular his instructive gradual development of standard mereology, which shows how stronger and stronger principles may be added gradually to a minimal core, until we arrive at the full-strength theory of standard mereology. Despite standard mereology's merits as a formal theory, however, it remains to be seen whether it is of any use to the metaphysician in characterizing ordinary mereological concepts, as they apply to our scientifically informed, common-sense ontology.

Keywords: standard mereology, common-sense ontology, Henry Leonard, Nelson Goodman, Stanislaw Leśniewski, mereological concepts

§I.1 Introductory Remarks

The objects we encounter in ordinary life and scientific practice—cars, trees, people, houses, molecules, galaxies, and the like—have long been a fruitful source of perplexity for metaphysicians. The purpose of this book is to give an analysis of those material objects to which we take ourselves to be committed in our ordinary, scientifically informed discourse. My focus will be on *material* objects in particular or, as metaphysicians like to call them, “*concrete particulars*”,[1] i.e., objects which occupy a single region of space-time at each time at which they exist and which have a certain range of properties that go along with space-occupancy, such as weight, shape, color, texture and temperature.

In giving an analysis of ordinary material objects, I want to focus in particular on the question of how the *parts* of such objects, assuming that they have parts, are related to the *wholes* which they compose. That most, or possibly all, ordinary material objects have parts I take to be an obvious intuitive datum.[2] We would commonly say, for example, that among the parts of a tree are its branches, its trunk, its leaves and its roots; among the parts of a table are its legs and its top; among the parts of an H_2O molecule are its two hydrogen atoms and its single oxygen atom. Let's call objects which have parts *mereologically complex*, **(p.10)** *compound* or *composite* objects, or *wholes*. Then, as I understand it, to ask the question, “What *are* ordinary material objects?”, is at least in part to ask, “How are these wholes related to their parts?”, or “What is the nature of the relation of *composition* for material objects?”.

§I.2 Standard Mereology

One prominent answer to these questions which has been embraced by three-dimensionalists and four-dimensionalists alike is that ordinary material objects are *mereological sums*, *fusions* or *aggregates*, according to a particular, standard conception of mereology.[3] The standard conception of mereology I have in mind is the family of systems which Simons (1987) calls “Classical Extensional Mereology” (CEM), and I shall follow him in this usage. The first formulation of CEM appears to have been given by Stanislaw Leśniewski, informally in Leśniewski (1916) and formally in Leśniewski (1927–30), though Simons speculates, based on some remarks by Russell in 1914, that Whitehead's mereology may actually have been developed not only independently of Leśniewski's but may also have preceded it (cf. Russell 1914, Simons 1987, p. 82). Leśniewski's system is not widely known to contemporary writers, due to the fact that it is based on his formal system, “Ontology”, which is generally found to be relatively inaccessible (but see Simons 1987, ch. 2, for a very clear and detailed exposition of Leśniewski's systems “Ontology” and “Mereology”). The classical statement of CEM in English, using the language of first-order predicate-logic, is Henry Leonard and Nelson Goodman's “Calculus of Individuals” (Leonard and Goodman 1940), of which the first version appeared in 1930 in Leonard's doctoral dissertation. Leonard and Goodman's (1940) Calculus of Individuals is formulated with appeal to set theory, as is Tarski's version of CEM in Tarski (1937) and (1956); but a nominalistic formulation of the same theory, in which reference to sets is replaced by reference to predicates, is given in Goodman (1977).[4, 5]

(p.11) §I.2.1 The Basic Concepts of Standard Mereology

The basic concepts of standard mereology are as follows:[6]

Proper Part:	$x < y$	"x is a *proper part* of y"
Proper or Improper Part:	$x \leq y$	"x is a *proper or improper part* of y"
Overlap:	$x \circ y$	"x *overlaps* y"
Disjointness:	$x \wr y$	"x is *disjoint* from y"
Binary Product:	$x \cdot y$	"the *product* of x and y"
Binary Sum:	$x + y$	"the *sum* of x and y"
Difference:	$x - y$	"the *difference* of x and y"
General Product (Nucleus):	$\pi x\, [F(x)]$	"the *product* of all the x's which are F"
General Sum:	$\sigma x\, [F(x)]$	"the *sum* of all the x's which are F"
The Universe:	U	"the *Universe*"
Complement:	$U - x$	"the *complement* of x"
Atom:	$At(x)$	"x is an *atom*"

The relation of *proper part*, <, does not require much illustration, since it is firmly embedded in our ordinary way of conceptualizing the world; it holds, for example, between a man and his forearm. The most obvious formal properties of < are *transitivity*, *asymmetry* and hence *irreflexivity*:

Transitivity of Proper Parthood:	$(x < y \;\&\; y < z) \rightarrow (x < z)$
Asymmetry of Proper Parthood:	$(x < y) \rightarrow \sim(y < x)$
Irreflexivity of Proper Parthood:	$\sim(x < x)$

In other words, if one object is a proper part of another and the second is a proper part of a third, then the first is a proper part of the third as well; if one **(p.12)** object is a proper part of another, then the second is not also a proper part of the first; and, finally, nothing is a proper part of itself. Thus, the relation of proper parthood is a *strict partial ordering*. However, as Simons points out, not every strict partial ordering can be described as a relation of parthood; thus, the question arises of what further formal properties distinguish proper parthood from other strict partial orderings. We shall turn to this question shortly. Though there are some writers who entertain the possibility of a relation of parthood which does not even satisfy these minimal properties,[7] I side with Simons in taking transitivity, asymmetry and irreflexivity to be partially constitutive of the relation of proper parthood; thus, any relation which fails to satisfy these rather weak formal requirements by its very nature ought not to be counted as a genuine notion of proper parthood. To illustrate, if a particular cell is a proper part of a particular body and a particular nucleus is a proper part of the cell, then I take it to be obvious that the nucleus is also a proper part of the body, even if there are plenty of *other*, more loaded, relations in the vicinity which cannot be so easily extended to hold both between the nucleus and the cell and between the cell and the body.

If identity is taken as given, then the relation, $\leq$, of *proper or improper parthood* can be understood in terms of identity and proper parthood: “$x \leq y$” holds just in case x is either a proper part of y or x is identical to y. Like the relation “is less than or equal to”, to which it is formally analogous, $\leq$ is *transitive*, *non-symmetrical* and *reflexive*:

Transitivity of Proper or Improper Parthood:	$(x \leq y \ \& \ y \leq z) \rightarrow x \leq z$
Non-Symmetry of Proper or	$(\exists x)(\exists y)\ (x \leq y \ \& \ y \leq x)$ &
Improper Parthood:	$(\exists x)(\exists y)\ (x \leq y \ \& \ {\sim} y \leq x)$
Reflexivity of Proper or Improper Parthood:	$x \leq x$

In other words, if an object is a (proper or improper) part of another, and the second is a (proper or improper) part of a third, then the first is also a (proper or improper) part of the third; if an object is a (proper or improper) part of another, then in some cases the second is also a (proper or improper) part of the first and in other cases the second is not also a (proper or improper) part of the first; and, finally, any object is a (proper or improper) part of itself.

Two objects *overlap* just in case they have a (proper or improper) part in common; thus, “x°y” holds in any of the following scenarios: (i) x and y share a proper part; (ii) x and y are identical; (iii) x is a proper part of y; or (iv) y **(p.13)** is a proper part of x. The notion of overlap is *reflexive* and *symmetric*, but not *transitive* (since, for example, in scenario (i), objects x and y may share a proper part, as do y and z, without z sharing a proper part with x):

Reflexivity of Overlap:	$x^\circ x$
Symmetry of Overlap:	$(x^\circ y) \rightarrow (y^\circ x)$
Intransivity of Overlap:	${\sim}[(x^\circ y \ \& \ y^\circ z) \rightarrow (x^\circ z)]$

In other words, every object overlaps itself; if an object overlaps another, then the second overlaps the first; and, finally, it does not in general follow that if one object overlaps a second, and the second overlaps a third, that the first object also overlaps the third. Although the notion of overlap (along with that of disjointness and possibly even that of parthood itself) is easily taken to have *spatial* overtones, it is important to keep in mind that this is merely an artifact of the natural language expression that is used to render this formal relation in ordinary English; according to the original theory of CEM, all of the basic mereological vocabulary is intended to be understood in an entirely *neutral* fashion, to allow for application across a wide range of cases.

Two objects are *disjoint* just in case they do not overlap, or share no (proper or improper) part in common. Disjointness is *symmetric*, but neither reflexive nor transitive:

Symmetry of Disjointness:	(xʅy) → (yʅx)

Irreflexivity of Disjointness:	~(xⲅx)
Intransitivity of Disjointness:	~[(xⲅy & yⲅz) → (xⲅz)]

In other words, if an object is disjoint from another, then the second is also disjoint from the first; nothing is disjoint from itself; and if an object is disjoint from another, and the second disjoint from a third, it does not in general follow that the first is also disjoint from the third.

As can be seen from the occurrence of the definite article in the paraphrases above, the remaining items in the list—*product*, *sum*, *difference*, *universe* and *complement*—are all used to form *singular terms* (with the exception of “At”, which plays the role of a *predicate*). The singular term “x·y”, which denotes the (binary) *product* of x and y, denotes that object which is part of both x and y, and which is such that any common part of both x and y is a part of it. Such an object will only exist, of course, if x and y have a common part; if they lack a common part, then “x·y” is a non-referring singular term and can be dealt with in whatever manner is chosen to apply to other non-referring singular terms. The notion of product is the mereological analogue of set-theoretic intersection, with the exception that two disjoint sets always have an intersection, viz., the null-set, whereas most mereologies want no truck with such a thing as the “null-object” (which would be defined as that object which is part of everything). The notion **(p.14)** of binary product can be generalized to apply to the infinite case by means of the variable-binding operator, π, so that “πx [F(x)]” denotes the product or nucleus (if there is one) of all the objects satisfying the predicate in question. The singular term “x + y”, which denotes the (binary) *sum* of x and y, denotes that object which is such that something overlaps it just in case it overlaps at least one of x and y. The notion of sum is the mereological analogue of set-theoretic union. Here, no proviso for non-referring singular terms or null-objects is needed, since it is a central thesis of CEM, and possibly its most notorious claim, that any two objects, no matter how disparate and dissimilar, have a sum. Again, the relation of binary sum can be generalized to the infinite case by means of the variable-binding operator, σ, so that “σx [F(x)]” denotes the object which is the sum of all the objects satisfying the predicate in question.

The singular term “x − y”, which denotes the *difference* of x and y, denotes the largest object contained within x which has no part in common with y. This difference exists only if x is not a part of y; if x and y overlap and x is not a part of y, then x − y is a proper part of x.

If arbitrary sums exist (that is, if any collection of objects has a sum), then there exists an object which is the sum of all objects whatsoever; this object, of which all other objects are part, is the *Universe*. Since CEM endorses not only the *existence* of arbitrary sums, but also their *uniqueness*, it also follows that there is only *one* such object, *the* Universe. The Universe functions algebraically as the Boolean unit element. In a non-classical system, in which the existence of arbitrary sums is not guaranteed, the existence of the Universe would have to be postulated separately. Assuming that differences and the Universe exist, then the singular term “U − x” denotes the *complement* of x, i.e., that object (if there is one) which comprises the remainder of the Universe outside of x.

Our final piece of basic mereological vocabulary consists of the notion of an *atom*: the predicate “At(x)” applies to an object just in case the object has no proper parts, i.e., the object is indivisible from the point of view of the theory. Anything may be taken as an atom for the purposes of the theory, whether or not it in fact has parts. (Compare, for example, the case of sentential logic versus predicate-logic: in sentential logic, sentences are taken as atomic for the purposes of the theory, even though we in fact take them to have parts. Predicate-logic, in turn, represents sentences as non-atomic, but construes the objects over which the variables range as atomic, even though, again, many, most or all of them in fact have parts.) Thus, to be a mereological atom simply means to be treated as indivisible by the theory. Whether there *in fact* are any atoms is an open question; certainly, the objects physicists call “atoms” have turned out not to be atoms in the mereological sense, since they have, for example, electrons and protons as parts. Mereology as such is neutral on the question of atomism; but a mereology can be explicitly turned into an *atomic*, *atomless* or *non-atomic* system, by means of further assumptions: **(p.15)**

Atomicity:	$(\forall x)(\exists y)\ (At(y)\ \&\ y \leq x)$
Atomlessness:	$(\forall x)(\exists y)\ (y < x)$
Non-Atomicity:	$(\exists x)(At(x))\ \&\ (\exists x)\ (\forall y)\ (y \leq x \rightarrow (\exists z)(z < y))$

An *atomic* mereology requires that every object either is itself an atom or is composed of atoms. An *atomless* mereology requires that every object is infinitely divisible into further proper parts. A *non-atomic* mereology requires that, among the objects over which it ranges, some are atomic and some are atomless. In an atomic mereology, the cardinality of the domain can be determined on the basis of the cardinality of atoms: for *n* atoms, there are $2^n - 1$ objects. Atomless and non-atomic mereologies of course have infinite domains.

§I.2.2 The Basic Principles of Standard Mereology

CEM is a very simple, elegant and surprisingly powerful theory. It requires only a single primitive notion in terms of which the remainder of the mereological concepts just introduced (along with others, if so desired) can be defined. In its standard formulations, CEM consists of a mere three axioms; all other statements of the theory follow as theorems from the definitions and axioms of the system. The single primitive can be chosen to be parthood (either < or ≤), overlap, disjointness or sum; the other notions are definable in terms of whichever one is taken as primitive. Identity is either assumed as given or (more controversially) as definable in terms of the primitive mereological notion. Although some formulations of CEM make use of set theory, reference to sets can be avoided, as can be seen, for example, from the definitions given above as well as from the formulation of CEM proposed in Goodman (1977). Algebraically speaking, while parthood is a mere partial ordering, CEM has the strength of a complete Boolean algebra, with the zero element deleted.

Historically, the development of CEM was motivated, first, by a desire to avoid the paradoxes of naive set theory and, secondly, by a desire to formulate a thoroughly nominalistic system. It is important to keep in mind, however, that especially the second goal is associated with mereology merely by historical accident and is in no way intrinsically connected with mereology as such; this is an important theme in Simons (1987) and is also visible in the work of others, most prominently perhaps that of Husserl, whose mereology is steeped in modal and other notions which would cause traditional nominalists great discomfort (cf. Husserl's third *Logical Investigation*, 1900–1). Both of these major goals of CEM are achieved by having the variables of the system range over entities of only a single, viz., the lowest, logical type; these entities are referred to by Leonard and Goodman (1940) as *individuals*. (Thus, somewhat misleadingly, the reference of Leonard and Goodman's term “individual” includes mereological sums, i.e., objects which have proper parts.) CEM itself, however, remains completely **(p.16)** neutral as to what is taken to be an individual for the purposes of the theory, as the closing passage from Leonard and Goodman (1940) reminds us:

> . . . [The Calculus of Individuals] performs the important service of divorcing the *logical* concept of an individual from metaphysical and practical prejudices, thus revealing that the distinction and interrelation of classes and wholes is capable of a purely formal definition, and that both concepts, and indeed all the concepts of logic, are available as neutral tools for the constructional analysis of the world. Then, for example, it becomes clear that the practice of supposing that *things* are what the x's and y's of *Principia mathematica* denominate and that qualities are necessarily to be interpreted as logical predicates thereof, rather than vice versa, is purely a matter of habit. The dispute between nominalist and realist as to what actual entities are individuals and what are classes is recognized as devolving upon matters of interpretative convenience rather than upon metaphysical necessity.
>
> (Leonard and Goodman 1940, p. 55)

Whatever may have become of Leonard and Goodman's further ambitions for their theory, the basic point of this passage is surely correct: like any formal system, CEM itself of course makes no pronouncements as to what its own variables range over, and hence what gets to count as an individual with respect to the theory. Thus, the theory may in principle be applied to anything which we are willing to regard as an individual and which can be appropriately characterized by means of mereological concepts.

Leonard and Goodman's version of CEM, which is called the “Calculus of Individuals”, uses as its single primitive the relation, $\wr$, of disjointness; identity is assumed (as defined independently, in accordance with the method given in *Principia Mathematica*). Then, “parthood”, “overlap”, “sum” and “product” can be defined in terms of disjointness as follows:

Definition of Parthood:	$x \leq y \equiv_{def} (\forall z)(z \wr y \rightarrow z \wr x)$
Definition of Proper Part:	$x < y \equiv_{def} x \leq y \ \& \ x \neq y$
Definition of Overlap:	$x^{\circ}y \equiv_{def} (\exists z)(z \leq x \ \& \ z \leq y)$
Definition of Sum:	$xFu\alpha \equiv_{def} (\forall z)((z \wr x) \leftrightarrow (\forall y)(y \in \alpha \rightarrow z \wr y))$
Definition of Product:	$xNu\alpha \equiv_{def} (\forall z)((z \leq x) \leftrightarrow (\forall y)(y \in \alpha \rightarrow z \leq y))$

In other words, an object is a *(proper or improper) part* of another object just in case anything that is disjoint from the second is also disjoint from the first. An object is a *proper part* of another just in case the first is a (proper or improper) part of the second and they are not identical. Two objects *overlap* just in case they have a (proper or improper) part in common. An object *fuses* a set, α, just in case everything that is discrete from the fusion is also discrete from every member of the set and vice versa. An object is the *product* or *nucleus* of a set, α, just in case everything that is a (proper or improper) part of the product is also a (proper **(p.17)** or improper) part of every member of the set, and vice versa. The notions of “difference”, “universe” and “complement” can also be defined straightforwardly in terms of those already cited; for the sake of brevity, I omit these definitions since they will not be of immediate concern to us in what follows.

We can now state the three axioms of the Calculus of Individuals, assuming any axiom system sufficient for first-order predicate-logic with identity and set theory:

Axiom 1 (Fusions):	$(\exists x)(x \in \alpha) \rightarrow (\exists y)(yFu\alpha)$
Axiom 2 (Parthood):	$(x \leq y \ \& \ y \leq x) \rightarrow x = y$
Axiom 3 (Overlap):	$x^{\circ}y \leftrightarrow \sim(x \wr y)$

The first axiom, which insures the *existence* of fusions (for all non-empty sets), is perhaps the most notorious among the three axioms; though the second axiom, which guarantees their *uniqueness*, has also generated some interesting discussion. The controversy surrounding both of these axioms will concern us further below. The third axiom merely lays down the formal properties of overlap in relation to the primitive notion of disjointness.

A very accessible formulation of CEM, which is slightly different from, but formally equivalent to, that of Leonard and Goodman (1940), is also given in Lewis (1991), where the three basic axioms of standard mereology are stated informally as follows:

> Axiom 1 (Unrestricted Composition): Whenever there are some things, then there exists a fusion of those things.
> Axiom 2 (Uniqueness of Composition): It never happens that the same things have two different fusions.
> Axiom 3 (Transitivity): If x is part of some part of y, then x is part of y.

In what follows, I will, whenever convenient, refer to the first two axioms of CEM using Lewis' terminology, as “Unrestricted Composition” and “Uniqueness of Composition”.[8]

§I.2.3 A Gradual Statement of the Theory

Even though, as we have seen, the full-strength theory of CEM can be stated in a very economical way in terms of the definitions and axioms given above, it is actually quite instructive to lay out the theory in a more round-about fashion, by gradually adding stronger and stronger principles to a minimal core, until we arrive at the full-strength version of CEM. Such a gradual exposition of CEM is given in Simons (1987, Sect. I.4). Its purpose is to bring out, for the benefit **(p. 18)** of those who do *not* view CEM as the ontologically harmless theory it is often advertised to be, how much mereology they can embrace before they arrive at the full-strength principles of CEM which they may find controversial. In what follows, I will present only some of the most important landmarks in Simons' gradual statement of the theory; for a full development, the reader is referred to my source.

We begin by assuming any set of axioms sufficient for first-order predicate-logic with identity; in order to preserve neutrality on the question of whether identity can and should be defined in terms of parthood, we take identity as given. We assume as our single primitive notion proper parthood, $<$. Since proper parthood (or so at any rate we presuppose) is at least a strict partial ordering, we assume that any mereology must accept the *asymmetry* and *transitivity* of $<$,

Axiom 1 (Asymmetry):	$x < y \rightarrow \sim(y < x)$
Axiom 2 (Transitivity):	$(x < y \,\&\, y < z) \rightarrow x < z$

from which the *irreflexivity* of proper parthood follows. To capture the characteristics of proper parthood, however, more is needed than what is already encapsulated in Axioms 1 and 2. For one thing, Axioms 1 and 2 are satisfied by models in which an object has only a *single* proper part. And while not all writers agree on this point, Simons at least takes it to be *constitutive* of the notion of proper parthood that an object cannot have merely a single proper part:

> How could an individual have a *single* proper part? That goes against what we mean by "part". An individual which has a proper part needs other parts in addition to *supplement* this one to obtain the whole.
>
> (Simons 1987, p. 26; his italics)

Since there are different ways of expressing this point formally, Simons proposes a series of what he calls "Supplementation Principles", of increasing strength. Two such principles, which in Simons' view are clearly too weak, are as follows:

Overly Weak Supplementation Principle I:	$(x < y) \rightarrow (\exists z)(z < y \,\&\, z \neq x)$
Overly Weak Supplementation Principle II:	$(x < y) \rightarrow (\exists z)(z < y \,\&\, \sim(z \leq x))$

The first principle is too weak, because it does not rule out models in which there is an infinitely descending linear chain of objects; and while each of these objects has more than a single proper part, these are themselves proper parts of its other proper parts. The second principle is too weak because it does not rule out models in which all proper parts overlap each other. To rule out all three sorts of models, we require a principle of at least the strength of the "Weak Supplementation Principle" (WSP), **(p. 19)**

Axiom 3 (Weak Supplementation Principle):	$(x < y) \rightarrow (\exists z)(z < y \,\&\, z \wr x)$

which requires that an object which has a proper part has at least another proper part disjoint from the first. While this axiom rules out the three models just considered, it still permits models in which distinct objects are made of exactly the same parts, which contradicts the second axiom of standard mereology. In order to exclude this possibility, one must assume either the "Proper Parts Principle" (PPP) or the "Strong Supplementation Principle" (SSP), from which both PPP and WSP follow:

Axiom 4 (Proper Parts Principle):	$((\exists z)(z < x) \,\&\, (\forall z)((z < x) \rightarrow (z < y))) \rightarrow x \leq y$
Axiom 5 (Strong Supplementation Principle):	$\sim(x \leq y) \rightarrow (\exists z)(z \leq x \,\&\, z \wr y)$

The axiom system which results from assuming Axioms 1, 2 and 5 still falls well short of CEM, in part because it does not guarantee the existence of unique products. Since the assumption that any two overlapping objects have a unique product appears plausible in an *extensional* mereology, i.e., one which has already accepted, in accordance with PPP or SSP, that no two distinct objects can be made of exactly the same proper parts, it would be natural to supplement Axioms 1, 2 and 5 with a further principle to this effect:

Axiom 6 (Products):	$(x \circ y) \rightarrow (\exists z)(\forall w)((w \leq z) \leftrightarrow (w \leq x \,\&\, w \leq y))$

In this stronger context, SSP can now be derived from Axioms 1, 2 and 6. Simons refers to the axiom system consisting of Axioms 1, 2 and 6 as "Minimal Extensional Mereology" (MEM). That MEM still has not reached the strength of CEM can be seen from the fact that MEM does not guarantee the conditional or unconditional existence of arbitrary sums, not only in infinite models (since MEM lacks provisions for infinitary operators) but also in small finite models. According to CEM, for example, there is only a single seven-element model (which is built up from three atoms), whereas according to MEM there are many such models (twenty-eight, to be precise). Thus, the remainder of Simons' gradual exposition consists in adding stronger and stronger principles to MEM which concern the conditional or unconditional existence of sums, binary or generalized (as well as the weaker notion of “upper bound”, which we can ignore here), such as the following:

Axiom 9 (Conditional Binary Sums):	(x°y) → (∃!)(x + y)
Axiom 14 (Unconditional Binary Sums):	(∃!) (x + y)
Axiom 16 (Universe):	(∃x)(∀y) (y ≤ x)
Axiom 18 (Conditional General Sums):	(∀x)(∀y)(((F(x) & F(y)) → (x°y)) → (∃!)(σx[F(x)]))

(p.20) Eventually, with Axiom 24 or the “General Sum Principle” (GSP), we reach the full strength of CEM:

Axiom 24 (General Sum Principle):	(∃x)(F(x)) → (∃x)(∀y)((y°x) ↔ (∃z)(F(z) & (y°z)))

GSP states that for any of the objects that satisfy the predicate in question, there exists a sum of these objects (provided that the predicate has a non-empty extension). Once Axiom 24 is added to Axioms 1, 2 and 3, the resulting system is formally equivalent to CEM and the intermediary stages have thereby become redundant.[9]

§I.3 The Application of Standard Mereology to Ordinary Material Objects

Given its simplicity and strength, CEM no doubt has its attractions as a theory characterizing such formal notions as <, ≤, °, ʅ, +, ·, −, π and σ. But whether CEM in fact correctly characterizes our *ordinary* mereological concepts is by no means obvious. For, whatever its merits as a formal theory, it is of course a further question whether the variables of CEM ought to be interpreted as ranging over anything to which we take ourselves to be committed in our ordinary, scientifically informed discourse or which is of any interest to metaphysicians. To anticipate, my own answer to these questions will be negative, and our next goal will be to motivate and defend the thesis that standard mereology does *not* provide the correct tool for the analysis of ordinary material objects. However, while a small minority of philosophers would agree with this assessment (e.g., Armstrong 1978, 1986, 1989, 1991, 1997; Fine 1982, 1994a, 1999; Harte 2002; Husserl 1900–1; Johnston 2002; Simons 1987; van Inwagen 1981, 1987, 1990a, 1993, 1994, 2002), it is fair to say that the vast majority would protest that insofar as we have any understanding of the notions of parthood and composition at all, this understanding derives from standard mereology. The following passage from Lewis (1986a) will do as a representative expression of this sentiment; it is **(p.21)** taken from a context in which Lewis is concerned primarily with the Uniqueness of Composition and Armstrong's work on *structural universals* (by “mereology”, Lewis means what we have called CEM; and by “mereological composition”, he means “composition” in the sense specified by CEM):

> My objection [to the idea that there are several different, non-standard senses of composition] is that I do not see by what right the operations are called *combining* operations. An operation applies to several universals; it yields a new universal. But if what goes on is unmereological, in what sense is the new one *composed* of the old ones? In what unmereological sense are they present in it? After all, not just any operation that makes new things from old is a form of composition! There is no sense in which my parents are parts of me, and no sense in which two numbers are parts of their greatest common factor; and I doubt that there is any sense in which Bruce is part of his unit set. [. . .] . . .[If the friend of “*sui generis* composition”] does insist that his unmereological composition is nevertheless composition, in a perfectly literal sense, then I need to be told why. Saying so doesn't make it so. **What is the *general* notion of composition, of which the mereological form is supposed to be only a special case? I would have thought that mereology already describes composition in full generality.** If sets were composed in some unmereological way out of their members, that would do as a precedent to show that there can be unmereological forms of composition; but I have challenged that precedent already.[10]

Thus, in Lewis' view, there is only one genuinely mereological notion of composition and it is that specified by CEM. And while I have tried to be as neutral as possible in my exposition of standard mereology, the reader has perhaps already noticed that there are some reasons for thinking that a skeptical attitude towards Lewis' stance might be justified. For one thing, we have seen that basically *any* assumption concerning the question of which axiom system correctly characterizes the logic of parthood is surrounded by controversy, down to even the seemingly most innocuous requirement that proper parthood be characterized formally as a strict partial ordering: for every assumption concerning parthood that has appeared obvious to some, there are others in the literature who have been willing to challenge it. Moreover, as Simons' gradual exposition of CEM brings out, provided sufficient independent motivation is given, there are various places in the evolving axiom system, short of the full-strength theory of CEM, at which one could stop and still end up with something which arguably deserves to be called a mereology; a weaker mereology of this sort will also come with an associated weaker sense of “composition”, which may nevertheless deserve to be viewed as genuinely mereological. We will in what follows encounter reasons for thinking that an adequate analysis of ordinary material objects dictates precisely such a strategy. Without going into Lewis' arguments in any detail at this juncture, it thus seems reasonable to believe that there is at the very least room for other genuinely mereological notions of composition besides that of CEM.

(p.22) But suppose, for the moment, that Lewis is right in thinking that standard composition is the only genuinely mereological form of composition there is. We began by taking it as an intuitive datum that ordinary material objects are *wholes* composed of parts; and everyone except the Nihilist (who believes that nothing has proper parts) will concur. If we combine this intuitive datum with Lewis' thesis that standard composition is the only genuinely mereological notion of composition, then we of course get the result that ordinary material objects must be wholes in the standard sense of composition, i.e., that ordinary material objects must be *mereological sums*.

The thesis that ordinary material objects are mereological sums has been remarkably popular among three-dimensionalists and four-dimensionalists alike. From a three-dimensionalist perspective, perhaps the most well-known defense of this approach can be found in Judith Jarvis Thomson's influential article "Parthood and Identity Across Time" (Thomson 1983). Among the four-dimensionalist tradition, the arguments provided by David Lewis, especially in Lewis (1986b) and (1991), have had a wide following; some of Lewis' main arguments in defense of standard mereology have also been adopted and elaborated in creative ways in Theodore Sider's recent book, *Four-Dimensionalism: An Ontology of Persistence and Time* (2001). Thus, we shall turn next to Thomson, Lewis and Sider's arguments in favor of the thesis that ordinary material objects are best viewed as mereological sums, in the standard sense.

Notes:

(1) *Concrete* is typically taken to contrast with *abstract*; *particular* with *universal*. Although it is difficult to make precise exactly what is meant by these distinctions, it is sufficient for present purposes to proceed with the rough and ready characterization given above. Thus, I understand "concrete" as entailing space-occupancy and the possession of a certain range of physical properties that we take to go along with space-occupancy. Since the defining feature of universals is typically taken to be that they are multiply located, i.e., that they are simultaneously present in their entirety in each of their instances, we can take particulars, in contrast, to be capable of being wholly present in only a single region of space-time at each time at which they exist. Due to my appeal to such notions as "space-occupancy", "being an instance of" and "being wholly present", I don't take anything I have just said to be particularly illuminating or *definitive* of the "concrete/abstract", "particular/universal" distinctions; I hope nevertheless that what I have said will give the reader at least a rough idea of the starting point of my analysis. As will become clear shortly, the current inquiry is not meant to answer the question, "What concrete particulars are there?", but assumes as given an ontology of material objects to which we take ourselves to be committed in ordinary life and scientific discourse.

(2) Though one which would be denied by the Nihilist, who holds that nothing composes anything, i.e., that the world consists of mereological simples. I take

on a particular version of the Nihilist position, as defended recently by Cian Dorr (e.g., in Dorr 2005), in Koslicki (2005b).

(3) To avoid confusion, I shall use the term "mereology" neutrally to mean what it literally means, viz., the study of parts and wholes; according to this usage, *any* theory concerning the logic of the part/whole relation is *a* mereology. The terms "whole", "part" and "composition" are to be understood in an equally non-theory-specific way: a (non-trivial) whole is simply any object which has parts; a part is that which (if it is a *proper* part) composes a whole and is non-identical with it. What it means to be a whole or a part may be spelled out differently by different mereologies; i.e., wholes and parts will have whatever properties the particular mereology specifies for its relations of parthood and composition. Finally, I shall use the terms "sum", "fusion" and "aggregate", interchangeably; unless otherwise indicated, I shall reserve these terms for the composite objects described by the particular theory I call "standard mereology"; what I mean by "standard mereology" will be explained shortly.

(4) Mereology can also be formulated by means of plural quantification, as illustrated for example in Lewis (1991) or van Inwagen (1990a) and (1994).

(5) Other work that deals more or less directly with mereology includes (in alphabetical order): Bostock (1979); Bunt (1985); Cartwright, H. M. (1996); Casati and Varzi (1999); Chisholm (1973, 1975, 1976); Clarke (1981); De Laguna (1922); Eberle (1970); Fine (1982, 1983, 1992, 1994a, 1994c, 1999, 2003); Harte (2002); Hudson (2000, 2001); Husserl (1900–1); Lejewski (1982); Lewis (1991); Markosian (1998a, 1998b); Menger (1940); Merricks (1993, 2003); Moltmann (1997, 1998); Needham (1981); Oliver (1994); Plantinga (1975); Rea (1998, 2002); Scaltsas (1990); Sharvy (1980, 1983); Sider (1993, 2001); Smith (1982, 1997); Smith and Varzi (2000); Thomson (1977, 1983, 1998); Tiles (1981); Tranöy (1959); van Benthem (1983); van Inwagen (1981, 1987, 1990a, 1993, 1994, 2002); Varzi (2000); Whitehead (1919, 1920, 1929); Wiggins (1979); and Zimmerman (1995). (Again, the reader is referred to Simons 1987, ch. 2, for a detailed discussion and explicit comparison of several alternative mereological systems, some classical, some non-classical, and some of which extend mereology into the realm of topology.)

(6) This section follows very closely Simons (1987, ch. 1), which should be consulted for a more detailed exposition of the basic mereological vocabulary.

(7) See, for example, Rescher (1955), Lowe (1989, p. 94, n. 9), Moltmann (1997, 1998) and Johnston (2002), for approaches that question whether parthood is in general transitive.

(8) In the Leonard/Goodman Calculus of Individuals, the transitivity of parthood follows as a theorem from the axioms and definitions of the system.

(9) In the context of disputing Lewis' mereological interpretation of set theory in Lewis (1991), Oliver (1994) quite rightly points out that it is doubtful whether even the full-strength system of CEM has really succeeded in *formal* terms in capturing what is characteristic of mereology. For just as not all mere partial orderings are plausibly interpreted as genuine relations of parthood, so similarly not all axiom systems that have the strength of a complete Boolean algebra minus the zero element are plausibly interpreted as being genuinely mereological in character. Oliver gives as an example to illustrate this point any finite set of prime numbers, together with their products: it is not obvious in a case of this sort that a number is *part* of another *merely* because the former *divides* the latter. Thus, it should be kept in mind that it is no objection against weaker systems of mereology that they fail to capture what is genuinely mereological about the relation of parthood in purely formal terms, since the same objection can arguably be launched against the stronger systems as well.

(10) Lewis (1986a, p. 97); his italics, my bold-face; page numbers come from the reprinted version in Lewis (1999).