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Regime changes in Bitcoin GARCH volatility dynamics

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ABSTRACT

We test the presence of regime changes in the GARCH volatility dynamics of Bitcoin log–returns using Markov–switching GARCH (MSGARCH) models. We also compare MSGARCH to traditional single–regime GARCH specifications in predicting one–day ahead Value–at–Risk (VaR). The Bayesian approach is used to estimate the model parameters and to compute the VaR forecasts. We find strong evidence of regime changes in the GARCH process and show that MSGARCH models outperform single–regime specifications when predicting the VaR.

1. Introduction

The last ten years have witnessed the spectacular development of cryptocurrencies. Nakamoto (2008) designed the first decentralized cryptocurrency based on the blockchain technology, namely Bitcoin. Bitcoin was created to facilitate electronic payments between individuals without going through a (trusted) third party. Despite its anonymity and reduction in transaction costs (see Kim, 2017), and while providing some diversification benefits (see Corbet et al., 2018), Glaser et al. (2014) and Baek and Elbeck (2015) show that Bitcoin is mostly used for speculative purposes, causing extreme volatility and bubbles (see also Cheah and Fry, 2015; Dyhrberg, 2016; Corbet et al., 2017a; Hafner, 2018). For investors, it is therefore key to gauge the risks related to an investment in Bitcoin.

Recent studies have focused on the volatility dynamics of the Bitcoin returns. Chu et al. (2017) find evidence of volatility clustering and show that GARCH–type specifications provide the best in–sample performance. Using asymmetric GARCH models, Bouri et al. (2017), Katsiampa (2017), Baur et al. (2018) and Stavroyiannis (2018) investigate the response of the conditional variance to past positive and negative shocks and find an inverted leverage effect. Further, they measure a high degree of persistence in the volatility process (i.e., unit root process). These findings are also observed by Catania and Grassi (2017) using the recent class of GAS models. Phillip et al. (2018) use a stochastic volatility model and also find evidence of long–memory in the volatility dynamics.

Other researchers have investigated properties in Bitcoin returns that can potentially lead to biases in the estimation or to the poor forecasting power of traditional models such as GARCH–types. Ignoring regime changes in the volatility dynamics can bias estimation results and have significant effects on the precision of the volatility forecast (see, e.g., Lamoureux and Lastrapes, 1990; Bauwens

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et al., 2014). Bariviera (2017) finds that Bitcoin returns exhibit some form of regime change, suggesting that regime–switching models could more adequately capture the volatility dynamics. In the same spirit, Balcombe and Fraser (2017) find that Bitcoin exhibits regime–switching behavior. Using a Bayesian change–point approach, Thies and Molnár (2018) identify several structural breaks in the volatility process of Bitcoin. Corbet et al. (2017b) attribute the structural breaks to changes in interest rate and quantitative easing policy made by central banks. Given these findings, it is thus key for proper estimation and risk forecasting to determine if regime changes are present in the GARCH dynamics and estimate the models accordingly.

This note tests whether Markov–switching GARCH (MSGARCH) models capture any regime changes in the Bitcoin volatility dynamics and outperform single–regime GARCH specifications in Value–at–Risk (VaR) forecasting. Extending the work of Katsiampa (2017), we consider various scedastic functions, error distributions, and specifications for up to three regimes, leading to a total of 18 models estimated using a Bayesian approach. Our results identify regime changes in the GARCH dynamics of Bitcoin. A two–regime specification for an asymmetric GARCH model with skewed and fat–tail conditional distribution leads to the best fit in–sample. The inverted leverage effect is observed in all volatility regimes. We also find that MSGARCH specifications outperform single–regime models for VaR forecasting.

2. Methodology

Let y_t be the daily Bitcoin log–return (in percentage) at time t. We focus on the volatility dynamics and therefore assume that $\mathbb{E}[y_t] = 0$ and $\{y_t\}$ is serially uncorrelated.[1] Following Ardia et al. (2018), we specify an MSGARCH model as:

$$y_t \mid (s_t = k, \mathcal{I}_{t-1}) \sim \mathcal{D}(0, h_{k,t}, \xi_k),$$

where $\mathcal{D}(0, h_{k,t}, \xi_k)$ is a continuous distribution with a zero mean, a time–varying conditional variance $h_{k,t}$ in regime k, and a vector ξ_k of additional shape (e.g., tail and asymmetry) parameters. The state variable s_t evolves according to a first–order homogeneous Markov chain with a finite number of states K. Finally, $\mathcal{I}_{t-1}$ denotes the information set available up to $t-1$.

We follow Haas et al. (2004) and, conditionally on regime $s_t = k$, specify the GARCH–type conditional variance $h_{k,t} \equiv h(y_{t-1}, h_{k,t-1}, \theta_k)$ as a function of past returns and the additional regime–dependent vector of parameters θ_k. Thus, the GARCH–type processes evolve independently and the model does not face the path–dependency problem (see Ardia et al., 2018).

Two different specifications for the conditional variance are considered. We use the symmetric GARCH(1,1) model of Bollerslev (1986):

$$h_{k,t} \equiv \omega_k + \alpha_k y_{t-1}^2 + \beta_k h_{k,t-1},$$

and the asymmetric GJR(1,1) model of Glosten et al. (1993):[2]

$$h_{k,t} \equiv \omega_k + (\alpha_k + \gamma_k I\{y_{t-1} < 0\}) y_{t-1}^2 + \beta_k h_{k,t-1},$$

where $I\{\cdot\}$ is the indicator function. For the innovations, we use the Normal, the (standardized) Student–t, and the skewed (standardized) Student–t (see Fernández and Steel, 1998). We consider up to three regimes (i.e., $K = 3$) thus leading to 18 models overall.

We estimate the model parameters with a Bayesian approach (via MCMC simulations). This allows drawing interesting probabilistic statements on (nonlinear functions of) the model parameters, such as the leverage effect and the unconditional volatility in each regime (see Ardia, 2008).[3] We use diffuse priors and rely on the adaptive random–walk Metropolis sampler of Vihola (2012) to generate draws from the posterior.[4] We ensure positivity and stationarity of the conditional variance in each regime during the estimation (see Trottier and Ardia, 2016). Moreover, we impose constraints on the parameters to ensure that volatilities under the MSGARCH specification cannot be generated by a single–regime specification. Both are achieved through the prior specification; see Ardia et al. (2017) for details on the implementation.[5]

3. Empirical results

We use daily Bitcoin mid–prices in USD downloaded from Datastream. The time period ranges from August 18, 2011, to March 3, 2018, for a total of 2355 observations. Summary statistics are reported in Table 1. The mean and median are both positive with values of 0.29% and 0.23%, respectively. The standard deviation is 5.36% (i.e., $102\% = \sqrt{365} \times 5.36$, on an annual basis). The largest price drop is -66.39% and the largest price increase is 44.55%. Data exhibit a moderate negative asymmetry and a very large excess kurtosis.

[1] In the empirical study, we ensure this by pre–filtering the data with an AR(1) model.

[2] Alternative asymmetric models such as EGARCH were considered and led to similar results.

[3] Also, Ardia (2008) illustrates that MCMC simulation provides a robust optimization approach compared to the traditional Maximum Likelihood, and Ardia et al. (2018) show that the predictive distribution for MSGARCH models obtained with the Bayesian framework provides more accurate risk forecasts than the frequentist approach.

[4] We use 5000 burn–in draws and build the posterior sample of size 1000 with the next 5000 draws keeping only every 5th draw to diminish the autocorrelation in the chain. We performed several sensitivity analyses to assess the impact of the estimation setup. In all cases, the conclusions remained qualitatively similar.

[5] Positivity in the GJR case is ensured by setting $\alpha_k > |\gamma_k|$ in the prior.

Table 1

Summary statistics. The table reports the descriptive statistics for the daily log–returns of Bitcoin in percentage from August 19, 2011, to March 2, 2018. It presents the number of observations, the mean, the median, the standard deviation, the skewness, the kurtosis, the maximum and the minimum values, the historical Value–at–Risk at both 5% and 1% risk levels (i.e., empirical percentiles) and the p–value of the Jarque–Bera Normality test.

Statistic	Value
Observations	2355
Mean (%)	0.29
Median (%)	0.23
Std (%)	5.36
Skewness	−1.36
Kurtosis	26.14
Maximum (%)	44.55
Minimum (%)	−66.39
VaR 5% (%)	−7.14
VaR 1% (%)	−17.45
JB p–value	< 0.01

3.1. In–sample analysis

We first consider an in–sample analysis, where we fit the 18 models to the full history of data. As we are interested in the volatility dynamics, we demean the series and remove autoregressive effects in the data using an AR(1) filter and estimate the models on the residuals.[6]

To evaluate the goodness–of–fit of the models, we use the Deviance information criterion (DIC) obtained from the Bayesian estimation (Spiegelhalter et al., 2002). The DIC is not intended for identification of the correct model, but rather merely as a method of comparing a collection of alternative formulations and determining the most appropriate. Berg et al. (2004) and Ardia (2008, Chapter 7) have illustrated the potential advantages of this information criterion in determining the proper stochastic volatility and MSGARCH model, respectively.

In Table 2, we report the DIC of the various models. We notice that, for all volatility and distribution specifications, the two–regime MSGARCH models offer a better trade–off between fitting quality and model complexity than their single–regime counterparts. When we compare three–regime versus two–regime models, we note that gains are only observed for the Normal conditional distribution. In this case, a third regime is required to capture the large unconditional kurtosis in the data. Further, we note that for all regime–specifications, a skewed and fat–tailed distribution is preferred.

We now consider the parameter estimates of the best in–sample model, that is the two–regime Markov–switching GJR skewed Student–t model. In Table 3, we report the median and 25–75th percentiles of the posterior distribution for the model parameters. For a given regime k, $(\omega_k, \alpha_k, \beta_k, \gamma_k)$ are the GJR parameters, where γ_k measures the asymmetry (i.e., leverage effect), and η_k is the tail parameter and ξ_k the asymmetry parameter of the skewed Student–t distribution. For each regime, we also report the probability of the leverage effect to be negative, the annualized unconditional volatility, and the regime persistence's probability. Finally, we provide estimates for the overall model leverage effect and unconditional variance (by marginalizing over the regimes), and the persistence of the squared and absolute returns (i.e., aggregated autocorrelation up to lag 50) implied by the models. For comparison, we also report results for the single–regime specification.

Estimation results for the single–regime model confirm the presence of an inverted leverage effect, with a posterior median for γ_1 at -0.07. The posterior probability of an inverted leverage is 100%. The posterior median of the annualized unconditional variance implied by the model is 73%.

For the two–regime Markov–switching process, we observe an inverted leverage effect in both regimes, with probabilities of 99% and 78%, respectively. The regimes are clearly associated with low and high unconditional volatilities, with posterior median at 26% and 202%, respectively. Both regimes are highly persistent, with posterior probabilities p_{11} and p_{22} at 94% and 93%, respectively. This regime–persistence is illustrated in Fig. 1, where we display the smoothed probabilities, $\mathbb{P}[s_t = k \mid \mathcal{I}_T]$, for the low– and high–volatility regimes ($k = 1$ and $k = 2$, respectively). When marginalizing over regimes, we obtain a posterior median for the leverage effect at -0.02 and a posterior probability of it being negative of 98%, both smaller than for the single–regime model. The posterior median of the annualized unconditional volatility is 91%, higher than for the single–regime model, and more in line with the empirical volatility at 102%. Also, we note that for both the squared and absolute returns, the volatility persistence implied by the

[6] Filtering the log–returns using an AR(p) model with p selected from AIC leads to similar results.

Table 2
Deviance information criterion. The table reports the Deviance information criterion of the 18 models (two scedastic specifications, three conditional distributions, and up to three regimes). In bold are highlighted the two–regime MSGARCH models that outperform their single–regime counterparts and the three–regime MSGARCH models that outperform their two–regime counterparts.

	$\mathcal{N}$	$\mathcal{S}$	$s\mathcal{S}$
Single–regime			
GARCH	13153.70	12454.45	12455.17
GJR	13156.12	12450.94	12450.67
Two–regime			
GARCH	**12515.66**	**12388.57**	**12370.78**
GJR	**12543.11**	**12393.72**	**12356.79**
Three–regime			
GARCH	**12451.82**	12397.03	12370.86
GJR	**12472.24**	12764.04	12621.17

Table 3
Parameter estimates. The table reports the median and the 25–75th percentiles (in squared parentheses) of the posterior sample for the GJR skewed Student–*t* model with one and two regimes. The GJR specification in regime k is expressed as $h_{k,t} \equiv \omega_k + (\alpha_k + \gamma_k I\{y_{t-1} < 0\})y_{t-1}^2 + \beta_k h_{k,t-1}$, where $I\{\cdot\}$ is the indicator function. The (standardized) skewed Student–*t* conditional distribution has tail parameter η_k and asymmetry parameter ξ_k. The regime's k persistence probability is $p_{kk} \equiv \mathbb{P}[s_t = k \,|\, s_{t-1} = k]$, the probability of the leverage effect to be negative is $\mathbb{P}[\gamma_k < 0]$, and the annualized unconditional volatility is UV_k. We also report quantities inferred from the overall model. The measure ρ_{sqr} is the sum of autocorrelations of the squared returns, and ρ_{abs} the sum of autocorrelations of the absolute returns, both up to lag 50. Estimates are obtained from the posterior sample of 1000 draws.

	Single–regime	Two–regime
Regime $k = 1$		
ω_1	0.30 [0.27, 0.33]	0.06 [0.05, 0.07]
α_1	0.22 [0.21, 0.23]	0.08 [0.08, 0.09]
γ_1	−0.07 [−0.08, −0.05]	−0.06 [−0.07, −0.05]
$\mathbb{P}[\gamma_1 < 0]$	1.00	0.99
β_1	0.81 [0.81, 0.82]	0.92 [0.92, 0.93]
η_1	3.35 [3.25, 3.47]	2.68 [2.57, 2.84]
ξ_1	1.00 [0.99, 1.01]	1.11 [1.09, 1.14]
p_{11}	1.00	0.94 [0.93, 0.94]
UV_1	73.06 [67.92, 79.90]	26.35 [23.76, 29.40]
Regime $k = 2$		
ω_2		1.96 [1.87, 2.08]
α_2		0.21 [0.19, 0.24]
γ_2		−0.03 [−0.06, 0.00]
$\mathbb{P}[\gamma_2 < 0]$		0.78
β_2		0.79 [0.77, 0.81]
η_2		5.00 [4.60, 5.47]
ξ_2		0.83 [0.81, 0.86]
p_{22}		0.93 [0.92, 0.94]
UV_2		202.35 [178.00, 233.21]
Overall		
γ	−0.07 [−0.08, −0.05]	−0.02 [−0.03, −0.02]
$\mathbb{P}[\gamma < 0]$	1.00	0.98
UV	73.06 [67.92, 79.90]	91.30 [86.13, 97.08]
ρ_{sqr}	2.88 [1.97, 4.02]	2.60 [1.89, 3.60]
ρ_{abs}	6.17 [4.39, 8.54]	5.82 [4.60, 7.76]

model is lower than for the single–regime model, indicating that the regime–switching specification is able to deal with the spurious integrated GARCH effect observed with single–regime models in the presence of breaks.

3.2. Out–of–sample analysis

We now turn to an out–of–sample analysis where we compare the ability of the 18 models to correctly forecast the one–day ahead Value–at–Risk (VaR). A "good" model for VaR forecasting should provide an accurate VaR coverage, that is, a correct percentage of VaR violations and their correct independent distribution over time. First, we backtest the accuracy of the VaR following Christoffersen (1998) and compute the conditional coverage test statistics. Second, we use the Dynamic Quantile test (DQ) of both

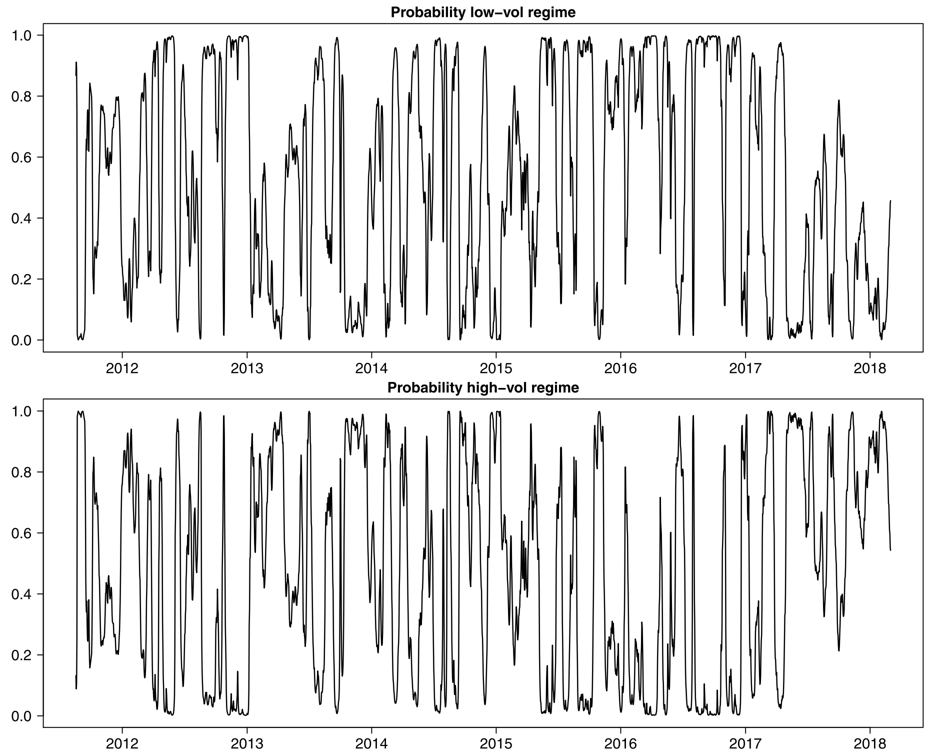

Fig. 1. Smoothed probabilities. The figure displays the smoothed probabilities for the two–regime skewed Student–*t* GJR model.

independence and coverage of Engle and Manganelli (2004).

We use 1000 log–returns for the rolling–window estimation and we run the backtest over 1355 out–of–sample observations for a period ranging from June 13, 2014, to March 2, 2018. The model parameters are updated every tenth observation and the VaR forecasts are computed at a daily frequency.

In Table 4, we report the one–day ahead VaR results at the 5% and 1% risk levels. According to the CC and DQ tests, the single–regime specifications do not forecast the VaR accurately at both risk levels. On the other hand, we cannot reject the null hypothesis of correct VaR forecasting for two–regime and three–regime MSGARCH models for the 1% VaR. Overall, the three–regime MSGARCH provides the most accurate VaR forecasts at both risk levels.

As a robustness check, we performed in– and out–of–sample analyses with constrained specifications where only the parameters of the variance equation can switch. Results (not reported) lead to similar conclusions to those obtained with unconstrained models.

Table 4

Accuracy of VaR predictions. The table reports the *p*–values of the conditional coverage test (CC) of Christoffersen (1998) and the dynamic quantile test (DQ) of Engle and Manganelli (2004) for the one–day ahead 5% and 1% VaR. In bold, we highlight *p*–values below 5%.

		CC test			DQ test		
		$\mathcal{N}$	$\mathcal{S}$	$s\mathcal{S}$	$\mathcal{N}$	$\mathcal{S}$	$s\mathcal{S}$
VaR 5% risk level							
Single–regime	GARCH	**0.01**	**0.01**	**0.01**	**0.01**	**0.02**	**0.02**
	GJR	**0.00**	**0.01**	0.05	**0.00**	**0.02**	0.23
Two–regime	GARCH	**0.02**	0.13	**0.02**	**0.03**	0.44	0.10
	GJR	0.05	**0.02**	0.06	0.08	0.11	0.15
Three–regime	GARCH	0.09	0.37	0.08	0.14	0.70	0.33
	GJR	**0.00**	0.06	**0.02**	**0.00**	0.12	**0.03**
VaR 1% risk level							
Single–regime	GARCH	**0.00**	**0.02**	**0.01**	**0.00**	**0.03**	**0.01**
	GJR	**0.00**	**0.00**	**0.03**	**0.00**	**0.00**	**0.02**
Two–regime	GARCH	0.30	0.32	0.55	0.42	0.37	0.99
	GJR	0.34	0.28	0.82	0.58	0.40	0.98
Three–regime	GARCH	0.86	0.39	0.39	0.97	0.88	0.94
	GJR	0.25	0.78	0.78	0.07	0.98	0.93

4. Conclusion

Given the growing interest in the Bitcoin cryptocurrency, it is of primary importance to choose reliable models to forecast the risk in such an investment. To that purpose, so far, most studies focus on GARCH–type models. While extensively used in academic research and practice, these models can lead to poor risk forecasts in the presence of regime changes in the conditional variance process.

By using specifications which can account for structural breaks in GARCH, namely Markov–switching GARCH models, we show that Bitcoin daily log–returns indeed exhibit regime changes in their volatility dynamics. In our sample period, a two–regime MSGARCH model exhibits the best in–sample performance with an inverted leverage effect in both low– and high–volatility regimes. When forecasting the one–day ahead Value–at–Risk, Markov–switching specifications clearly outperform standard single–regime GARCH models.

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Supplementary material

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