Numerical analyses of the effect of heterogeneities on rock failure process

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ABSTRACT: While heterogeneities in strength and deformation properties are thought to play an important role in the failure processes of rocks and rock masses, they are rarely explicitly introduced in numerical models. This paper present the results obtained by introducing heterogeneities in a Finite Element Modeling tool (Phase2™). Particularly the effect of heterogeneities in rock modulus and strength are investigated at the laboratory test sample scale. Limited modulus variability (coefficient of variation smaller than 1.5%) is sufficient to generate rock behaviour that is highly affected by induced tensile stress conditions. This variability in modulus reduces the peak strength and the post-peak strength drop: with increasing heterogeneity in deformability, the rocks become less brittle (i.e., more strain-softening). However, the brittleness in the low confinement range, leading to spalling behaviour, is enhanced by this heterogeneity. Various loading case and geometries are investigated highlights the influence of rock and rock mass heterogeneities. Modulus heterogeneities generate tensile conditions and damage when taking core samples from relatively high stress conditions. This may influence the samples strength and lead to an underestimation of the in-situ rock strength at depth. Heterogeneities influence the failure pattern around opening, with practical implication on ground support requirements. An equivalent homogeneous properties concept, as used for example by the GSI system, doesn't properly capture the failure pattern generated by the presences of heterogeneities, suggesting that the approach of "equivalent" homogenous material could be inadequate and that heterogeneities should be introduced explicitly in numerical analyses of geomechanics problems.

1. INTRODUCTION

The failure of brittle rocks under low overall compressive conditions is often dominated by tensile microcrack propagation and coalescence (e.g. [1]). Indeed, even in an overall compressive macroscopic stress field, tensile conditions are created in heterogeneous rocks (e.g. [2, 3]). For this reason, heterogeneities are thought to play a primordial role in the failure processes of rocks and rock masses.

Heterogeneities in rocks are due to: (i) mechanical contrast between the mineralogical components, (ii) grain shape and size, and (iii) the presence of micro-fractures. One or more of these aspects are almost always present in rocks making rock and rock masses rarely homogeneous and hence emphasizes the importance if accounting for heterogeneities in the modeling of the rock failure process.

Rock mass classification systems implicitly consider the effect of heterogeneities on the strength of rock masses. For example the use of the GSI system with the general Hoek-Brown failure criteria allow to derive equivalent homogeneous failure envelop to be used in numerical modeling for intrinsically heterogeneous rock masses. Refinement of this system for hard brittle rocks has been developed to account for rock texture, mineralogical composition and microstructure ([4]).

However, heterogeneities are rarely explicitly introduced when numerically modeling rocks failure processes, thus preventing a key characteristic of rock failure, the introduction of tensile damage initiation, accumulation and coalescence, failure localization, etc. Fig. 1 show the results of the modeling of a compressive loading test with the finite element method (FEM) code Phase2™ [5] using homogenous material properties: at each stage along the loading path, stresses are homogenous throughout the sample until they exceed the prescribed strength and failure occurs simultaneously in shear for all elements in the model. Even if the overall strain-stress curve would properly reflect the prescribed modulus and strength properties, the failure process and pattern is unrealistic.

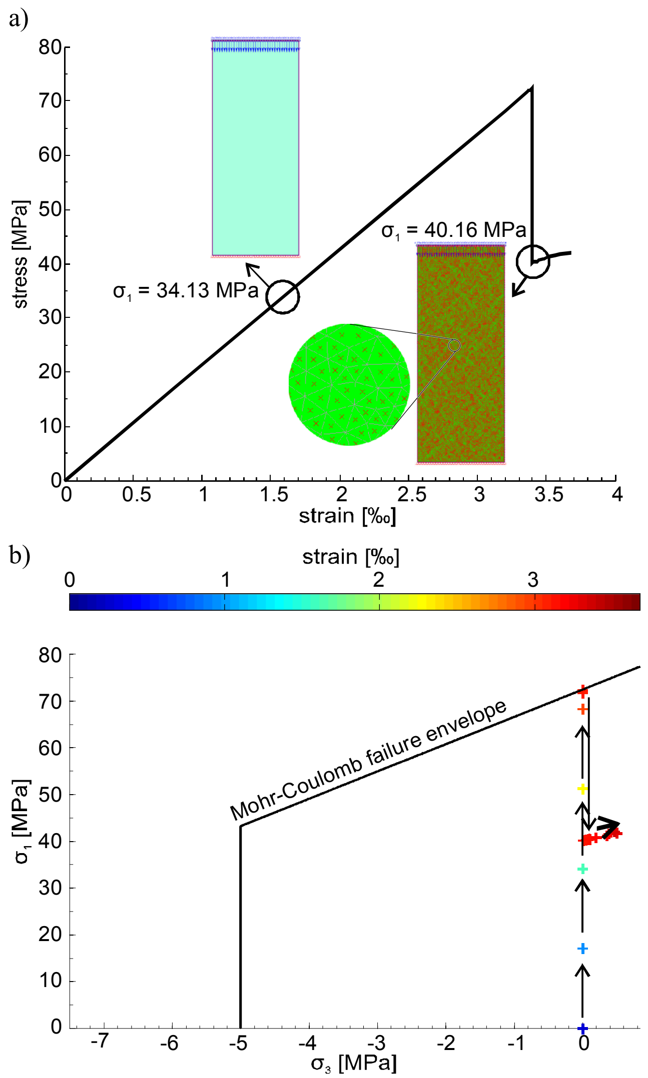

Fig. 1: Simulation of an unconfined compressive loading test using phase2[TM]. Model assumption: sample size: 50 x 125 mm; plain strain; loading applied by displacement boundary condition of the upper end of the sample; homogeneous Mohr-Coulomb material properties: $E = 20$ GPa, $\nu = 0.25$, $\sigma_T = 5$ MPa, $c = 15$ MPa, $\phi = 45°$, $\sigma_{ci} = 72.4$ MPa, $c_r = 10$ MPa and $\phi_r = 37°$. a) strain-stress curve; strain is overall strain computed from the imposed displacement boundary condition; stress is overall stress obtained by averaging σ_1 over the entire model; second sample model screenshot is entirely failed in shear (red x); b) stress path followed in this compression loading test.

Advanced numerical studies with particle flow code (PFC, e.g. [6]), Rock Failure Process Analyses Code (RFPA, e.g. [7]), elasto-plastic cellular automaton (EPCA, [8]) or Universal Distinct Element Code (UDEC, [9]) show that heterogeneities are a fundamental requisite to explain failure localisation processes and the complexity in failure mechanism and patterns of rocks and rock masses.

Result from [7] show that the homogeneity index is the most important factor influencing the rock failure in their implementation of the finite element method with a smeared crack model (RFPA). The results from [6] give insight in the brittle failure of rock. The spalling limit is defined as a constant σ_1/σ_3 ratio above which unstable tensile crack propagation dominates. Increase in heterogeneities influence the spalling limit toward higher σ_1/σ_3 ratio, i.e. more brittle behaviour. [8] investigated the influence of heterogeneities on scale and shape effects on failure. They show that elastic mismatch at the platen/rock interface is critical for the scale and shape effect to develop and were able to reproduce numerically shape effect observed on laboratory samples. Modelling from [9] highlights the importance of micro-heterogeneities on the micromechanical as well as the macroscopic response of rock subjected to uniaxial compression loading.

The examples of explicit modeling of heterogeneities cited above involve research tools available to a limited number of users. The results presented in this study concerns the implementation of explicit heterogeneities in the widely available commercial finite element code Phase2[TM]. It includes an evaluation of the advantages and limitations of the proposed approach and potentially could lead to a more widely applied explicit consideration of heterogeneities in numerical modeling for practical rock mechanics problems.

2. METHOD

Phase2[TM] is a stress analysis numerical tool based on a finite element method (FEM). In order to resolve stress and displacement throughout the model, a mesh of triangular or quadrilateral element is built. The approach used in this paper to introduce heterogeneities explicitly, is to randomize material properties on an element base as presented in Fig 2a. Each element is considered to be a "grain" of the rock fabric. This method imposes a certain number of constraints:

- Grain shape is controlled by the element type in use, triangle in this case. Also meshing algorithm tends to generate close to equilateral triangles.
- Grain size is imposed by the FEM meshing constraints. To obtain accurate results, the mesh must be relatively fine and the element size distribution fairly uniform. Fig. 3 display a typical element sizes distribution for the loading tests simulation presented below.
- There is an upper limit of 40 different materials that can be used in Phase2[TM]. Thus variability has to be discretized in a maximum of 40 bins.
- Because element properties are attributed in a random manner, no control over the spatial correlation of heterogeneities is introduced.

Another potential issue with this approach is to insure an

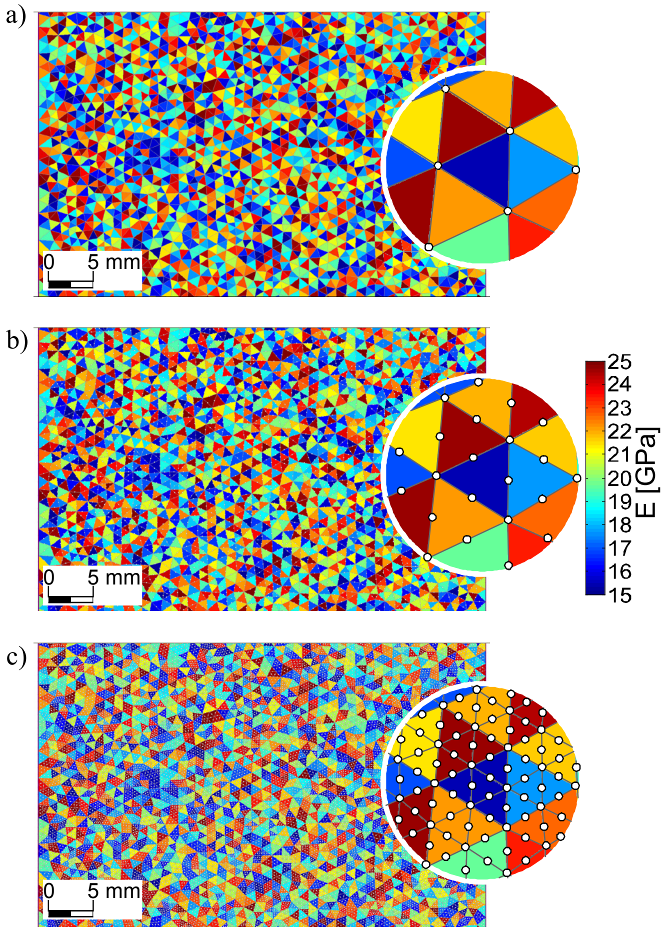

Fig. 2: Implementation of material properties heterogeneities by randomizing properties (here colour reflects elastic modulus variability) on triangular element base. a) using a first order triangular mesh; b) using a second order triangular mesh; c) using subdivided second order triangular mesh.

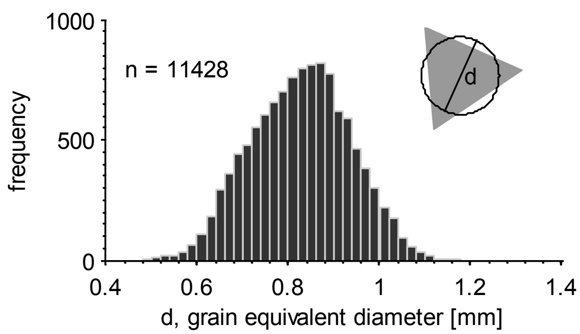

Fig. 3: Distribution of element size in the model used for compression loading simulations. Equivalent diameter, *d*, is the diameter of a disc with same area of the triangular element.

accurate stress approximation within the grains and the related selection of appropriate element type. Phase2™ offers two types of triangular elements: 3-noded (first order, Fig. 2a) or 6-noded (second order, Fig. 2b) triangles. An alternative solution to increase the accuracy of the calculation is to subdivide each element in multiple sub-elements (Fig 2c).The three approaches illustrated in Fig. 2 have been evaluated. The approach with 3-noded elements generated significantly different results than the other approaches and suffers from inaccuracies. Results from the 6-noded mesh and the subdivided 6-noded mesh are comparable. For this reason and in order to optimize model building easiness and computation time, non-subdivided 6-noded (Fig. 2b) elements have been used.

The remainder of this paper consists of an evaluation of the method and provides insight into the effect of heterogeneities on rock deformability and strength behaviour. First, results from simulation of compression loading are used to investigate the effect of various heterogeneities parameters on models behaviour. Finally other load cases and geometries are evaluated.

3. RESULTS

3.1. Unconfined compression loading simulation

The unconfined compression loading simulation presented in Fig. 1 was repeated using heterogeneous material properties. Material properties used where identical to those of the homogenous model except for the Young modulus: it was varied uniformly in 40 bins (identical probability for each bins) from $E = 15$ to 25 GPa. Fig. 4a show the strain-stress curve for each element (overlapping gray lines) and averaged over the entire model (thick black line). Due to the heterogeneities, yield initiate at about 30% of peak strength with tensile mechanisms dominating. Accumulation of damage reduces the final peak strength from 72.4 MPa to 55.3 MPa (24% reduction). Fig. 4b show the stress state for each element in the model at loading step 9 ($\varepsilon = 0.17\%$). The effect of stiffness heterogeneities induces dispersion of the stress level in the sample, particularly for σ_3 where even if the overall stress boundary condition are compressive 50% of the sample is in a tensile stress state. Tensile stress magnitudes increase with increasing axial load to the point that the initial yielding mechanism is tensile. Accumulation of damage is initially distributed but it localises at peak and post-peak stages (Fig. 5).

3.1.1. Effect of variation of mapping heterogeneities

Changing the seed value of the random number generator which distributes material properties in the sample allows the variation of the spatial distribution (mapping) of the heterogeneities without varying modulus distribution. In details, a list of uniformly distributed random numbers is generated and used to attribute modulus to the elements of the model. By using different lists of random numbers (change of the seed value), individual elements will have different modulus attributed to them (different mapping), but the overall statistic throughout the model will remain the same (same modulus distribution). The effect of mapping on sample behavior was evaluated by creating ten of such similar but not identical samples. All model behaved in a very similar manner. They all provided almost identical strain – stress curves, peak strength, stress paths and very similar failure pattern. Thus, in the presented cases, the overall behaviour is independent on the detail of the material mapping.

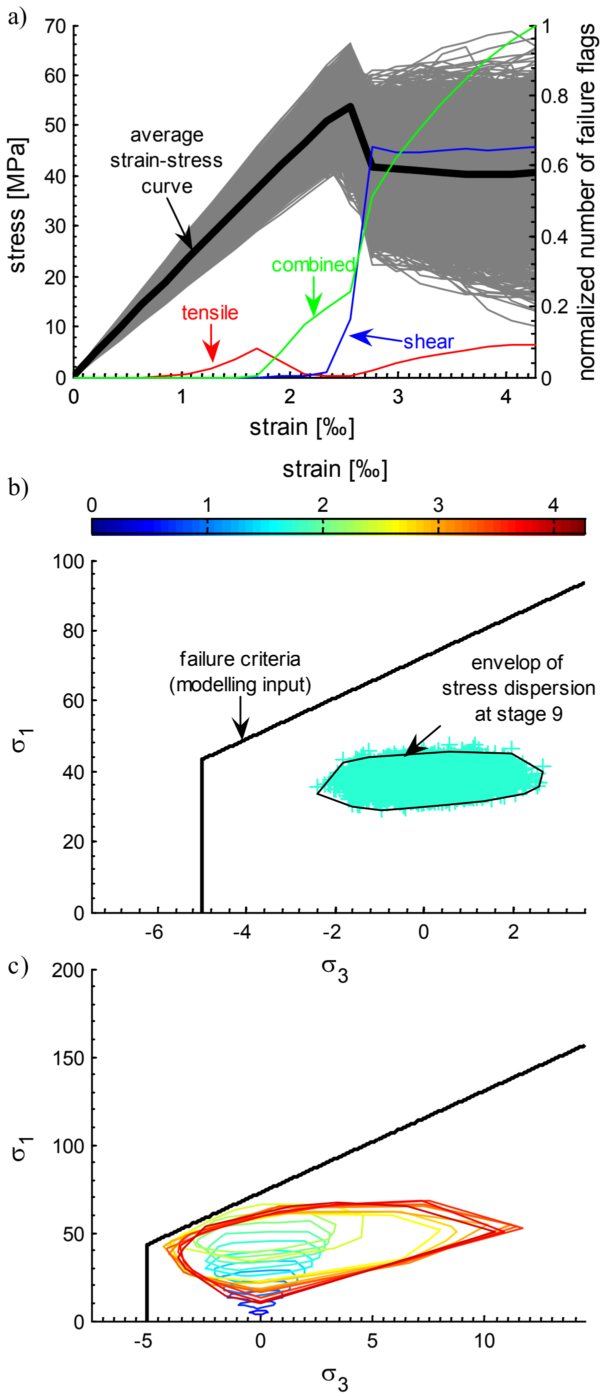

Fig. 4: a) strain stress curve for each elements (gray lines) and for their average (black line) as well as relative number of failure flag types (tensile, shear or combined). b) stress state of each elements at model stage 9, where overall strain is equal to 1.7‰. The thin black line is the envelop of stress state for every elements, including the entire stress variability (here called dispersion) induced by the modulus heterogeneities c) stress state envelopes for every stages.

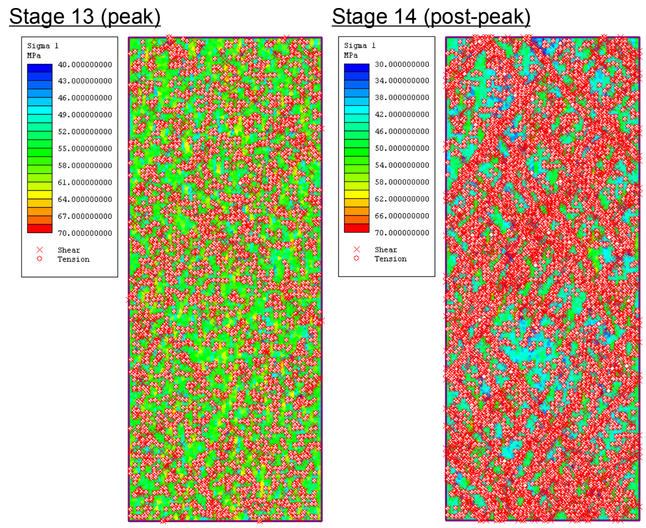

Fig. 5: Failure pattern at the peak stage and at the first post-peak stage.

3.1.2. Failure pattern

The localization of failure generates a failure pattern. A typical failure pattern obtained in the compressive loading model is shown in Fig. 5b: the main feature is a conjugate set of shear bands. One would expect that the angle between conjugate shear bands to be controlled by the friction angle, ϕ, of the material ($\phi = 45°$ in our case). Mohr-Coulomb theory predicts that the angle between the conjugate shears and the angle of friction is related by the following equation:

$$\phi[°] = 90 - 2\theta[°] \qquad (1)$$

where θ is the half angle between the conjugate set of shears.

Measurements of the conjugate shears angle for the ten models with same material properties but different heterogeneities mapping lead consistently to the same $\theta = 36°$ corresponding to an unexpectedly low $\phi = 18°$. Series of models with various friction angles were run and systematically, the friction angle calculated from Eq. 1 was low. Additional model runs were performed using different meshes and elements types (e.g. regular meshes, quadrilateral elements), all leading to consistent but unexpected results of low corresponding friction angle which discards the hypothesis that the mesh geometry is the controlling factor of the failure pattern. A possible explanation of the discrepancy between friction angle and observed failure pattern could be that the predominance of tensile states of stress in the models apparently brings effective state of stress to a level where frictional resistance cannot fully develop.

3.1.3. Effect of the amplitude of the variability

Compression loading simulations were repeated with modulus variability ranging from 20±0.001 GPa to 20±4 GPa, i.e. 0.005% to 20% of the mean modulus.

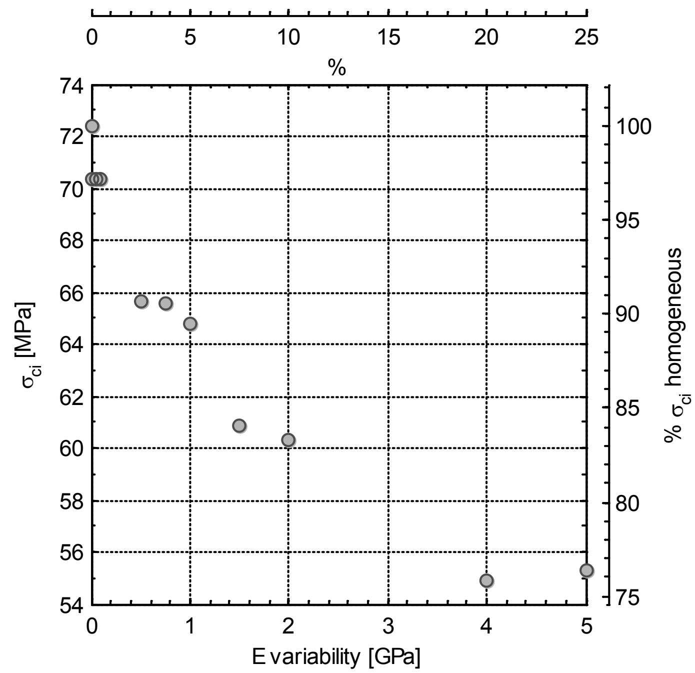

Fig. 6: influence of modulus variability on peak strength.

Results suggest that below a certain amount of variability, i.e. ±0.5 GPa in this case, the model behaves similarly to a homogeneous model. Above this threshold, variability starts to influence the results; particularly the peak strength as well as the brittleness (the magnitude of the post-peak stress drop) decrease with increasing variability. Fig. 6 illustrates the decrease of peak strength with increasing variability.

3.1.4. Variability following beta distributions

Up to this point, the assumed variability followed a uniform distribution. In reality, however, extreme values would be much less frequent than the median value (bell-like distribution). Beta distributions are convenient to use for rock properties like strength or modulus, because they allow the generation of bounded bell-like distributions. Contrary to the normal distribution for example, beta distributions are bounded, which avoids unrealistic events such as negative strength or modulus values. The shape of the beta distribution is controlled by two parameters: $\beta(a,b)$. Here only symmetrical distributions ($a = b$) are considered. With $a = b = 1$, a uniform distribution is generated. Increasing the distribution parameters reduces the dispersion of the distribution and thus generates more homogeneous models (see Fig. 7). In addition to the dispersion parameter, the data range must be specified. A symmetric beta distribution is then completely described using three parameters: $\beta(a, l, h)$ with a dispersion parameter a, a lower bound l, and an upper bound h.

Generally speaking, the effect is similar that the one obtained with the uniform distributions: increase in

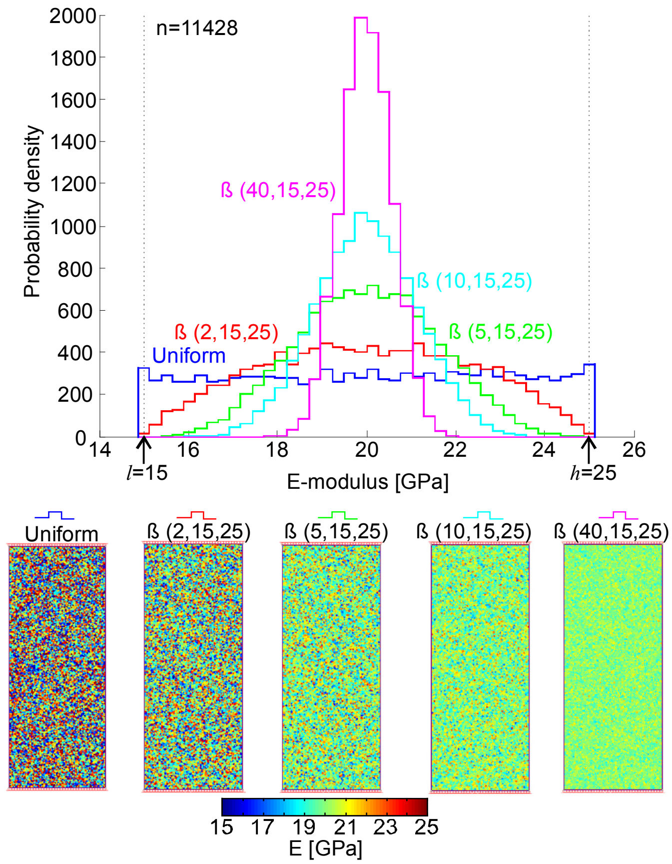

Fig. 7: Change of modulus variability dispersion in samples using beta distributions.

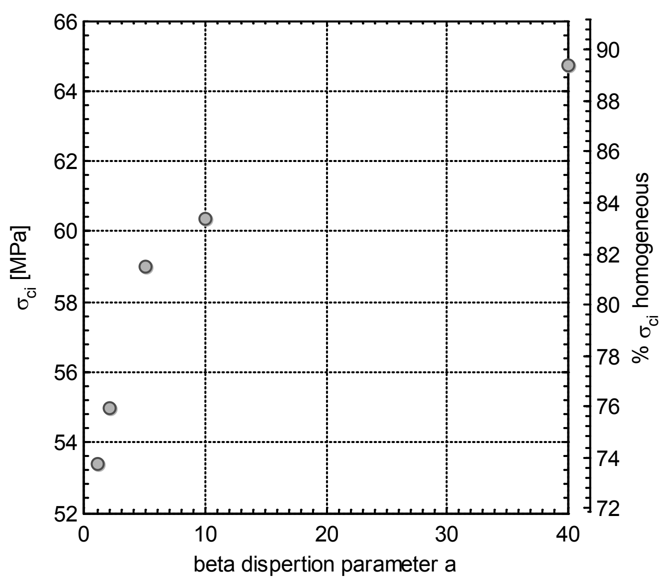

Fig. 8: Effect of heterogeneity dispersion on peak strength

sample heterogeneities decreases peak strength as can be seen on Fig. 8.

3.1.5. Bimodal distribution and strength parameters variability

In all tests presented up to this point, only E was varied. All inputs parameters though are variable. In practice, it would be fairly difficult to properly characterize the variability of each parameter and their correlations (e.g. correlation between strength of modulus). Also rock are most of the times made of three or more mineralogical components which will lead to combined distributions (bimodal, trimodal, …). An attempt to include this sort of complexity was made for one of the model run, where variability in modulus, cohesion, residual cohesion and tensile strength was included, all following bimodal distributions (see Fig. 9).

Results of this model run generated qualitatively similar result than by varying E only, but with some noticeable differences:

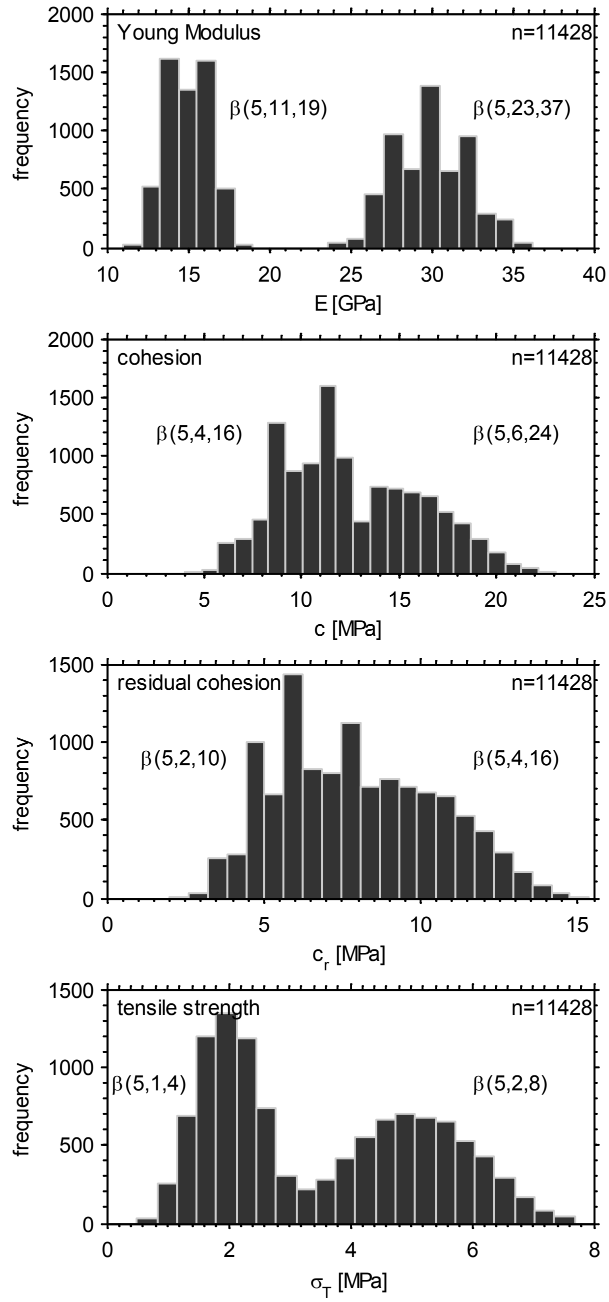

Fig. 9: Modulus, cohesion, residual cohesion and tensile strength distributions for bimodal models.

- Tensile fracturing is the predominant process, probably due to the relatively low selected tensile strength.
- The peak strength is about 50% of the average σ_{ci} for the weaker material of the bimodal distribution.
- Non-linearity (softening) precedes peak-strength and reduction to residual strength is progressive

These characteristics will be very dependent of the modelling input and thus cannot be generalised for any complex combinations of input parameters.

3.2. Confined compression loading simulation

Confined compression test were performed by adding constant pressures on the model's vertical boundaries. This geometry and loading conditions are similar to triaxial tests, but not exactly equivalent due to the 2D (plain strain) assumption. The out of plane stress in the presented model is always the intermediate stress. Two set of models were run:

(i) Homogenous models with identical properties than the model presented in Fig. 1.

(ii) Heterogeneous models with same properties than homogenous models except for the Young modulus which varies uniformly between 15 and 25 GPa.

Confinement level where applied from 0 to 70 MPa. Typical failure pattern for heterogeneous (ii) sample is presented on Fig. 10: with increasing confinement, tensile failure vanishes and failure is controlled by shear mechanisms. Also failure is localised into narrower shear bands.

Strain-stress curves for both model and all confinement pressures are presented in Fig. 11. Heterogeneities don't only reduce the peak strength but also influence the post-peak behavior, rendering it more ductile. Note that in the case of the homogeneous model, sudden failure of all elements simultaneously generates numerical instabilities in the post peak domain. When displaying the peak strength for each confinement in the σ_3 - σ_1 space, the input parameters (c=15 MPa, Φ=45° and σ_{ci}=72.4 MPa) are recovered as expected for the homogeneous model (Fig. 12). For the heterogeneous model, a Mohr-Coulomb (linear trend of blue squares on Fig. 12) still applies, but with reduced properties (c=13.1 MPa, ϕ=18° and σ_{ci}=53.9 MPa, obtained by best llinear fit through the data of Fig. 12), even if the individual elements have the same strength characteristics as those of the homogeneous model: heterogeneities on modulus alone generate a significant reduction of the overall strength of the model throughout the confinement range.

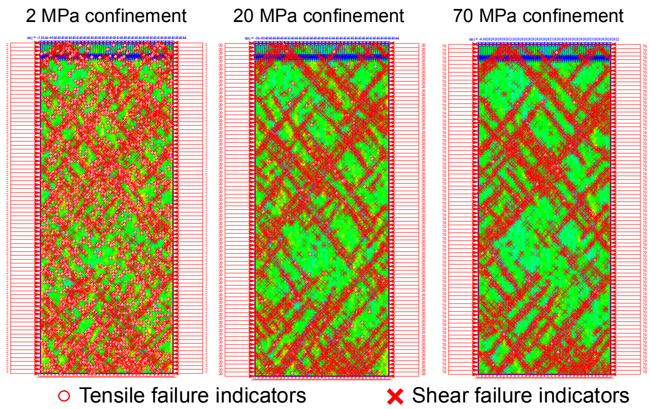

Fig. 10: Failure pattern in heterogeneous samples (ii) under 2 MPa, 20 MPa and 70 MPa confinements.

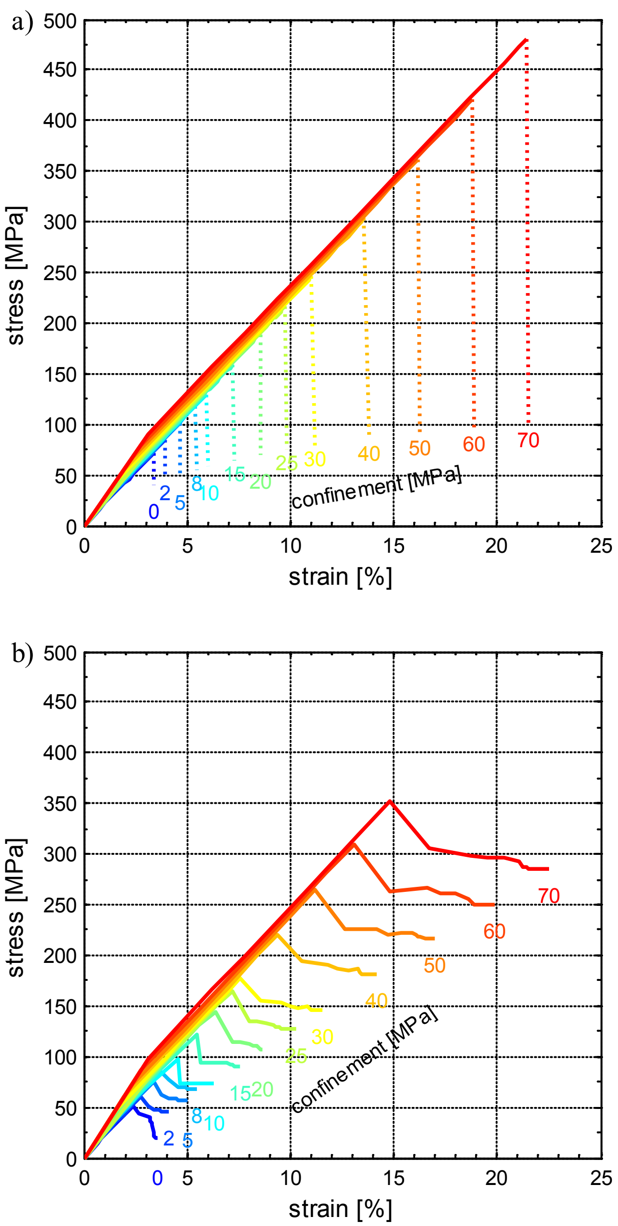

Fig. 11: Strain-axial stress curves for a) homogeneous models (i) and b) heterogeneous models (ii).

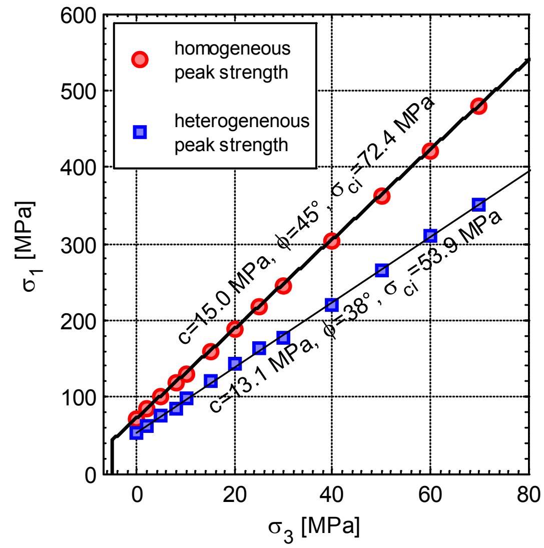

Fig. 12: Peak strength at various confinement for the homogeneous model (red circles) and heterogeneous model (blue squares). Material parameters (c, Φ and σ_{ci}) are obtained by evaluating the linear best fit through the data series.

3.3. *Circular opening*

The effect of heterogeneities on the stresses and failure distribution around a circular opening was also examined. A Phase2™ model with a 100 mm circular opening (borehole) was built, including concentric zoning in order to control the mesh homogeneity. Three material characteristics were used:

- **(i)** Homogenous material with identical properties than presented in Fig. 1.
- **(ii)** Heterogeneous material with same properties than homogenous models except for the Young modulus which varies uniformly between 15 and 25 GPa.
- **(iii)** A homogenous material with properties equivalent to the heterogeneous material as derived on Fig. 12, e.g. $c = 13.1$ MPa and $\phi = 38°$.

Initial stress conditions where $\sigma_1 = 40$ MPa, $\sigma_z = 35$ MPa, $\sigma_3 = 30$ MPa. Excavation was simulated by reducing internal pressure to zero over 20 steps.

The results of these analyses are presented on Fig. 13. The presence of modulus heterogeneities influences the stress pattern by promoting tensile conditions around the opening. This change in stress condition affects also the failure pattern. Interestingly, the depth of yield doesn't increase in the heterogeneous case, but the overbreaks opening angle almost doubles, covering most of the excavation circumference. This behaviour corresponds well with observed pattern in spalling of brittle rocks and has practical consequence on the support strategy (bolting length and pattern) [10, 11].

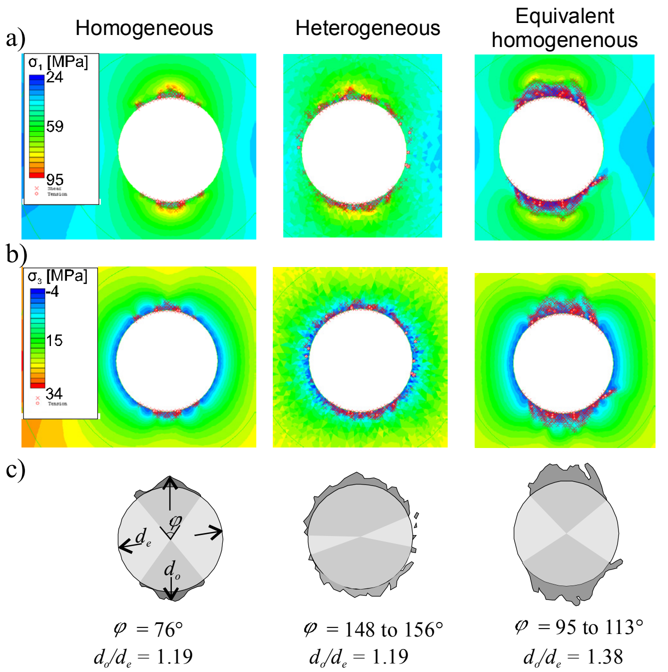

Fig. 13: Results of circular opening modeling for, from left to right, material parameters (i), (ii) and (iii): a) Maximum principal stress magnitude contours; b) Minimum principal stress magnitude contours; and c) yield geometry.

Another interesting output from this analysis is the test of the equivalent material properties concept. Rock mass classification scheme like GSI use this concept to derive strength parameters for the rock mass: with increasing joints and alteration, i.e. heterogeneities, the strength of the rock mass is reduced from intact properties to properties which should be equivalent to those of the rock mass. However, comparison of the yield pattern for the heterogeneous and equivalent homogeneous model (Fig. 13) shows significant discrepancies. For the equivalent homogenous model, the overbreaks opening angle increase moderately, but the depth of failure increases significantly. This significant difference of behavior between both models suggests that the equivalent material properties concept used by the GSI classification system could be flawed when brittle (tensile) failure process occur: heterogeneities and their effect on rock behavior cannot be captured by equivalent homogeneous models.

The practical consequence is that classification system like GSI must be used with extreme care when applied to brittle rocks with large modulus heterogeneities promoting tensile failure conditions. In such conditions, it will be more appropriate to explicitly model the presence of heterogeneities. Fortunately, current development of hardware and software, provide tools that allow for the explicit modeling of heterogeneities, either by simple approach as presented here, but also with more sophisticated approaches as suggested in [13, 14].

3.4. *Sample unloading*

The last case simulated for this paper concerns the unloading of confined rock sample to zeros stresses. This stress path is achieved each time a core is retrieved from depth. It is well known than when retrieving cores from high stress conditions, tensile damage occurs in the core potentially generating discing [12]. Heterogeneities are believed to significantly contribute to the creation of tensile conditions in the core leading to core discing.

The model presented here start in homogenous initial stress conditions $\sigma_1 = 55$ MPa, $\sigma_z = 55$ MPa, $\sigma_3 = 25$ MPa, with stress boundary conditions. Stress at the model boundaries are then decreased to zeros over 20 stages. Material properties (ii) were used. Due to the modulus heterogeneities, dispersion of the stress conditions in a similar fashion than in the loading case, eventually generate sufficient tensile conditions to generate failure.

The state at the last but one unloading stage is presented in Fig. 14. The modulus heterogeneities generated sufficient tensile stress conditions to initiate tensile yield. Tensile yielding zone are oriented perpendicular to the initial maximum stress orientation. However, the complex stress path ahead, at the tips and within the core barrel is not captured in this analysis. Hence, the observed damage pattern would significantly differ in a core. Nevertheless, when sampling in relatively high stress conditions, heterogeneities alone may generate sufficiently tensile stresses to damage the sample as was observed by [12]. Thus in these conditions one must expect that sample strength underestimates the actual in-situ rock strength.

4. DISCUSSION AND CONCLUSIONS

The introduction of heterogeneities into numerical analyses allows the capture of important mechanism like

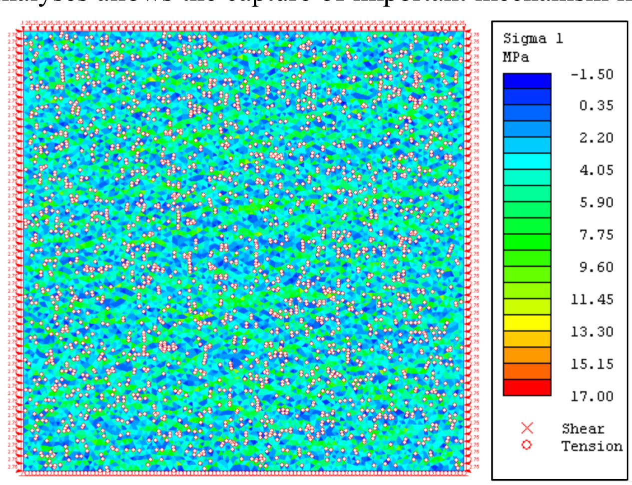

Fig. 14: Maximum stress magnitudes and yield pattern at the last but one unloading stage (boundary stresses $\sigma_1 = 2.75$ MPa, $\sigma_3 = 1.25$ MPa). Initial stress conditions were σ_1 left to right and σ_3 top to bottom.

damage initiation, accumulation and localization that homogeneous models fail to reproduce. The few case examples presented here suggest that the effects of heterogeneities are significant and have major practical consequence such as, for example, on ground support strategies or rock strength determination. The approach developed here is simple enough that the consideration of heterogeneities in numerical analyses could not be restricted to research tools. However, to systemically introduce heterogeneities in the design process, our ability to characterize and quantify heterogeneities needs to be enhanced such that realistic parameter variation models can be employed.

The approach adopted here also has some evident limitations inherent to the FEM code used as base (Phase2™). Particularly it doesn't capture properly tensile crack propagation which leads to unexpected failure pattern in compression loading simulation test. Some other restriction of the current approach (e.g. control on the mapping of heterogeneities) could easily be improved by applying methods developed for example in [9].

However, even with the rather simplistic approach presented here, some important conclusions can drawn on the effect of heterogeneities on rock behavior:

- Heterogeneities in modulus alone are sufficient to generate significant strength reduction of the rock.
- Limited modulus variability (coefficient of variation smaller than 1.5%) is sufficient to generate rock behaviour that is highly affected by induced, internal tensile stress conditions.
- Heterogeneities in modulus and/or peak strength parameters don't affect the peak strength only, but have important consequences on the post-peak behavior (strain softening characteristics).
- Heterogeneities generate a change in the dominating damage accumulation and failure process from tensile mechanism at low confinement to shear mechanism at higher confinement.
- The geometry of yield around openings is significantly affected by heterogeneities. This may influence the appropriate ground support strategy.
- The homogeneous equivalent concept to simplify the modeling of heterogeneities as used for example in rock mass characterization approaches such as GSI could be flawed.
- Modulus heterogeneities alone are sufficient to generate tensile damage when taking core samples from relatively high stress conditions. This may influence the samples strength and lead to an underestimation of the in-situ rock strength at depth.

5. NOMENCLATURE

Symbol	Description	Unit
d	Grain equivalent diameter	mm
d_e	Nominal excavation diameter	m
d_o	Excavation diameter including overbreaks	m
φ	Overbreaks opening angle	°
$\beta(a,l,h)$	Symmetric Beta distribution parameterized by its dispersion a, its lower bound l, and its upper bound h	-
E	Young Modulus	GPa
ε	Strain	‰
ν	Poisson ratio	-
c	Peak cohesion	MPa
c_r	Residual cohesion	MPa
ϕ	Peak friction	°
ϕ_r	Residual friction	°
θ	half angle between conjugate set of shears	°
σ_{ci}	Uniaxial compressive strength of intact rock	MPa
σ_T	Tensile strength	MPa
σ_1	Major principal stress	MPa
σ_3	Minor principal stress	MPa
σ_z	Out of plane stress (plain strain assumption)	MPa

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