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Technical Note/

Confined Flow into a Tunnel during Progressive Drilling: An Analytical Solution

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Abstract

A convolution integral is developed to evaluate transient, drilling speed–dependent discharge rates into a tunnel gradually excavated in a homogeneous infinite aquifer. Comparison with the classical instantaneous-drilling evaluation commonly used in practice reveals drastically reduced early-time discharge rates, higher maximum rates, and similar long-term rates. Dimensionless-type curves are provided to help assess total discharge sensitivity to drilling time and predict safer maximum flow rates.

Introduction

Prediction of ground water discharge rates into tunnels excavated in saturated geological materials constitutes an important aspect of the design and safety issues that must be resolved prior to excavation. Such predictions can be attempted by means of numerical models when regionalized data are available, or more quickly using analytical models based on simple idealized systems. Due to the relatively high investments required by a numerical approach, and given the recurrent scarcity and uncertainty of hydrogeological data affecting large-scale results, analytical approaches are often preferred in practice.

By accounting for typical flow configurations and boundary conditions, a large variety of formulas have thus been developed that help predict tunnel discharge rates and assess their sensitivity to system parameters (see, e.g., Goodman et al. 1965; Chisyaki 1984; El Tani 2003). However, these analytical solutions generally only yield relatively small equilibrium steady-state values. The only solution available for confined, transient conditions is that of Jacob and Lohman (1952), which assumes a tunnel installed instantaneously over its whole length and results in unrealistically high initial inflow values.

In reality, a tunnel is drilled progressively. Discharge increases from an initial value at zero to a maximum followed by a period of recession. However, due to the lack of relevant analytical solutions, this drilling speed–dependent process can only be evaluated at present using relatively time-consuming, nonexhaustive numerical approaches (see, for example, Molinero et al. 2002).

The solution presented in this article is meant to complete the analytical modeler's toolbox with appropriate closed-form solutions and dimensionless-type curves that address this issue. This allows an estimation of realistic, nonempirical drilling speed–dependent design and safety maxima that may be anticipated during excavation.

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Instant-Drilling Solution

An analytical formula for the transient discharge rates at a well under constant drawdown was published by Jacob and Lohman (1952), who applied the diffusion equation solution of Smith (1937) to ground water dynamics. These authors assumed an idealized, infinite, homogeneous aquifer of finite thickness with perfect radial flow toward a fully penetrating well and hydrostatic initial conditions.

As illustrated in Figure 1a, this formula can also be used to predict analogous transient inflows into a tunnel intersecting a subvertical, permeable layer over a finite length L (see, for example, Maréchal and Perrochet 2003). In this context, the total discharge is

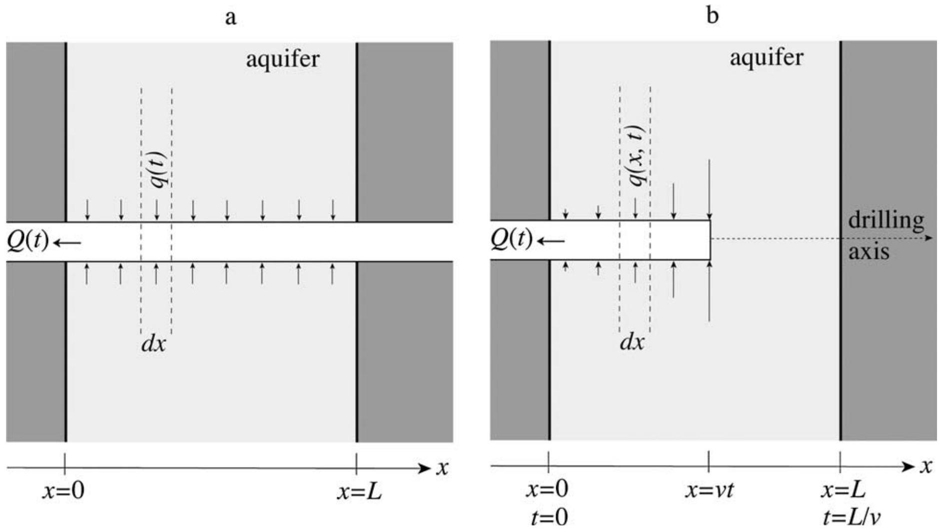

Figure 1. Tunnel of length L through a subvertical aquifer. (a) Uniform specific discharge assuming instantaneous drilling. (b) Nonuniform specific discharge resulting from progressive drilling.

$$Q(t) = 2\pi K L s_o G(\alpha), \quad \alpha = \frac{Kt}{S_s r_o^2} \tag{1}$$

where the symbols stand for aquifer hydraulic conductivity (K), specific storage coefficient (S_s), time (t), tunnel radius (r_o), specified drawdown at the tunnel (s_o), and tunnel length over which a permeable zone is encountered (L). In Equation 1, the production function $G(\alpha)$ (dimensionless discharge) is

$$G(\alpha) = \frac{4\alpha}{\pi}\int_0^\infty u e^{-\alpha u^2}\left\{\frac{\pi}{2} + \tan^{-1}\left(\frac{Y_o(u)}{J_o(u)}\right)\right\} du \tag{2}$$

where α is dimensionless time, J_o and Y_o are first- and second-kind, zero-order Bessel functions, respectively, and u is a dummy variable.

This can be, with a very good approximation, replaced by

$$G(\alpha) = \frac{1}{\ln(1 + \sqrt{\pi\alpha})} \tag{3}$$

resulting from the alternative solution of Perrochet (2005), and providing excellent accuracy over the whole range of dimensionless times (errors less than ~10^{-3} on dimensionless discharge).

Equation 1 is frequently used in practice and implies the uniform specific discharge

$$q(t) = \frac{Q(t)}{L} = \frac{2\pi K s_o}{\ln\left(1 + \sqrt{\frac{\pi K t}{S_s r_o^2}}\right)} \tag{4}$$

over the distance L, as illustrated in Figure 1a. From the initial time onward, it predicts a monotonically decreasing discharge rate into a tunnel drilled instantaneously and is therefore believed to provide conservative overestimates. As shown subsequently, this assumption is certainly accurate at early times, but it is erroneous at later times. Moreover, starting with an infinite initial value, Equation 1 does not allow the prediction of an objective maximum.

Insights were also given in Perrochet (2005) as to the time during which Equation 1 may hold. This time must be smaller than the time needed for the drawdown perturbation to reach a known aquifer boundary at a distance R from the tunnel axis (i.e., impervious layer, surface water body, etc.). For relatively large systems ($R \gg r_o$), this limiting time can be evaluated by

$$t_{\lim} = \frac{1}{\pi e}\frac{S_s R^2}{K} \tag{5}$$

For larger times, the flow conditions depart from those assumed initially and Equation 1 gradually loses accuracy.

Progressive-Drilling Solution

The simplified analytical expression of the specific discharge in Equation 4 enables further mathematical manipulations such as that required to evaluate the temporal evolution of discharge rates during drilling.

Consider the progressive drilling of a permeable zone at an average drilling speed v, as illustrated in Figure 1b. At time t, the drilling front is located at the distance vt, and the time at which a position $x < vt$ was reached is x/v ($x > 0$, $v > 0$). Hence, the time elapsed since that position was reached and during which inflows occurred at that location is $t - x/v$, making transient specific discharge a nonuniform function of space over the distance vt.

Assuming now that perfect radial symmetry of the flow is preserved at all times and that the aquifer is not significantly perturbed beyond the drilling front, the specific discharge at any location $x < vt$ along the tunnel axis can be expressed from Equation 4 by

$$q(x,t) = \frac{2\pi K s_o}{\ln\left(1 + \sqrt{\frac{\pi K}{S_s r_o^2}\left(t - \frac{x}{v}\right)}\right)}, \quad t - \frac{x}{v} > 0 \tag{6}$$

indicating gradually increasing inflows from the entry point of the permeable zone ($x = 0$) to the vicinity of the drilling front ($x = vt$). The two aforementioned simplifying assumptions may hold particularly well in case of subvertical zones of relatively high permeability intersecting the tunnel at a high angle or when a reasonably steady drilling speed is maintained.

The total discharge into the tunnel during and after excavation through the permeable zone may then be expressed as

$$Q(t) = \int_0^{vt} \frac{2\pi K s_o \mathrm{H}(L-x)}{\ln\left(1+\sqrt{\frac{\pi K}{S_s r_o^2}\left(t-\frac{x}{v}\right)}\right)} \mathrm{d}x \qquad (7)$$

where the Heaviside step function $\mathrm{H}(L - x)$ has been introduced to account for the finite size of the permeable zone and for the assumption that specific discharge vanishes at locations $x > L$.

In contrast to Equation 1, the cumulative integral in Equation 7 indicates that transient total discharge monotonically increases from the initial time to the drilling time L/v where a maximum is reached. Beyond this time, the total discharge decreases monotonically since the drilling of the permeable zone has been completed.

The time during which Equation 7 may hold can also be taken as t_{lim} defined in Equation 5. Beyond this limiting time, the specific discharge at locations $x < v(t - t_{\mathrm{lim}})$ starts to be gradually influenced by possible boundary effects at the distance R from the tunnel axis. Defining the associated temporal variables in a manner similar to that used for α in Equation 1 gives

$$\alpha_d = \frac{K}{S_s r_o^2}\frac{L}{v}, \quad u = \frac{K}{S_s r_o^2}\frac{x}{v} = \alpha_d \frac{x}{L}, \quad \alpha_{\mathrm{lim}} = \frac{K t_{\mathrm{lim}}}{S_s r_o^2} = \frac{1}{\pi e}\frac{R^2}{r_o^2} \qquad (8)$$

where α_d is the dimensionless drilling time, u is the dimensionless time at which the location x is reached, and α_{lim} is the dimensionless limiting time.

Substituting the previous definitions in Equation 7 and dividing by $2\pi K L s_o$ yields the dimensionless discharge rate

$$Q^*(\alpha) = \frac{Q(\alpha)}{2\pi K L s_o} = \frac{1}{\alpha_d}\int_0^{\alpha} \frac{\mathrm{H}(\alpha_d - u)}{\ln\left(1+\sqrt{\pi(\alpha-u)}\right)} \mathrm{d}u \qquad (9)$$

as a convolution of the production function $G(\alpha)$ and the Heaviside step function.

Integration of the right-hand side of Equation 9 results in

$$Q^*(\alpha) = \frac{F(\alpha)}{\alpha_d}, \quad \alpha < \alpha_d \qquad (10)$$

$$Q^*(\alpha) = \frac{F(\alpha)}{\alpha_d} - \frac{F(\alpha-\alpha_d)}{\alpha_d}, \quad \alpha > \alpha_d \qquad (11)$$

$$F(\alpha) = \int_0^{\alpha} G(u)\mathrm{d}u = \frac{2}{\pi}\left(\mathrm{Ei}(2\ln(1+\sqrt{\pi\alpha})) - \mathrm{Ei}(\ln(1+\sqrt{\pi\alpha})) - \ln(2)\right) \qquad (12)$$

where Ei is the exponential integral function.

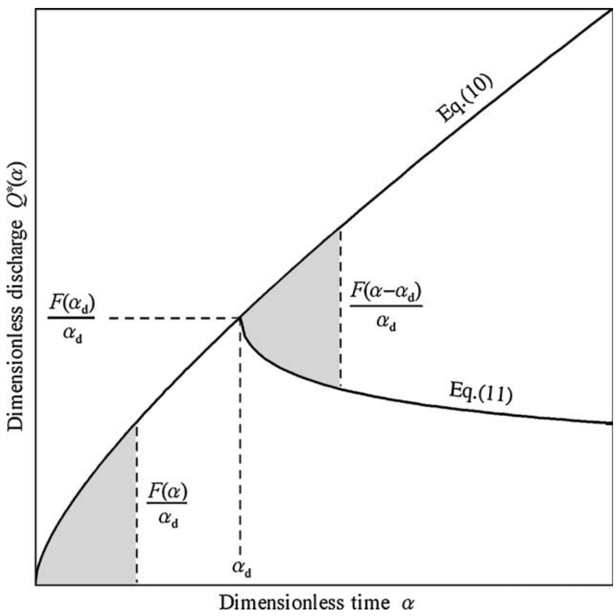

Figure 2. Graphical representation of the dimensionless discharge $Q^*(\alpha)$ in Equations 10 and 11 with maximum at $\alpha = \alpha_d$.

The temporal evolution of $Q^*(\alpha)$ is represented in Figure 2, with maximum discharge $F(\alpha_d)/\alpha_d$ occurring when penetration of the permeable zone is completed. The two equal-size shaded areas suggest that the process could also be addressed by means of the superposition principle, with two tunnels of infinite length. In this case, the second tunnel would be activated with negative drawdown after time α_d. The superposition principle could also be used to account for a permeable zone with several sections of contrasted hydrodynamic parameters and drilling speeds.

Type curves are provided in Figure 3 for a range of dimensionless drilling times. The production function

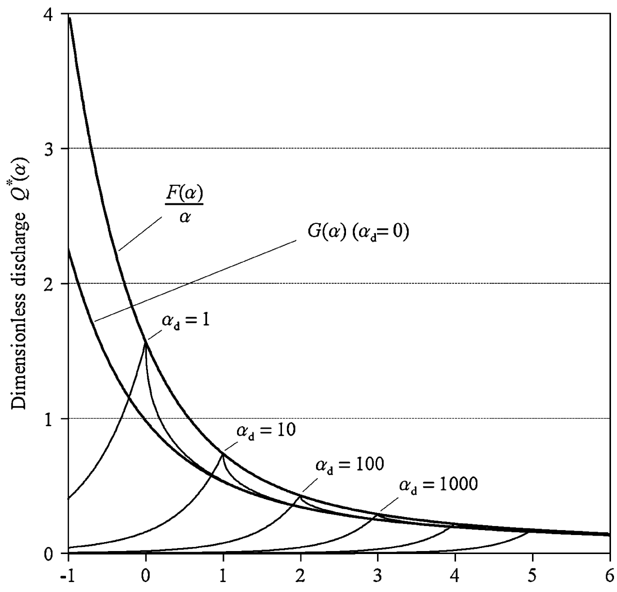

Figure 3. Dimensionless-type curves for estimating transient and maximum discharge into a tunnel for various drilling times α_d.

$G(\alpha)$ corresponding to instant drilling ($\alpha_d = 0$) is also shown for comparison as well as the line joining the drilling speed–dependent maxima. The latter is expressed by

$$Q^*_{\max}(\alpha) = \frac{1}{\alpha}\int_0^{\alpha} G(u)\mathrm{d}u = \frac{F(\alpha)}{\alpha} \qquad (13)$$

Notwithstanding the relatively crude additional assumptions enforced in Equation 6, the type curves in Figure 3 indicate transient discharge rates that are more realistic than those obtained with the classical instantaneous-drilling approach.

In the beginning of tunnel drilling, the length of the tunnel is zero, so the discharge rate is zero. This contrasts with the results obtained from the classical instantaneous-drilling approach, which predicts infinite initial discharge. Instead of the very large short-time inflow rates predicted by the classical approach, the progressive-drilling solution generates values that increase more or less rapidly depending on the penetration rate, as would be expected.

After about four-fifth of the drilling time, discharge rates start to exceed those predicted at the specified dimensionless time value using the classical method. At the time when drilling is completed ($\alpha = \alpha_d$), this excess is about one-third at short drilling times and decreases gradually for larger drilling times. Hence, the flow rates predicted by the classical approach using time values corresponding to the expected duration of tunnel drilling are underestimated during drilling of the last fifth of the tunnel length and thereafter.

Drilling speed–dependent maxima are given by the function $F(\alpha)/\alpha$. This function allows the designer to evaluate objective and more realistic upper limits and to anticipate the total discharge rate, in addition to assessing their sensitivity to drilling times.

Concluding Remarks

A new analytical solution has been developed to estimate total discharge flowing into a homogeneous tunnel during drilling. Based on simple assumptions, this solution should replace the classical approach as it yields transient results based on gradual drainage activation; this is much more compatible with the real drilling process. The problem of large unrealistic early-time discharge rates encountered in the classical approach is overcome as well as the inability to find reasonable maximum inflow rates. Moreover, the approach shows that drilling speed–dependent maxima occur when the drilling of the permeable zone is completed and thus enables straightforward evaluation of these maxima.

Further analytical work is presently in progress to account for composite tunnel sections with varying drilling speeds and hydrodynamical parameters. First applications to exploratory sections of new Alpine tunnels confirm the validity of the approach and will be reported in due time.

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