Sample coordination methods and systems for establishment surveys*

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Abstract

Sample coordination has been a topic of interest in the world of establishment surveys, from well before the first International Conference on Establishment Surveys (ICES-I), where Ohlsson (1995) summarised the state of the art of methods using permanent random numbers (PRNs). A range of procedures have been proposed in the literature for sample coordination (divided into PRN and non-PRN methods), and these have given rise to several implementations in different countries. The national statistical offices of different countries currently use so-called 'sample coordination systems'. 'Sample coordination methods' and 'sample coordination systems' represent, in our opinion, two different concepts. The existing literature does not distinguish between them. Moreover, a definition of a sample coordination system has not yet been provided, while the term is widely used. This chapter aims to review these two concepts and to underline the similarities and distinctions between them. First, we review the main existing methods for sample coordination, and highlight their strengths and weaknesses. Next, we enumerate the components of a coordination system and review some of those currently being used in different countries. Finally, we distinguish 'sample coordination methods' and 'sample coordination systems'.

1 Introduction

Establishment statistics are important tools for economic management, and the need to obtain information as the basis for such statistics has led to legislation in many countries making re-

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sponse to establishment surveys compulsory. The converse of this requirement has been the management of survey burdens, a concept articulated by Bradburn (1978), so that the information requirement from government is limited. Burden management has manifested in two ways. First, through the reduction of numbers of surveys, sample sizes and numbers of questions, all aiming to reduce the overall burden; and second, through procedures to spread the burden so that any individual establishment is selected in as few surveys as possible at a given moment. A major tool in this second approach has been the use of sample coordination approaches.

Sample coordination refers to methods used to optimize the overlap of two or more samples. The samples can be selected on different occasions in a repeated survey, or over different surveys. If a survey's objective is to estimate change across time, or to reduce the costs associated with recruiting a new sample unit, the overlap between samples should be maximized. Sometimes one would like to reduce the probability of having the same sampling unit selected across different samples, and therefore to limit its response burden. In such cases the objective is to minimize the overlap between samples selected over time or over different surveys. In the first case, one advocates *positive sample coordination*, while in the second one, *negative sample coordination*.

Sample coordination started with the pioneer works of Keyfitz (1951) and Raj (1956). As Ernst (1999) underlined, different names have been used in the literature such as 'overlapping maps' (Raj, 1956), 'optimal integration of surveys' (see for instance Pathak and Fahimi, 1992), and more recently 'sample coordination', 'sampling coordination' or 'coordinated sampling'.

Various procedures have been proposed for sample coordination (such as permanent random number procedures, methods based on mathematical programming, etc.), and these have given rise to several implementations in different countries. The national statistical offices of different countries currently use so-called 'sample coordination systems'. 'Sample coordination methods' and 'sample coordination systems' represent, in our opinion, two different concepts. The existing literature does not distinguish between them. Moreover, a definition of a sample coordination system has not yet been provided, while the term is widely used. This chapter aims to review these two concepts and to underline the similarities and distinction between them. To do this, we proceed as follows. In Sections 2–3 we review the main existing methods for sample coordination, and highlight their strengths and weaknesses. Section 4 provides the components of a sample coordination system; we review some of them currently being used in different countries in Section 5. In Section 6 we distinguish 'sample coordination methods' and 'sample coordination systems'. In Section 7 we provide our conclusions.

2 Sample coordination

2.1 Notation and definitions

Establishment survey frames define the populations to be surveyed. These populations change over time: some units disappear from the populations (they are called 'deaths'), other units appear ('births'), still others are split into several other units ('splits'), merge, or are taken over ('acquisitions'). Probability samples are selected from these frames as the basis for collecting information, and some businesses are included repeatedly in surveys because they are large enough that their contribution is vital (these are completely enumerated), or because including the same units increases period to period correlation of the estimates and reduces the variance of estimates of change. Sometimes, to reduce the response burden, some units are rotated out of the sample at each transition, and replaced with new units.

Let us consider a simple case of two overlapping finite populations of units denoted by U_1 and U_2, respectively. From U_1 one selects a sample s_1, and from U_2 a sample s_2, using the sampling designs p_1 and p_2, respectively. The expected sample sizes for s_1 and s_2 are denoted respectively by n_1 and n_2. A bivariate sample $s = (s_1, s_2)$ is selected from $U_1 \times U_2$, using a joint sampling design p, with marginal sampling designs p_1 and p_2. The samples s_1 and s_2 are said to be *coordinated* if

$$p(s_1, s_2) \neq p_1(s_1)p_2(s_2),$$

that is, the samples are not drawn independently (see Cotton and Hesse, 1992b; Mach et al., 2006). The main goal of sample coordination methods is to optimize the overlap between samples.

The size of the *overlap* between s_1 and s_2, denoted by c, represents the number of units common to s_1 and s_2. It is in general a random variable having expectation

$$E(c) = \sum_{k \in U} \pi_{k,12},$$

with $\pi_{k,12} = P(k \in s_1, k \in s_2) = \sum_{(s_1,s_2), k\in s_1, k\in s_2} p(s_1, s_2), \forall k \in U$, and $U = U_1 \cup U_2$ is the so-called 'overall population'. In positive coordination, the goal is to maximize $E(c)$, while in negative coordination it is to minimize it. In negative coordination, it is expected that minimization of $E(c)$ will reduce the perceived response burden incurred by units which are potentially selected in both samples, by minimising the probability to be selected in both samples, with a concomitant reduction in the probability to be selected in neither sample.

Let $\pi_{k1} = P(k \in s_1)$ and $\pi_{k2} = P(k \in s_2)$ be the first-order inclusion probabilities of unit $k \in U$ in the first and second samples, respectively. In order to manage the births in $U \setminus U_1$ and the deaths in $U \setminus U_2$, we assume that $\pi_{k1} = 0$ if $k \notin U_1$ and $\pi_{k2} = 0$ if $k \notin U_2$.

2.2 Methods for sample coordination

Various sample coordination methods have been proposed in the literature. For an overview on sample coordination methods, one can see, for instance, Ernst (1999), Mach et al. (2006), and the references therein. See also Nedyalkova et al. (2008), Nedyalkova et al. (2009), Grafström and Matei (2015). The extensive literature on sample coordination covers a wide range of sampling designs, from coordination for simple random samples without replacement to coordination for balanced samples (Tillé and Favre, 2004), and spatially balanced samples (Grafström and Matei, 2018).

An important approach in sample coordination is the use of *permanent random numbers* (PRNs). PRNs were introduced by Brewer et al. (1972) to coordinate Poisson samples, and have been widely used afterwards in other, different methods (Ohlsson, 1995). Brewer et al. (1972) provided the following method: one associates a uniform random number u_k drawn independently from the $U(0,1)$ distribution to each unit $k \in U$, which is used in the selection process for samples across all surveys or different time occasions. The numbers u_k are called 'permanent random numbers' since they are kept over time and over surveys for units which persist in the population. For a 'birth', a new PRN is assigned; for a 'death', the unit and its associated PRN are deleted from the corresponding survey frame.

One can roughly divide sample coordination methods into two categories: methods using PRNs, and methods that do not use PRNs. The main methods based on PRNs and some non-PRN methods are reviewed below. In their descriptions, we consider that s_1 and s_2 are selected with the same sampling method. However, one can also provide sample coordination using two different sampling schemes for s_1 and s_2.

2.3 Methods based on PRNs

Methods based on PRNs are divided below in two categories (*Poisson sampling with PRNs and its extensions* and *fixed ordered procedures with PRNs*) to overview the most important. Other PRN methods have also been developed in the literature; for instance, spatially balanced sampling with PRNs (Grafström and Matei, 2018).

a. Poisson sampling with PRNs and its extensions

- In *Poisson sampling with PRNs* (Brewer et al., 1972), a unit $k \in U_1$ is selected in the sample s_1 if $u_k < \pi_{k1}$; for a positive coordination between s_1 and s_2, $k \in U_2$ is selected in s_2 if $u_k < \pi_{k2}$. To achieve negative coordination between s_1 and s_2, $k \in U_2$ is selected in s_2 if $1-u_k < \pi_{k2}$. Using Poisson sampling with PRNs gives a high degree of coordination, as the bounds on the expected overlap size are reached; see Section 3.1. However, Monte-Carlo studies showed that the variance of the sample overlap can be large (see, for instance, Grafström and Matei, 2015, 2018, , for positive coordination). The main drawback of the method is that it provides samples of random size that increase the variance of the estimators compared to simple random sampling without replacement. Null sample sizes are also possible. If the inclusion probabilities are equal ($\pi_{k1} = n_1/N_1, \forall k \in U_1, \pi_{\ell 2} = n_2/N_2, \forall \ell \in U_2$), one obtains *Bernoulli sampling with PRNs.*
- *Collocated sampling* (Brewer et al., 1972, 1984) is a variant of Poisson sampling with PRNs, used to reduce the sample size variability and to avoid null sample sizes. For the first sample s_1, a random permutation $R = (R_k)'$ of the units in U_1 is provided, where R_k is the rank of unit k in the permutation. A single random number $\varepsilon \sim U(0,1)$ is generated. To each unit $k \in U_1$, the random number $t_k = (R_k - \varepsilon)/N_1$ is associated, where N_1 is the size of U_1. If $t_k < \pi_{k1}$, unit k is selected in s_1. Thus, t_k replaces u_k in the original Poisson with PRNs scheme. This modification allows a uniform spread of the population units, removing any potential clustering of u_k that can occur in small populations. The sample size becomes almost not random because the R_k are equally spaced in $[0,1]$. If $U_1 = U_2$, in positive coordination, selection of the second sample uses the same t_k, with updated probabilities $\pi_{k2}, k \in U_2$; for negative coordination, k is selected in s_2 if $1 - t_k < \pi_{k2}, k \in U_2$. Hesse (1999) pointed out that 'it is difficult to use this technique for several samples at the same time, unless they adopt the same stratification and the same definition of scope'. Brewer et al. (1984) discussed how to manage births and deaths that can occur between the two sample selections.

  Ohlsson (1995, p. 161) introduced *PRN collocated sampling.* The PRNs u_k are sorted in ascending order and the rank is addressed to each one of them. As before, a single random number $\varepsilon \sim U(0,1)$ is generated. For each unit $k \in U_1$, the random number $t_k = (Rank(u_k) - \varepsilon)/N_1$ is associated. If $t_k < \pi_{k1}$, unit k is selected in s_1 (t_k replaces u_k in Poisson with PRNs). For a positive coordination, the second sample is drawn using the same u_k and ε but with updated ranks and probabilities π_{k2} (for a negative coordination, $1-u_k$ and ε with updated ranks and probabilities π_{k2}). The coverage of births and deaths with PRN collocated sampling is discussed by Ernst et al. (2000).

- *Poisson mixture (PoMix) sampling* is a mixture of Bernoulli and Poisson sampling introduced by Kröger et al. (1999). The authors noted that it is 'suitable for business surveys with its [sic] often highly skewed populations'. Let $a \in [0, n_1/N_1]$ be a starting point. The PoMix inclusion probabilities are defined as

$$\widetilde{\pi}_{k1} = a + \left(1 - a\frac{N_1}{n_1}\right)\pi_{k1}, k \in U_1. \tag{1}$$

A unit $k \in U_1$ is selected in s_1 if one of the following conditions is fulfilled:

a. $0 < u_k \leq a$;
b. $a < u_k \leq 1$ and $\widetilde{\pi}_{k1} \geq (u_k - a)/(1-a)$.

The quantities $\widetilde{\pi}_{k1}$ are greater than a to avoid small $\widetilde{\pi}_{k1}$ values. If $a = 0$, one obtains Poisson sampling; if $a = n_1/N_1$, one obtains Bernoulli sampling. Other values of a yield a Poisson-Bernoulli sampling mixture, that is in fact Poisson sampling obtained by using the probabilities $\widetilde{\pi}_{k1}$ instead of π_{k1}. As in the case of Poisson/Bernoulli sampling, the drawback of this method is its random sample size. Kröger et al. (1999) conducted Monte Carlo simulation studies and showed that such mixtures provide estimates with smaller variance than Poisson sampling. The next sample s_2 is drawn as in Poisson sampling with PRNs, using the same u_k in positive coordination ($1-u_k$ in negative coordination) and the same value of a, but with updated $\pi_{k2}, k \in U_2$, n_2, and N_2 (the size of U_2).

- *Conditional Poisson sampling* (Hájek, 1964) is a fixed size sampling design having important theoretical properties, such as maximizing the entropy in the class of sampling designs with given first-order inclusion probabilities and fixed sample size.

The entropy of a sampling design $\tilde{p}(.)$ is defined as $-\sum_{\tilde{s}} \tilde{p}(\tilde{s}) \log(\tilde{p}(\tilde{s}))$, with the convention that $0 \log(0) = 0$, and $\tilde{s}$ being a possible sample (Hájek, 1981). A sampling design with high entropy speeds up the convergence of the Horvitz-Thompson estimator of the population total towards the normal distribution (and allows the construction of confidence intervals as usual; Berger, 1998). Furthermore, high entropy sampling designs allow the second-order inclusion probabilities to be approximated by the first-order inclusion probabilities, thus greatly simplifying the variance of the Horvitz-Thompson estimator.

Grafström and Matei (2015) introduced two methods to coordinate Conditional Poisson (CP) samples. The first method uses PRNs and a sequential implementation of CP sampling. The second method is a non-PRN method that uses a CP sample in the first selection and provides an approximate one in the second selection because the prescribed inclusion probabilities are not exactly respected. On the simulations performed by Grafström and Matei (2015), the second method provided results similar to Poisson sampling with PRNs for $E(c)$, but a smaller variance of c in positive coordination.

b. Fixed ordered procedures with PRNs refer to procedures using ordered quantities depending on PRNs, that draw samples of fixed size. We include in this category: simple random sampling without replacement with PRNs, synchronized sampling, order sampling, fixed size PoMix sampling and Ohlsson's exponential sampling.

- In *sequential simple random sampling* or *simple random sampling without replacement* (SRS-WOR) with PRNs the list of units is sorted in ascending or descending order of $u_k, k \in U_1$. The first sample is composed by the first or the last n_1 units in the ordered list. This method is described by Fan et al. (1962). A proof that this method generates a simple random sample without replacement is given for instance in Sunter (1977a) and in Ohlsson (1992). For positive coordination, the second sample is drawn using the same u_k, and sample size n_2; for negative coordination, the second sample is drawn using $1 - u_k$ and n_2.
- *Synchronized sampling* (Hinde and Young, 1984) assigns a PRN independently to each unit and selects units whose associated PRNs lie in an interval. The first sample of size n_1 is selected using SRS-WOR with PRNs. The used PRNs (in ascending order) lie in an interval $[b, e]$ (with b for 'beginning' and e for 'end'). For the next sample and positive coordination, the same interval is kept if the population doesn't

change. If births or deaths occur, or the sample size changes for s_2, the interval is adjusted: e is increased, extending the interval to the right to include more units, or b is increased to reduce the interval from the left to exclude units, until the desired sample size is obtained. This process can increase survey rotation rates, and the increase can be substantial if births and deaths are frequent. Negative coordination is achieved by giving samples fixed, disjoint selection intervals within the range of the the PRNs, and the selections of the samples take place wholly within these intervals (so the population must be sufficiently large to support the required sample sizes within these intervals). Brewer et al. (2000) comment that if the two surveys have different stratification, 'positive coordination can only be achieved by a manual choice of the selection intervals'.

- *Order sampling* (Rosén, 1997a,b) is a class of sampling designs with fixed sample size. Let $\lambda_{k1}, k \in U_1$ be the target inclusion probabilities, usually proportional to a size measure. To each unit $k \in U_1$ a continuous random variable $X_k = H^{-1}(u_k)/H^{-1}(\lambda_{k1})$ is associated, where $H(.)$ is a cumulative distribution function (cdf) defined on $[0, \infty)$. The first n_1 units in increased order of X-values form the first sample. The random variables $X_1, \ldots, X_k, \ldots, X_N$ each follow the same type of distribution, but are not necessarily identically distributed. Order sampling yields SRS-WOR samples when X_k are identically distributed.

  Various types of order sampling arise when the X_k follow different distributions:

  1. uniform order sampling or sequential Poisson sampling (Ohlsson, 1990, 1998), where $X_k = u_k/\lambda_{k1}$, and H is the cdf of the $U(0,1)$ distribution.
  2. exponential order sampling or successive sampling (Hájek, 1964; Rosén, 1997b), where $X_k = \ln(1-u_k)/\ln(1-\lambda_{k1})$, and H is the cdf of an exponential distribution with parameter 1.
  3. Pareto order sampling (Saavedra, 1995; Rosén, 1997a,b), where

     $$X_k = \frac{u_k/(1-u_k)}{\lambda_{k1}/(1-\lambda_{k1})},$$

     and H is the cdf of the Pareto distribution with shape parameter 2, and scale parameters u_k in the numerator and λ_{k1} in the denominator.

  Note that the inclusion probabilities π_{k1} are not equal to the quantities λ_{k1}. If λ_{k1}

are proportional to a size measure known for all units $k \in U_1$, the resulting order sampling is only asymptotically πps sampling, with a faster convergence for Pareto sampling (Rosén, 1997a).

For all types of order sampling, positive coordination uses the same u_k but $\lambda_{k2}, k \in U_2$ instead of λ_{k1} in the X_k definition to select s_2; in negative coordination, u_k is replaced by $1 - u_k$. For an application of Pareto sampling with PRNs, see Scholtus and van Delden (2016).

- *Fixed size PoMix sampling* (Kröger et al., 2003) is a mixture between order sampling and PoMix sampling. It was introduced to obtain a fixed sample size and to preserve the good properties of the PoMix sampling concerning the variance in skewed populations compared to Poisson sampling. The PoMix inclusion probabilities are defined as in (1). The method to obtain a sample is similar to order sampling. Different types of sampling arise by changing the parameter a and the values, depending on u_k, to be ordered. Ordered values of $u_k/\widetilde{\pi}_{k1}, k \in U_1$ and $a = 0$ gives sequential Poisson sampling; the same ordered values and $a = n_1/N_1$, gives SRS-WOR with PRNs. Finally, ordered values of

$$\frac{u_k/(1-u_k)}{\widetilde{\pi}_{k1}/(1-\widetilde{\pi}_{k1})}, k \in U_1$$

and any $a \in [0, 1]$, gives Pareto sampling. Note that the quantities $\widetilde{\pi}_{k1}$ are not equal to the inclusion probabilities π_{k1}.

The second sample is drawn using the same u_k for positive coordination ($1 - u_k$ for negative coordination), and $\pi_{k2}, k \in U_2$.

- *Ohlsson's exponential sampling* (Ohlsson, 1996) is a procedure that follows the same idea of ordering values depending on the PRNs. It is a method to select only one unit in a sample. It is assumed that $\sum_{k \in U_1} \pi_{k1} = \sum_{k \in U_2} \pi_{k2} = 1$. The quantities $\xi_k = -\log(1-u_k)/\pi_{k1}, k \in U_1$ are computed. The first unit in the sorted order of ξ_k is selected in s_1. The name of the procedure is given by the distribution of the ξ_k, which is exponential with parameter $1/\pi_{k1}$; see also Ernst (2001). For positive coordination, s_2 is selected using u_k ($1 - u_k$ for negative coordination) and $\pi_{k2}, k \in U_2$.

2.4 Non-PRN methods

The non-PRN methods can be divided into *methods using mathematical programming* and *other methods*. Among methods based on mathematical programming, one can include the approaches of Raj (1956, 1968); Arthnari and Dodge (1981); Causey et al. (1985); Ernst and Ikeda (1995); Ernst (1996); Ernst (1998); Ernst and Paben (2002); Mach et al. (2006); Matei and Skinner (2009); Tiwari and Sud (2012); and Şchiopu-Kratina et al. (2014). In general, methods based on mathematical programming try to optimize the overlap between samples, under some constraints. Mathematical programming was also used to minimize the expected respondent burden as in Perry et al. (1993). Among the methods using mathematical programming for sample coordination the transportation problem offers an attractive framework (Causey et al., 1985). *Other methods* include the approaches of Keyfitz (1951), Kish and Scott (1971), Sunter (1989), Tillé and Favre (2004), Matei and Tillé (2005) and Grafström and Matei (2015, method II).

Some non-PRN methods are reviewed below.

- *Keyfitz's method*: the method of Keyfitz (1951) can be applied for positive coordination when one unit per sample (or per stratum) is selected. It is assumed that $\sum_{k\in U}\pi_{k1} = \sum_{k\in U}\pi_{k2} = 1$, and $U_1 = U_2 = U$. A unit $\in U$ is selected in s_1 with probability π_{k1}. Let $I = \{k \in U | \pi_{k1} \leq \pi_{k2}\}$ be the set of 'increasing' units, and let $D = \{k \in U | \pi_{k1} > \pi_{k2}\}$ be the set of 'decreasing' units. Assume that unit k was selected in s_1 with probability π_{k1}. If $k \in I$, it is selected in s_2 with probability 1, else with probability π_{k2}/π_{k1}. If k was not selected in s_2, a new unit $\ell \in I$ is selected in s_2 with probability proportional to $\pi_{\ell 2} - \pi_{\ell 1}$. The method can be applied for two sampling designs with the same stratification. Kish and Scott (1971) extended Keyfitz's method to the case where units change strata in the second selection.

- *Sunter's method*: Sunter (1989) introduced a method for positive coordination of two general samples, inspired by Keyfitz's method. This method uses the selection probabilities of samples (that is, sets of selected units), that need to be computed under both the original and new designs. There are two time occasions; a sample s is selected on the first occasion with probability $p_1(s)$ and the probability that the same sample is realised on the second occasion is $p_2(s)$. If s_1 is selected on the first occasion, it is retained with probability 1 on the second occasion if $p_2(s_1) > p_1(s_1)$, and with probability $p_2(s_1)/p_1(s_1)$, otherwise. If s_1 was retained on the second occasion, the procedure ends. If not, it repetitively continues until a new sample is retained as follows: a new sample s_2 is selected with probability $p_2(s_2)$. If $p_2(s_2) < p_1(s_2)$, s_2 is rejected, else it is retained with probability $1 - p_1(s_2)/p_2(s_2)$.

- *Transportation problem*: let $\mathcal{S}_1$ and $\mathcal{S}_2$ denote the sets of all possible samples on the first and second time occasion/survey, respectively, with $m = |\mathcal{S}_1|$, $q = |\mathcal{S}_2|$, and $|.|$ denoting the cardinality of a set. Let $s_{i1} \in \mathcal{S}_1$ and $s_{j2} \in \mathcal{S}_2$. It is assumed that $c_{ij} = |s_{i1} \cap s_{j2}|, p_{i1} = p_1(s_{i1}), p_{j2} = p_2(s_{j2}), i = 1, \ldots, m, j = 1, \ldots, q$ are known. Given that s_{i1} was already selected, the goal is to draw s_{j2}, conditionally on s_{i1}, by optimizing their overlap size as much as possible. Causey et al. (1985) optimized $E(c) = \sum_{k \in U} \pi_{k,12} = \sum_{i=1}^{m} \sum_{j=1}^{q} c_{ij} p_{ij}$ in positive coordination as a transportation problem (a well-known linear programming problem) given as

$$\max \sum_{i=1}^{m} \sum_{j=1}^{q} c_{ij} p_{ij}, \tag{2}$$

subject to the constraints

$$\left| \begin{array}{l} \sum_{j=1}^{q} p_{ij} = p_{i1}, i = 1, \ldots, m, \\ \sum_{i=1}^{m} p_{ij} = p_{j2}, j = 1, \ldots, q, \\ p_{ij} \geq 0, i = 1, \ldots, m, j = 1, \ldots, q, \end{array} \right.$$

where $p_{ij} = p(s_{i1}, s_{j2})$. Once the solution p_{ij} is obtained, a sample s_{j2} is selected with the conditional probability $p(s_{j2}|s_{i1}) = p_{ij}/p_{i1}$, noting that $p_{i1} > 0$, $i = 1, \ldots, m$, since $\mathcal{S}_1$ contains only *possible* samples. Since the marginal constraints are fulfilled by solving Problem (2), $p_{j2} = \sum_{i=1}^{m} p(s_{j2}|s_{i1}) p(s_{i1})$ is respected and thus also $\pi_{k2}, \forall k \in U_2$.

To optimize $E(c)$ in negative coordination, use $\min \sum_{i=1}^{m} \sum_{j=1}^{q} c_{ij} p_{ij}$ instead of $\max \sum_{i=1}^{m} \sum_{j=1}^{q} c_{ij} p_{ij}$ as the objective function in (2), and keep the same constraints. When $c_{kk} = 1$ and $c_{k\ell} = 0, \forall k \neq \ell, k, \ell \in U$, with $U_1 = U_2 = U$, the problem reduces to the method of Raj (1956, 1968), where a sample is formed by a single unit.

The transportation problem (TP) is limited to samples s_{i1} and s_{j2} that are selected sequentially. The use of a TP is computationally intensive since all possible samples on the two occasions/surveys must be enumerated, and their selection probabilities known. The method is impractical if U is large. For stratified designs, it can be solved, however, separately in each stratum. The method can be directly applied to consecutive stratified

SRS-WOR designs. If the stratifications differ, then the approximate solution might use the stratum intersections. When both samples are selected by SRS-WOR, Mach et al. (2006) showed how to reduce the dimension of the initial TP to a much smaller TP. The main advantage of using a TP is to provide the best solution in terms of optimizing $E(c)$, for given p_{i1} and $p_{j2}, i = 1, \ldots, m, j = 1, \ldots, q$.

3 Comparing sample coordination methods

3.1 Measures used in sample coordination

Different measures can be used to check the quality of a coordination method. The main ones are based on the bounds of the expected overlap size and on the response burden associated with a unit $k \in U$.

1. **Bounds of the expected sample overlap**

   Any $\pi_{k,12}$, $k \in U$ has associated limits provided by the Fréchet bounds on the joint probabilities:

$$\max(0, \pi_{k1} + \pi_{k2} - 1) \leq \pi_{k,12} \leq \min(\pi_{k1}, \pi_{k2}). \tag{3}$$

   By taking the sum over U in both sides of (3), one obtains bounds for $E(c)$:

$$\sum_{k \in U} \max(0, \pi_{k1} + \pi_{k2} - 1) \leq E(c) = \sum_{k \in U} \pi_{k,12} \leq \sum_{k \in U} \min(\pi_{k1}, \pi_{k2}). \tag{4}$$

   These bounds are used to quantify the performance of sample coordination methods when the overlap optimization is considered as the main criterion. Yet, there are few methods in the literature able to reach these bounds; only Poisson sampling with PRNs, SRS-WOR with PRNs and Keyfitz's method reach them among the methods presented in Section 2.2. For more details, see also Matei and Tillé (2005). Note that the solution to (2) does not necessarily meet the bounds in (4), because of the constraints on p_{i1} and p_{j2}.

   Conditions to reach the bounds in (4) for given marginal sampling designs p_1 and p_2 have been provided by Matei and Tillé (2005). These conditions are, however, very restrictive, and hard to fulfill if the given sampling designs p_1 and p_2 are, for instance, unequal probability sampling designs with fixed sample sizes.

2. **Measuring the response burden**

   We point out that the reduction of the response load of a unit is a consequence of minimizing overlap size between samples, rather than a direct objective of sample coordination.

   Response burden is a complex concept, that is difficult to quantify (Haraldsen et al., 2013). Response burden influences non-response in surveys, and thus the quality of data collection. An easy definition of response burden is considered in Sunter (1977b). Let us use a general framework, where a unit i is exposed to several surveys $j = 1, 2, \ldots, M$ having associated $U_1, U_2, \ldots, U_M$ populations of units, and $U = \cup_{j=1}^{M} U_j$ represents the overall population corresponding to all the surveys. Each survey questionnaire is assessed for its response 'load'. Sunter (1977b) emphasized about the response load that: 'this would be expressed conveniently as a money equivalent to the time and effort required to complete the questionnaire'. Let β_j be the response load imposed by the jth survey for all units selected to participate in this survey. The *response burden* of unit i in M surveys is a random variable $RB_i = \sum_{j=1}^{M} \beta_j \times I_{ij}$, with $I_{ij} = 1$ if $i \in s_j$ and 0 otherwise (Sunter, 1977b). Its expectation is given by

   $$E(RB_i) = \sum_{j=1}^{M} \beta_j \pi_{ij}, i \in U,$$

   where $\pi_{ij} = P(i \in s_j)$, and s_j is the jth survey sample, $j = 1, \ldots, M$. The response burden is usually associated with a negative coordination of samples: it is expected that RB_i is reduced if unit i is less often selected to answer the survey questionnaires, so when the overlap size is minimized. Haraldsen et al. (2013, p. 251), comments that 'sample coordination does not affect the expected response burden, but is rather intended to control for excessive burdens and to allocate burdens in a fair way'.

3.2 Criteria for sample coordination

It is possible to use different criteria to measure the quality of a coordination procedure. For example, Tillé (2020, p. 175) advocates the following 4 criteria:

1. 'the procedure provides a controllable overlap rate,
2. the sampling design is satisfied for each sample,
3. for each unit, a period out of a sample is satisfied,

4. the procedure is easy to implement.'

The first criterion in the list of Tillé (2020) is obviously less strong than the criterion related to the optimality of a coordination method, where the bounds in (4) are reached. The third criterion refers to the rotation of units in repeating surveys.

We use the following list of criteria inspired by Ernst (1999) to characterize the coordination methods reviewed in this chapter:

1. uses PRNs/mathematical programming/other methods;
2. maximizes or minimizes the overlap size, or both;
3. the bounds given in Expression (4) can be reached (the method is optimal from this point of view);
4. allows sequential or simultaneous selection of samples, or both (sequential selection is useful for sample coordination over time, while simultaneous selection is more for sample coordination over surveys);
5. any constraint on the number of units to be selected in a sample (or stratum, in the case of stratified designs);
6. allows restratification, in the case of stratified designs;
7. allows independence of sampling from stratum to stratum (on each occasion, all strata are sampled independently of each other; for a single sample s this means that $P(i \in s, j \in s) = P(i \in s)P(j \in s)$ whenever units i and j are in different strata; this is important for variance estimation);
8. number of surveys to be coordinated;
9. allows the creation of a coordination system;
10. the main drawback of the method.

The criterion 'the method allows the creation of a coordination system' is a result of the 'number of surveys that can be coordinated'. In some cases, a method allows only two samples to be coordinated, and thus its use over time is excluded. Many methods for sample coordination control the sample overlap, however, optimality (in the sense that the bounds on $E(c)$ given in

Expression (4) are reached) is not always achieved. Table 1 provides an overview of the sample coordination methods presented in Section 2.2, on the basis of the above criteria.

We introduce below our own features of a coordination system, distinguishing between *sample coordination methods* and *sample coordination systems*.

Table 1: Properties of sample coordination methods described in Section 2.2. For the definitions of the columns, see the text.

Method	1	2	3	4	5	6	7	8	9	10
Poisson with PRNs	PRNs	both	yes	both	no limit	yes	yes	no limit	yes	random sample size
Collocated	PRNs	both	no	both	no limit	no	yes	no limit	yes	doesn't allow restratification or redesign of the survey
Pomix	PRNs	both	no	both	no limit	yes	yes	no limit	yes	random sample size
CP sampling:										
method I	PRNs	both	no	both	no limit	yes	yes	no limit	yes	less good coordination degree than Poisson with PRNs
method II	other	both	no	sequential	no limit	yes	not yet evaluated	2	no	only the first sample is CP
SRS-WOR with PRNs	PRNs	both	yes	both	no limit	yes	yes	no limit	yes	only equal inclusion probab. of units
Synchronised sampl.	PRNs	both	no	both	no limit	yes	yes	no limit	yes	for different stratifications, positive coordination must be done 'manually'
Ordered sampling (unif., Pareto, exp.)	PRNs	both	no	both	no limit	yes	yes	no limit	yes	nearly πps sampling
Fixed size PoMix	PRNs	both	no	both	no limit	yes	yes	no limit	yes	nearly πps sampling for Pareto
Ohlsson	PRNs	both	no	both	1	yes	yes	no limit	yes	select only 1 unit
Keyfitz	other	maxim.	yes	sequential	1	no	yes	2	no	select only 1 unit
Sunter	other	maxim.	no	sequential	no limit	no	yes	2	no	only for 2 samples
Causey, Cox & Ernst	math. program.	both	no	sequential	no limit	yes	yes	no limit	no	unpractical for large U/large stratum

4 Sample coordination systems

A range of procedures has been proposed for sample coordination, as summarised in Section 2.2. These methodological approaches do not, however, provide enough structure to deal with a range of different surveys with various designs and rotation patterns, and with the resulting range of positive and negative coordination characteristics. In order to manage this wider problem, we need a *sample coordination system*, and there has been a range of implementations, based on several of the available sample coordination methods, in different countries. A general system must deal with longitudinal surveys (including rotating panels); Nedyalkova et al. (2009), based on Cotton and Hesse (1992b), used the framework of longitudinal designs within a coordinating system as follows. For M surveys, a unit $k \in U$ has an associated probability vector $\boldsymbol{\pi}_k = (\pi_{k,j})', j = 1, 2, \ldots, M$. A longitudinal sample associated with unit $k \in U$ is given by $(I_1, I_2, \ldots, I_M)$, where $I_j = 1$, if $k \in s_j$ or 0, otherwise, s_j being the jth sample, $j = 1, 2, \ldots, M$. Sample coordination itself generally has well-defined targets for optimisation, but there are two incompatible strategies in a system which develops longitudinally (Nedyalkova et al., 2009):

- 'to choose the cross sectional design and try to get the best coordination
- to choose a longitudinal systematic design and accept a progressive loss of control over the cross-sectional design'.

To produce an implementation of a sample coordination system suitable for use in a national statistical institute generally means deciding on one of these strategies, and on one or more corresponding methods of sample coordination, and then developing the data requirements, procedures and software which support the use of these methods. The sample coordination system includes the implementation and the data requirements together with the messy practical details.

It is challenging to define precisely what a sample coordination system consists of, but here is an attempt to enumerate its components:

a. One or more methods for sample coordination with defined goals (it will generally be most practical to use a single sample coordination method);

b. The system must be sequential to deal with changes in the survey portfolio (the set of surveys whose samples are to be coordinated using the system), and changes in sample design/stratification etc.;

c. The target measures to be optimised; in particular the burden should be spread as evenly as possible over the sample units with specific total selection probabilities;

d. Deals satisfactorily with many surveys and potentially many successive periods for each survey;

e. Allows both positive and negative coordination.

In addition there are features which are not *required* for a sample coordination system, but which simplify its implementation:

f. Common stratification variables and common stratum boundaries;

g. (Under the assumption that control of either the designs or the coordination gradually breaks down as described above) allows the system to be updated to a new baseline, preserving as many of the coordination and control properties as possible at the time of updating. Or allows for gradual updating (via a moving time window) so that control of the target measures holds within the window but not outside it.

4.1 Optimisation measures in sample coordination systems

In support of item c., Qualité (2009) points out that there are different ways in which positive and negative coordination can be defined, and we find it convenient to produce a typology so that we are able to discuss the different approaches. We therefore define:

p-coordination as a criterion based on the joint inclusion probability (named because it is defined with the inclusion **p**robabilities). If a unit k has inclusion probabilities π_{k1} and π_{k2} in two surveys (or, equally, two occasions of the same survey) respectively, then under independent selection its probability of being jointly included in both surveys is $\pi_k^{12} = \pi_{k1}\pi_{k2}$. If the achieved joint inclusion probability is $< \pi_{k1}\pi_{k2}$ then there is negative p-coordination, and if it is $> \pi_{k1}\pi_{k2}$ then there is positive p-coordination. We move therefore from the *coordination of samples* as underlined in Section 2.1 to *coordination of units* (see also Cotton and Hesse, 1992b, p. 27).

s-coordination as a criterion based on the units in the intersection of two designs (and named after the **s**ample size in the intersection). So if there are fewer units in the intersection than expected under independent sampling, that is $< \sum_{U_c} \pi_{k1}\pi_{k2}$, where U_c is the population of units common to both surveys, we say that there is negative s-coordination, and if there are more units than expected there is positive s-coordination.

b-coordination is a criterion based on equalising the survey burden (and named after the **b**urden), which may vary between or within surveys, and possibly between survey occasions. If the variability of the burden across units is lower than expected under independent sampling there is negative b-coordination, and if the variability is greater then there is positive b-coordination. b-coordination could be implemented as a weighted p- or s-coordination.

5 Overview of sample coordination systems

Systems for sample coordination have been of interest since the first developments in Australia and Sweden in the 1970s. Hesse (1999) reviewed sample coordination systems as part of an EU project, covering in detail Sweden, Australia, the Netherlands and France, where the the methods and approaches were well documented, and with some comments on the coordination systems in Canada, Finland, New Zealand, UK and USA. The only coordination methods which are suitable for generalisation to a system of surveys are those based on PRNs (Hesse, 1999), but even here several approaches have been used, and we give an overview of the main methods and their uses below. Brewer et al. (2000) also include a small comparison covering the same countries.

5.1 Coordinated Poisson sampling/conditional selection

Brewer et al. (1972) put forward a coordinated sampling approach based on PRNs in which the cross-sectional sample designs are Poisson, and therefore have variable sample sizes - Poisson sampling with PRNs from Section 2.2. Qualité (2009) notes that using Poisson designs makes coordination substantially easier to handle, particularly because

- the selection of any particular unit is not influenced by whether other units are selected or not;
- a substantial proportion of the loss in efficiency which arises from having a random sample size can be recovered by suitable calibration in the estimation phase.

However, the variable sample size has some undesirable features, since the probability of achieving a sample of size zero may be non-negligible when target sample sizes are small. The sample size may be substantially smaller than desired even if it is not zero, leading to increased variances. The risk of both zero and small sample sizes are compounded when the survey is subject to substantial nonresponse. This potentially restricts the usefulness of coordinated Poisson sampling in highly constrained designs, or where optimisation requires very specific sample sizes (for example, consider the interaction with numerical allocation for stratified sampling, in chapter Smith & Yung (this volume)).

Qualité suggests algorithms for coordinated Poisson sampling that meet many of the requirements for a system, and this is the basis of the system implemented in Switzerland (see Qualité, 2019), for both business and household surveys. The process has essentially worked, providing a lot of flexibility, though there are still some challenges in the practical details. For business surveys, after 127 selections (including the waves of rotating panels) the system still functions well for a business population of 600,000 units. For household surveys with around 8.5 million units, the procedures slowed and became unusable after around 200 selections, eventually being restricted to only negative coordination. The size of the calculations grows at least with the square of the number of surveys in the case where both positive and negative coordinatioon are implemented, so even with very efficient programming cannot be used for a very long time. The same method was developed independently in Australia (Bell, 2011) under the name of *conditional selection*, and is in use in Australia and New Zealand for coordination of the system of household surveys. The storage and processing requirements for a system based on this approach are substantial. Periodic resets of the system may therefore be needed to control them, and this has been done already in Switzerland.

The method of Qualité (2009) and Bell (2011) follows Poisson sampling with PRNs for the first two surveys/occasions in a system, and therefore achieves the optimum coordination for these surveys. Third and subsequent surveys/occasions are added conditional on the outcomes for earlier selections, so in these cases coordination is not in general so good. It is therefore necessary to specify an ordering to selections to give the best control in the outcome, and although Bell (2011) explores the challenges of specifying the priority order of surveys in a coordination system, both he and Qualité suggest this as a topic for further research. This requirement for a priority of coordination rules contributes to making this approach practical (as it is clear what action to take when not all of the constraints can be met simultaneously), but also means that not all the constraints in the system are met. Therefore, there needs to be some assessment and reporting of the cases where, despite the sample coordination, some units are selected in more surveys and/or on more occasions than expected. Situations where few or no unsampled units remain may also be problematic for survey selections later in a sequence, and especially where some guarantee about the number or frequency of selection is given to businesses. Such evaluation and monitoring are particularly important in this system, but also valuable monitoring tools to ensure that the implementation of other coordination systems does not produce unexpected results because of the conjunction of different designs. Bell (2011) also proposes a systematic selection approach within the same system, which induces a correlation between units which is not present in Poisson sampling with PRNs; Bell says 'In our simulations this correlation has no discern[i]ble effect on the selection probabilities for later surveys'.

Sample coordination systems based on Coordinated Poisson sampling/conditional sampling are based on the joint inclusion probabilities, and use p-coordination. They therefore do not explicitly take account of the realised burdens (although these could be used in the prioritisation of surveys/occasions), and rely on the control of the joint inclusion probabilities to spread the burden more equally. In an evaluation of the Swiss system applied to business surveys, Qualité (2019) suggests that this makes relatively little difference for the largest (most frequently selected) and smallest (least frequently selected) units (which is intuitively true in general for coordination systems, Guggemos and Sautory, 2012), but does spread the burden for medium-sized units.

The variable sample size has been seen as a substantial drawback, so considerable research in sample coordination has gone into generating approaches which give fixed-size cross-sectional designs, and these form the basis of the other sample coordination systems discussed below.

5.2 SAMU

The SAMU system has been used in Statistics Sweden for a long time (Ohlsson, 1992; Lindblom, 2003, 2014). It was instituted in the 1970s, and is based on coordinating groups of surveys by allocating them suitable starts within the PRN range and a direction for selection (that is, whether it should use u_k or $1 - u_k$ as described in Section 2.2). Each survey within the group uses the same starting point for all the strata. This is (stratified) sequential random sampling with PRNs and is an s-coordination system seeking to control the size of the overlap between surveys. The SAMU system also includes (stratified) Pareto πps sampling as an option (an extension since Lindblom, 2003). Roughly half of the business surveys at Statistics Sweden used this system in 2017 (Lindblom, 2019).

There are different ways to implement rotation within this system, but SAMU employs five rotation groups in strata with a sampling fraction < 0.1, where the PRNs in the rotation groups are shifted each year. This ensures a constant rate of rotation within strata, and is more easily managed in SAMU, which maintains the same starting values for survey groups, than the main alternative which involves updating the start values.

The stratification of surveys is based on a 'frozen' version of the business register; there are four versions during a year, based on an initial frozen register, with updates for births and deaths, and some reclassification.

5.3 Synchronised sampling

The system for coordination of business surveys in Australia is based on synchronised sampling (although conditional sampling – Section 5.1 – has been proposed, it has not yet been implemented for business surveys). The process works as described in Section 2.2 for positive coordination of panel samples, and rotation can be straightforwardly introduced by making the starting points of selection intervals increase according to expected or achieved rates, depending on which is the faster (see McKenzie and Gross, 2001, for details). In order to achieve negative coordination, different surveys use disjoint intervals, and control over this coordination is increased if the surveys use common stratification.

Essentially the same approach is used in the UK (the system is sketched in Smith et al., 2003), but their surveys are not confined to mutually exclusive intervals of PRN range, and therefore occasional intervention is needed to prevent selections for surveys overlapping instead of being negatively coordinated. Statistics Canada has also implemented synchronised sampling (along with Poisson sampling with PRNs) in its generalised system G-SAM, but here again it is necessary to set the start points of selection ranges manually to achieve negative coordination.

In both Australia and UK, a unit starts afresh when it changes to a different stratum (that is, it is treated as death in the old stratum and birth in the new one), so that control of coordination is lost in these cases. The UK updates the stratification variables annually (except in exceptional cases where this makes a large difference to the survey estimates), as in the Swedish system, to avoid a lack of control during a year. The loss of coordination when units change strata is a challenge when the sample design must be updated, but some limited control can be exerted by an appropriate choice of start point for the new selection interval, as described by McKenzie and Gross (2001). Having found the start point that maximises the required coordination (using positive indicators for positive coordination and negative indicators for negative coordination), the *next* element is chosen as the start, to reduce a bias in selection. More complex situations require an examination of the whole range of PRNs to identify a suitable range of values to start the redesigned survey from.

5.4 Burden-based coordination

The Netherlands sampling system was originally described (as EDS) by Van Huis et al. (1994) (although it had been developed earlier). EDS formed the basis for the development of a new system implemented in 2014 (Smeets and Boonstra, 2018) for coordination among groups of surveys. It is assumed that the surveys have common stratification, with some minor exceptions which allow division of strata into substrata. The basis of the method is to keep a PRN, an indicator of whether a unit was selected in each sample, and the cumulated burden for each unit. For a negative coordination, units are ordered at each selection by increasing cumulative burden, then by PRN to break ties in burden values. Then the first units in the resulting ordered list form the sample of units to be selected. For a positive coordination, units are ordered by the sample indicator for the survey with which positive coordination is desired; if necessary an adjustment can be made at this stage to induce a panel rotation by removing those units which have been in the survey the longest. Then the first units in the resulting ordered list again form the sample of units to be selected.

Some additional steps are needed to deal appropriately with births; when they join the population, they have no accumulated burden, and would be preferentially selected, leading to a sample containing too many recent births. To avoid this, the burden and the sample inclusion indicators should be imputed for a birth so that it has a typical value for the stratum. This is done by taking the values from the unit with nearest PRN within the stratum; then it will be treated appropriately in sampling.

A similar process occurs for stratum movers. First the position in the old stratum is calculated, with respect to a user-defined ordering of burden, indicators and PRN, as $r/(N_h + 1)$ where r is the rank with respect to this order in stratum h, and N_h is the size of stratum h. This position is maintained in the new stratum, and new cumulated burden and inclusion indicators are imputed from the unit with the nearest position in the new stratum. Finally the moving unit is assigned a new PRN (so in fact PRNs are not permanent in this system).

The Netherlands system therefore uses b-coordination explicitly, but within burden totals effectively uses sequential SRS-WOR with PRNs.

5.5 Coordination functions

The outline of the approach seems to have been developed by Frank Cotton at INSEE (France) in the late 1980s. It began with the idea of swapping PRNs between units so that previously selected units were not reselected (negative coordination), and the method was already implemented at this stage in software called OCEAN, for a single year. The paradigm of this approach is different from most other PRN methods (with the exception of the way rotation is implemented in the SAMU system) in that the PRNs are changed through a permutation of the random number line, while the selection interval remains fixed, always beginning from 0 and extending as far as is needed to achieve the required sample size for the survey being selected. The approach was extended to more general coordination functions which allowed different permutations of PRNs within different ranges in a series of papers. The full details were worked out in Cotton and Hesse (1992b) and summarised in Cotton and Hesse (1992a), then extended by Cotton and Hesse (1997) and Hesse (2001). This approach was implemented in two software packages, SALOMON and MICROSTRAT (Rivière, 2002) designed for use in EU member states. Despite this software availability, the method does not seem to have been used in earnest in France until the 2010s.

Notwithstanding the different selection paradigm, the approach is basically an implementation of sequential SRS-WOR with PRNs (Section 2.2), but the ability to permute the PRNs provides additional flexibility because it is not restricted to a single ordering. This also makes it more flexible for use in the case where the surveys within the system do not share the same stratification. By using a fixed end point for the selected PRN range it can also be used for Poisson sampling designs.

The approach through permutation of PRNs in particular allows some measure(s) of the accumulated burden to be taken into account. The permutations are defined by sorting the population elements according to a small number of variables, with ties in the first variable being broken according to the order of the second variable, and so on, so sorting can be done by accumulated burden (for negative coordination), though of course this must be ignored for positive coordination. Ties in burden are broken by sorting by the PRNs within burden. Coordination is strongest with the variable at the first level of the sort, and becomes weaker at lower levels, so it is important to define a hierarchy of coordinations, as in conditional selection (see Section 5.1). Because the coordination is based on the accumulated burden values, there is no control over the time in sample in rotating panels.

The method has been gradually extended, and Guggemos and Sautory (2012) described a system based on coordination functions which allows positive and negative coordination over a wider range of surveys; they set out the theory in detail, including the challenges of approximating the burden functions with step functions, and examine the effectiveness of the system through simulation. Gros and Le Gleut (2018) undertake a more extensive simulation based on the real designs of a range of surveys constituting 20 selections corresponding to a two-year period, and including both negative and positive coordination. The negative coordination is very effective, increasing the number of businesses selected in only one survey, and dramatically decreasing those in more than one compared with independent selections. The positive coordination is more effective than the previous approach described in Demoly et al. (2014).

A system based on coordination functions is basically an s-coordination, aiming to control the sample overlap, but it can be viewed at least partly as a b-coordination system when the accumulated burden values are specified as the sorting variables. This means that an extra process to calculate and update accumulated burden measures for all units is needed as part of the system.

6 Discussion

6.1 Distinguishing sample coordination methods and sample coordination systems

From these descriptions of sample coordination systems, we can deduce two properties that make a *system* distinct from a *sample coordination method*. The first is that coordination systems are much more concerned with *units* than with *samples*. Sample coordination aims to ensure maximum or minimum overlap between two samples, but a system focuses on whether a single unit is included in a disproportionate number of surveys, and the time in and out of a sample through rotation. Poisson sampling with PRNs is the sole member of the category of sample coordination methods that achieves unit coordination. This property makes this sample coordination method particularly easy to use as the basis of a system, which is why it has been introduced in Switzerland and (partially) in Australia.

The second difference is the explicit use of a measure of *survey burden* as part of the sample coordination system. None of the methods for sample coordination presented in Section 2.2 takes account of the accumulated burden that a unit has already expended in responding to surveys, but systems do. In some cases this process is implicit, through control of the number of surveys or using sample coordination methods to exert control indirectly, but some systems use burden measures explicitly. In this way, they manage the process of spreading a fixed total burden more fairly across the units in the population. The corollary of this is that units will be reselected more quickly than under independent selections, but in many cases (particularly for the smallest businesses with small inclusion probabilities) the effect will not be noticeable to businesses, and therefore their perceived burden will be lower.

6.2 Further challenges

There are several other challenges which present themselves in operating a sample coordination system. One particular issue is that businesses have hierarchical structures, and surveys may use more than one unit in the hierarchy for sampling. Several authors of systems have addressed this, most by making the PRN for a higher-level unit and its largest lower-level unit the same. This means that there is dependence between surveys at the two levels, but that some coordination between levels is possible. While this is not as well controlled as for surveys at the same level, there is evidence that it manages the burden better than independent selections in Lindblom (2003); Gros and Le Gleut (2018).

7 Conclusion

Sample coordination methods aim to maximize or minimize the overlap between samples. While the PRN methods are easier to use and thus more popular, theoretical properties of the expected overlap size $E(c)$ are in general difficult to establish. The performance of such methods are usually studied by Monte-Carlo simulation. On the other hand, some non-PRN methods also offer good solutions to optimize $E(c)$, such as methods based on mathematical programming. Nevertheless, they are hard to use if the population size (or stratum size) is large, but can be applied, for instance, for surveys that employ a multi-stage design where samples of primary sample units have to be coordinated over time.

Coordination systems use coordination methods. Sometimes, different goals have to be reached, so that *optimization of the overlap* becomes less important, and it is replaced only by a *control of the overlap* between samples.

The landscape of sample coordination systems for business surveys has been remarkably stable in the last 20 years. The same four countries (Sweden, Australia, the Netherlands and France) have continued as the principal proponents of systems, and have continued to operate systems based on the methods that were in use at the time of Hesse (1999)'s review. France has moved to a full implementation of the methods based on coordination functions, and Australia has considered but not implemented a change to conditional selection. The main development in sample coordination systems has been in Switzerland, where a system based on coordinated Poisson sampling has been introduced and run successfully.

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