The machine scenario. A computational perspective on alternative representations of indeterminism*

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(preprint version)

Abstract

In philosophical logic and metaphysics there is a long-standing debate around the most appropriate structures to represent indeterministic scenarios concerning the future. We reconstruct here such a debate in a computational setting, focusing on the fundamental difference between moment-based and history-based structures. Our presentation is centered around two versions of an indeterministic scenario in which a programmer wants a machine to perform a given task at some point after a specified time. One of the two versions includes an assumption about the future behaviour of the machine that cannot be encoded in any programming instruction; such version has models over history-based structures but no model over a moment-based structure. Therefore, our work adds a new stance to the debate: moment-based structures can be said to rule out certain indeterministic scenarios that are computationally unfeasible.

Keywords: Machine scenario – Open future – Representations of indeterminism – Thought experiments

1 Indeterminism, the open future and a computational perspective

There are various phenomena that might be interpreted as cases of indeterminism (e.g. ontic vagueness, quantum superposition, etc.); in this paper, we will focus on a phenomenon that is part of a picture of time many will find intuitive: the idea that the future is open. This idea can be expressed in many ways. First, we think of the future as partially unsettled (e.g., it is settled that we will not live eternally, but it is unsettled whether the next President of the US will be a woman). Secondly, we think that there are things we can do to affect how the future will unfold (e.g., finding a cure for cancer, acting in an ethically responsible manner). We decide, we create, we

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pray. All these common attitudes – and there are many more – presuppose a future that we can somehow influence.[1] Thirdly, and perhaps more radically, we may wonder whether the future will unfold (e.g., will our world still exist in a thousand years?).

Of course, one could object that the open future is a non-fundamental phenomenon. Some arguments taken from science, especially from contemporary physics, point in this direction. In particular, since the 'block universe' view of time, which is favored by physicists, is isotropic (space-time has no intrinsic direction) and fundamental laws of physics are time-reversal invariant (they do not distinguish the future-direction from the past direction), it appears that the open future is not to be found within the 'fundamental features' of reality. Two alternatives are then given: either (i) the open future is an illusion, due to the way in which our minds interact with reality, or (ii) the open future is a phenomenal feature of reality captured by certain branches of physics and not by others.

However, dismissing the open future as an illusion seems illegitimate: the fact that science fails to capture certain phenomena does not entail that these phenomena are illusional. As Norton (2010, p. 26) puts it: [w]hen we start to believe that [our theories of space and time are telling us all that can be said about time objectively], we begin to invert the reasoning" . For example, there are dozens of natural phenomena (e.g., northern lights, will-o'-the-wisps) that science has for centuries failed to account for, but that did not prevent generations of people from rightly believing that these phenomena were parts of objective reality. Furthermore, saying that the open future is a phenomenal feature of reality, captured only by certain branches of physics, opens the issue of how different branches of physics can be conceptually related. Most attempts in this sense involve the second law of thermodynamics, which states that the entropy of any isolated system tends to increase over time. However, thermodynamics is, as Callender (2016) defines it, a 'phenomenal science'; that means that its variables range over magnitudes, such as pressure, heat and entropy. These magnitudes arise from the collective behavior of many microscopic entities, which is governed by the time-reversal invariant laws of quantum mechanics. Therefore, even assuming that the increase of entropy can explain the open future, this at best postpones the problem: if there is no asymmetry in time in fundamental physics, where does the thermodynamic asymmetry in time come from?

Although many different theories wish to accommodate the intuition of an open future,[2] it seems that a *necessary condition* for all is rejecting *physical determinism*,

[1] However, it is worth noting that our belief that we can influence the future does not necessarily rely on an intuition of indeterminism; indeed, there is a recent debate in experimental philosophy concerning whether folk intuition can be regarded as *compatibilist*; to deepen this topic, see Lim and Chen (2017).

[2] Some philosophers tend to reduce the open future debate to the question of human abilities. They take the claim that 'the future is open' to express the idea that 'humans can affect what will happen' (though, as we pointed out at the beginning, the latter is just one of the possible ways of expressing the former). Such a characterization of the open future debate is problematic. First it seems that if the future is open, then it was also open prior to the existence of any human agent. For example, it might be argued that, one hundred million years ago, it was open whether dinosaurs would disappear and humanity would emerge. Secondly, there is at least one sense in which the future may be said to be open that does not involve any agent: time could come to an end, with no ontological commitment to future things standing in the way (Correia and Rosenkranz 2018, p. 99). The question of the open future seems thus to exceed what humans may claim to have power on,

i.e., the thesis that the future development of the world is nomologically necessitated by its past development. After all, if physical determinism is true and, therefore, if there is only one way the future can nomologically issue from the current state of the world, then it is unclear how the future can be still said to be open. In particular, if it is *necessary*, given how things are at some time t and what physical laws obtain, that the world will be a certain way at a time t' later than t, then it seems *settled* that it will be that way (van Inwagen 1975). Opposed to physical determinism is the idea that some events in the world happen by *chance*, that is, in accordance with a degree of physical probability strictly between 0 and 1 (and this notion of probability can be justified in alternative ways; for instance, in terms of a propensity of a certain situation to produce a certain outcome or in terms of the relative frequency of an event in a set of trials). In philosophical debates the notion of chance is sometimes equated with the notion of *randomness*: an event is said to be random if and only if it happens by chance.[3] However, a deeper analysis seems to reveal that chance is a broader notion than randomness: if we consider the possible outcomes in a sequence of tosses of a biased coin, then any of these outcomes can be said to happen by chance while being predictable to a certain extent. Therefore, there can be some resistance in regarding such an outcome as properly random. Anyway, in our presentation we will be dealing only with indeterministic scenarios in which chance can be actually equated with randomness. For rigorous accounts of randomness the reader is referred to Kolmogorov (1963) and Martin-Löf (1966); some critical remarks on the chance-randomness equation can be found in Eagle (2005).

It is also worth noting that, despite being considered necessary, the rejection of physical determinism is oftentimes deemed an *insufficient condition* for characterizing the open future. As Geach (1973, p. 208) metaphorically puts it, considering a book, "even if the text of later pages is not determined by the text of earlier pages, there may nevertheless be a completely fixed text on those pages which we have not yet turned over". The idea this metaphor expresses is quite clear: the future may be held to be fixed without being held to be nomologically determined by the current state of the world. Moreover, the rejection of physical determinism does not even guarantee the absence of *eternal facts*; indeed, there is theoretical room for a picture of the world in which facts exist in an eternal way while being mutable, due to events occurring in time; such a position is illustrated, for instance, in Correia and Rosenkranz (2012). In any case, the open future is associated with the idea that there are *contingent events* and this precludes not only physical determinism but also *fatalism*, namely the thesis that human actions are epiphenomenal, i.e., they have no impact at all on the way the future will unfold. To deepen all mentioned aspects of the debate around the characterization of the open future, we invite the reader to see, for instance, Barnes and Cameron (2009), Torre (2011) and Grandjean (2019).

In order to avoid problems due to attempts of providing a general definition of the open future in terms of physical (or even metaphysical) indeterminism, one can restrict the analysis to a specific class of scenarios. In the present article we will focus on scenarios in which the open future is a property arising from non-deterministic computational processes. We will therefore speak of a *computational perspective* on

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and should therefore not be reduced to the question of our abilities.

[3] See, for instance, the discussion in Lüthy and Palmerino (2016).

the open future. In these scenarios, at an instant t the future can be *construed as open* if and only if there is a set I of programming instructions that generates a set R_I of alternative possible runs of a machine after t. We will only deal with instructions that constitute a *randomized algorithm*, that is, an algorithm whose behaviour depends, at each stage of the computation, on a random number generator. Such an algorithm can be implemented in a machine whose possible computations are within the range of computations of a *probabilistic Turing machine*. The choice of this particular kind of algorithm does not affect the generality of our arguments, which could be restated in terms of a different computational framework. Anyway, given the concise exposition in this article, we refer the reader to Floyd (1967) for a broader analysis of algorithms to represent non-deterministic scenarios. What is important is that when a randomized algorithm allowing for a set of alternative runs can be implemented in a machine, the machine clearly represents an *indeterministic system*. Furthermore, another crucial aspect will be the difference between *expected* and *implementable* runs: given a set of expected runs of a machine R_{exp}, we can say that R_{exp} gives rise to an indeterministic scenario which is *computationally feasible* if and only if there is a set of programming instructions I able to generate *every possible run* in R_{exp}, namely if and only if there is some I such that $R_{exp} \subseteq R_I$. This is the same as saying that all runs in R_{exp} are implementable. Thus, R_{exp} gives rise to a computationally unfeasible scenario if and only if, for every set of programming instructions I, we have $R_I \subset R_{exp}$.

2 Alternative representations of indeterminism

One popular way to picture an indeterministic scenario involving the future consists in using a *tree-like structure of nodes*. In philosophical logic these nodes are traditionally called *moments* and will be here denoted by m, m', m'', etc. In a tree-like structure every node is assigned a particular *state* of a given system —that can be thought of as a set of true tense-free statements about the system— and, at a moment m, the past of the system up to m constitutes a single trunk, whose series of nodes is assigned a series of states, while the future of the system constitutes a *multiplicity of branches*, each a series of nodes to which a possible series of states is assigned. Moments and instants (or times) are not to be confused: two moments belonging to two different branches in a tree may represent alternative possibilities for the same instant. Of course, there can only be a unique *actual* way the system will be at an instant t' later than t; hence, only one of the moments representing alternative possibilities for t' will be actual when t' will become present.[4]

In tree-like structures construed from moments, which we can also call *moment-based structures*, if we consider a branch stemming from a given moment m, together with that branch's trunk, then this linear series of moments is called a *history* of the

[4]Notice that there are many problematic aspects of tree-like structures that will not be discussed in this paper. For example, tree-like structures might appear like a metaphysics in which it is perfectly settle how the future will be, but we just do not know where we will be in the future (cf. Rosenkranz 2013 and Cameron 2015). Moreover, tree-like structures seem to have difficulties in accounting for radical openness (i.e. the possibility that the world will not continue beyond a certain time) (cf. Cameron 2015), and for time-travel (cf. Miller 2005 and Norton 2018).

system analysed passing through m. We can denote histories passing through m as h_m, h'_m, h''_m, etc. Thus, in such structures moments are regarded as the primitive entities and histories are defined in terms of them. If an indeterministic scenario can be represented in terms of a moment-based structure, then we will say that the scenario at issue *has a model over* (at least) one of such structures. In a model over a moment-based structure, the sequence of states associated with a sequence of moments constituting a history gives a complete description of one course of temporal development of the system analysed. In particular, the sequence of states associated with the sequence of moments in a history h_m represents a course of temporal development of the system that is still possible at m.

However, a model over a moment-based structure is not the only possible way of representing a scenario in which the future is open. Indeed, one can also represent the set of possible developments of a system as a set of possible histories which are pairwise indistinguishable up to a certain time and diverge afterwards. Histories are in this case taken as the primitive entities and a linearly ordered set of moments is assigned to each of them; we can therefore say that these structures are *history-based structures*.

A natural question is whether the two sorts of structures mentioned —those in which moments are the primitive entities and those in which histories are the primitive entities— make any difference with respect to the representation of indeterministic scenarios: are there (im)plausible indeterministic scenarios that can be represented in terms of one kind of structure but not in terms of the other? This question has given rise to an intense philosophical debate over the last forty years. Nishimura (1979a, b) firstly showed that alternative structures to represent indeterministic scenarios are not always equivalent with respect to the satisfiability of formulas of a certain formal language. Authors have since then tried to provide a transposition of Nishimura's results in a metaphysical setting (see, e.g., Thomason 1984, Øhrstrøm and Hasle 1995 and Belnap et al. 2001). The focus of the debate has gradually moved to providing examples of indeterministic scenarios that are compatible with some but not all the alternative structures considered and analysing their conceptual plausibility.[5]

In the present article we contribute to the debate by comparing moment-based structures and history-based structures in a computational setting. We use two versions of a thought experiment that will be called *the machine scenario*. In one version, the *basic scenario*, a programmer asks a machine to perform a task after a specified time and as soon as a relevant event takes place (tails appear in a virtual coin toss); in the other version, called the *extended scenario*, we add the assumption that the machine *will eventually* perform the task. The basic version of the scenario has both models over moment-based structures and models over history-based structures; by contrast, the extended version of the scenario has only models over history-based structures. Given that the property which makes a crucial difference between the two scenarios (that the machine will at some point perform the task) cannot be encoded in any programming instruction, our analysis provides evidence for the fact that moment-based structures are able to rule out some computationally unfeasible scenarios and are therefore preferable in a computational setting.

[5] To deepen the logical side of the debate, the reader is referred to Reynolds (2002) and Zanardo (2006).

3 The machine scenario

Consider the following thought experiment: a programmer is about to specify instructions for a machine. The machine is built in such a way that it can in principle work forever, namely is not doomed to turn itself off due to lack of energy; moreover, let the machine measure time with a clock, so that it treats time as a *discrete* series of instants. The programmer wants the machine to have the possibility of performing a certain task at an *arbitrary* instant after a specified time. Let us take 6 p.m. of 10 December 2029 as the crucial time triggering this possibility and denote the task by T. The programmer expresses his/her request by ordering the machine to start tossing a virtual coin at every instant measured by the clock after 6 p.m. of 10 December 2029 and to keep doing that until tails appear, in which case T is performed. We can further assume that the coin toss and the performance of T are for all practical purposes instantaneous. Finally, we assume that the task T has never been performed before by the machine. The programmer's aim can be described in terms of the following two properties concerning the possible behaviour of the machine across time:

P1 for each moment m, there is some previous moment m' at which T has not yet been performed *and* the possibility that T will have been performed at some moment m'' immediately after m.

P2 at each moment m, T has not yet been performed *if and only if* there is the possibility that T will not have been performed either at some moment m' immediately after m.

P1 has the form of a conjunction (... *and* ...), whereas P2 has the form of a biconditional (... *if and only if* ...). P1 conveys the idea that T cannot be performed before 6 p.m. of 10 December 2029 and that, at an arbitrary instant following this time, if T has not been performed yet it *might* be performed immediately after. P2 conveys the idea that the performance of T can be indefinitely postponed after 6 p.m. of 10 December 2029 (left-to-right direction of the biconditional) and that every moment keeps track of a past performance of T (right-to-left direction of the biconditional). Let us say that P1 and P2 describe the *basic scenario* associated with the machine experiment. Notice that the basic scenario does not guarantee that the machine will eventually perform the task, since it is, at least in principle, *possible* that tails never obtain in the coin toss.[6]

The assumption that the machine will eventually perform the task can be conveyed by the following property:

P3 in each possible run of the machine, there is a moment m at which T has been performed.

The addition of P3 gives rise to what we will call the *extended scenario* associated with the machine experiment. The extended scenario is still indeterministic: while P3

[6]As observed by Belnap et al. (2001, p. 201), this is a matter of possibility, not of probability. Indeed, according to standard probability theory, the probability of an infinite sequence of tails in the machine scenario is zero.

settles *whether* the task will be performed at some point, it does not settle *when* the task will be performed (after 6 p.m. of 10 December 2029).

The two versions of the machine experiment, the basic scenario and the extended scenario, require discrete time and can be represented in terms of history-based structures. By contrast, moment-based structures can be used only for the basic scenario. Here we provide an informal sketch of how these structures can be construed. Hereafter we will use the expressions 'structure' and 'frame' as interchangeable.

A *discrete moment-based frame* is a tuple $\mathfrak{F} = \langle M, \prec \rangle$ where M is a set of moments (denoted by m, m', m'', etc.) and $\prec$ a binary relation over M such that, for any two moments m and m', $m \prec m'$ means that m *immediately* precedes m' in some possible way in which the world can develop. We assume that in a frame the relation $\prec$ and its transitive closure, denoted by $\ll$, satisfy the following properties (where $\neg$ stands for negation, $\wedge$ for conjunction, $\vee$ for disjunction and $\rightarrow$ for material implication):

- $\forall m, m', m''((m \prec m' \wedge m' \prec m'') \rightarrow \neg(m \prec m''))$
  (intransitivity of $\prec$);
- $\forall m \neg(m \ll m)$
  (irreflexivity of $\ll$);
- $\forall m, m'(m \ll m' \vee m' \ll m \vee \exists m''(m'' \ll m \wedge m'' \ll m'))$
  (backward connectedness of $\ll$).

These properties are sufficient to guarantee that every possible course of events has a direction going from the past to the future and which cannot be reversed (first two properties), as well as that every two distinct courses of events share their past up to a certain time and diverge afterwards (third property). Thus, the resulting structures have the shape of a tree branching towards the future. Furthermore, from these properties it follows that $\prec$ is asymmetric, irreflexive and backward functional, namely:

- $\forall m, m'(m \prec m' \rightarrow \neg(m' \prec m))$
  (asymmetry of $\prec$);
- $\forall m \neg(m \prec m)$
  (irreflexivity of $\prec$);
- $\forall m, m', m''((m' \prec m \wedge m'' \prec m) \rightarrow m' = m'')$
  (backward functionality of $\prec$);

Given a discrete moment-based structure $\mathfrak{F} = \langle M, \prec \rangle$ satisfying these properties, a *history* h in $\mathfrak{F}$ is any maximal subset of M linearly ordered by $\ll$. The set of all histories in $\mathfrak{F}$ is denoted by $H_{\mathfrak{F}}$. When we want to make reference to a particular moment m through which a history passes we can exploit the already introduced notation h_m. Furthermore, we can use H_m to denote the set of all histories passing through m in a frame.

Let p represent the proposition that task T has been performed by the machine; p will be the *only* propositional letter needed for our purposes. We can supplement our structures with a valuation for p, namely a function V associating p with a set of moments $V(p) \subseteq M$. The result of this addition is a model over a moment-based frame, or *moment-based model*, $\mathfrak{M} = \langle \mathfrak{F}, V \rangle$. Thus, $V(p)$ is the set of moments of the model $\mathfrak{M}$ in which it is true that the task has been performed.

A *discrete history-based frame* is a tuple $\mathfrak{F} = \langle H, g \rangle$ where H is a set of histories and g is a function that assigns to each $h \in H$ a linear ordering $(M_h, \prec_h)$ of a set M_h of moments. Notice that, in general, given two histories h and h', $M_h \cap M_{h'}$ might be non-empty; however, without loss of generality, we can restrict our attention to structures in which this is not the case. Therefore, history-based frames will be here graphically represented as sets of parallel lines. Furthermore, for our purposes we can denote moments in these frames as ordered pairs (n, i), where $n \in \mathbb{N}$ and $i \in \mathbb{Z}$. The intuition behind this notation is that n is a number associated with the history to which the moment belongs and i is a temporal index of the moment. A model over a history-based frame, or *history-based model*, is a tuple $\mathfrak{M} = \langle \mathfrak{F}, V \rangle$, where $\mathfrak{F}$ is the underlying frame and V is a function associating our propositional letter p with a subset of $\bigcup_{h \in H} M_h$.

In order to build temporal models for the two versions of the machine experiment, let us start by considering the case of history-based structures. For the sake of simplicity, we can assume that all histories in a structure of this kind are *isomorphic*; this means that given two histories h_n and h_k, it is possible to define a *bijective function* $f_{h_n \to h_k}$ between their moments which preserves the relation of temporal precedence (namely, for any $(n, i), (n, i')$ in h_n, $(n, i) \prec_{h_n} (n, i')$ iff $f_{h_n \to h_k}(n, i) \prec_{h_k} f_{h_n \to h_k}(n, i')$). In a history-based structure we can say that a moment (n, i) belonging to a history h_n *is possible in the immediate future* of a moment (k, j) belonging to a history h_k iff:

- for all moments $(n, i') \in h_n$ s.t. $((n, i') \ll_{h_n} (n, i)) \vee ((n, i') = (n, i))$, we have $(n, i') \in V(p)$ iff $f_{h_n \to h_k}(n, i') \in V(p)$;
- $f_{h_n \to h_k}(n, i) \prec_{h_k} (k, j)$.

When (n, i) and (k, j) belong to the *same* history (i.e. $n = k$), this definition boils down to the simple claim that $(n, i) \prec_{h_n} (k, j)$. An analogous definition can be used for a moment (k, j) that is *possible in the future* of a moment (n, i); one only needs to replace $\prec_{h_k}$ with $\ll_{h_k}$ in the second clause employed above.

We can now check which conditions are needed in history-based structures in order to satisfy properties P1-P3. P1 requires that for every moment (n, i) in a history h_n there is a moment (n, i') in h_n (i.e., in the same history) s.t. $(n, i') \ll_{h_n} (n, i)$ and p is false at (n, i') and a moment (k, j) in a history h_k s.t. $f_{h_n \to h_k}(n, i) \prec_{h_k} (k, j)$ and p is true at (k, j). P2 requires that for every moment (n, i) in a history h_n, p is false at (n, i) iff there is a moment (k, j) in a history h_k s.t. $f_{h_n \to h_k}(n, i) \prec_{h_k} (k, j)$ and p is false at (k, j). P3 requires that for every moment (n, i) in a history h_n there is a moment (n, i') in h_n (i.e., in the same history) s.t. $(n, i) \ll_{h_n} (n, i')$ and p is true at (n, i').

Take a history-based structure $\mathfrak{F} = \langle H, g \rangle$, where, for any $h_n \in H$, $g(h_n) = (M_{h_n}, \prec_{h_n})$ is such that:

- $M_{h_n} = \{(n, i) : n \in \mathbb{N} \wedge i \in \mathbb{Z}\}$;
- for every $(n, i), (n, i') \in M_{h_n}$, $(n, i) \prec_{h_n} (n, i')$ iff $i' = i + 1$.

In the values for i and n, let 0 stand for 6 p.m. of 10 December 2029, 1 for the instant immediately after, -1 for the instant immediately before (measured by the clock of the machine), etc. Given two histories h_n and h_k and a moment (n, i) in h_n, let $f_{h_n \to h_k}(n, i) = (k, i)$; the function $f_{h_n \to h_k}$ is an isomorphism. Expand $\mathfrak{F}$ with a valuation function V s.t. $V(p) = \{(n, i) : i > n\}$. Let $\mathfrak{M} = \langle \mathfrak{F}, V \rangle$ be the resulting temporal model. It can be easily verified that properties P1, P2 and P3 hold in $\mathfrak{M}$, which is thus a model for both the basic and the extended scenario. In particular, every history h_n in $\mathfrak{F}$ represents the possibility that task T is performed between instant n and instant $n + 1$.

Since the basic scenario is in principle distinct from the extended one, being not committed to P3, we also need a model for the basic scenario which is not a model for the extended scenario. This is obtained by taking a structure $\mathfrak{F}' = \langle H', g' \rangle$, where:

- $H' = H \cup \{h_\infty\}$;
- $h_\infty = \{(\infty, i) : i \in \mathbb{Z}\}$;
- for every $h_n \in H'$ and $(n, i), (n, i') \in M_{h_n}$, $(n, i) \prec_{h_n} (n, i')$ iff $i' = i + 1$.

Keeping the definition of $f_{h_n \to h_k}$, for any two histories h_n and h_k in H', we can then use a valuation function $V' = V$ to get a model $\mathfrak{M}'$ in which P1 and P2 hold, while P3 fails at any state of kind (∞, i).[7] Figure 1 is a sketched graphical representation of the model $\mathfrak{M}'$, where moments in black are those in which p is true and moments in red those in which p is false; history h_∞ is marked in red since p is false throughout it. A graphical representation of the model $\mathfrak{M}$ (in which P3 holds as well) can be obtained by removing history h_∞ from Figure 1.

$(0,0)$	$(0,1)$	$(0,2)$	$(0,3)$	h_0
$(1,0)$	$(1,1)$	$(1,2)$	$(1,3)$	h_1
$(2,0)$	$(2,1)$	$(2,2)$	$(2,3)$	h_2
$(3,0)$	$(3,1)$	$(3,2)$	$(3,3)$	h_3
$(\infty,0)$	$(\infty,1)$	$(\infty,2)$	$(\infty,3)$	h_∞

Figure 1: *A history-based model for the basic scenario.* Pairs of kind (x, y) denote moments. Black pairs are moments where p is true and red pairs are moments where p is false. Histories are labelled by h_0, h_1, h_2, etc. A label for a history is black if at some point in the history p becomes true, red otherwise.

We can now move to the case of moment-based structures. In these structures a moment m^* is possible in the (immediate) future of a moment m iff $m \ll m^*$ ($m \prec m^*$). Consider a structure $\mathfrak{F}'' = \langle M'', \prec'' \rangle$ where:

[7] In the case of P1, it is useful to observe that any moment $(n, i+1)$, with $i = n$, is possible in the immediate future of a moment of kind (∞, i) and p is true at $(n, i+1)$.

- $M'' = \{(j,i) : j,i \in \mathbb{Z}^- \wedge j = i\} \cup \{(j,i) : j,i \in \mathbb{N} \wedge j \leq i\}$;
- $(j,i) \prec'' (j',i')$ iff $(j = i \wedge j' = i' = i+1) \vee (j = j' \wedge i' = i+1)$.

A history is, as usual, a maximal chain of moments ordered by the transitive closure of $\prec''$. We can extend our structure with a valuation function V'' s.t. $V''(p) = \{(j,i) : j < i\}$, so as to get a model $\mathfrak{M}'' = \langle \mathfrak{F}'', V'' \rangle$. It can be easily verified that properties P1 and P2 hold in $\mathfrak{M}''$. On the other hand, property P3 fails, since there is one history including no state in which p is true. Such history can be denoted by h_∞ and is the set of all moments (j,i) where $j = i$. Figure 2 is a sketched graphical representation of the model $\mathfrak{M}''$, where moments in black are those in which p is true and moments in red those in which p is false; also in this case, history h_∞ is marked in red since p is false throughout it.

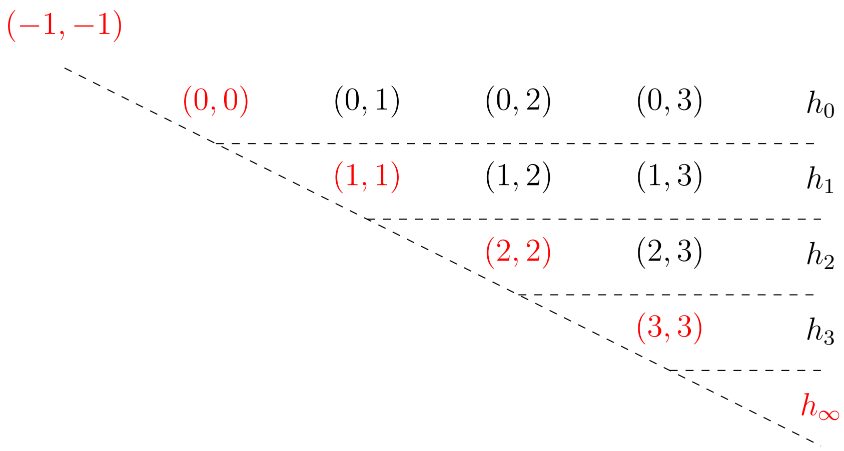

Figure 2: *A moment-based model for the basic scenario.* Pairs of kind (x,y) denote moments and histories are labelled by h_0, h_1, h_2, etc. The colours black and red are used as in Figure 1.

Not only this, but P1, P2 and P3 together have no model over a discrete moment-based structure. Indeed, consider a discrete moment-based structure $\mathfrak{F} = \langle M, \prec \rangle$. Let $\mathfrak{M}$ be an arbitrary model over $\mathfrak{F}$ in which P1 and P2 hold. Let m be any moment. By P1 we know that there is some earlier moment m^* at which p is false. Let m' be the moment immediately preceding m^*. Then since m^* is in the immediate future of m', by P2 p is false at m'. By iteration of this argument, we see that at all predecessors of m^* p is false. By P2 we also know that m^* has an immediate successor m'' at which p is false. Iterating this argument, we see that there is an infinite series of moments following m^* which, together with m^* and its predecessors, comprise a maximal linear series of moments, and therefore a history h, throughout which p is false, which contradicts P3. Nonetheless, at each moment in h there is, by P1, a moment in its immediate future at which p is true. Thus P1–P3 have no model together over any structure in which all maximal linearly ordered sequences of moments are histories, and in particular, any moment-based structure.

4 Discussion

In this work we have been dealing with the computational side of the debate around alternative representations of indeterministic scenarios. Such a debate encompasses several other areas, ranging from logic and semantics to metaphysics and epistemology; some surveys can be found in Øhrstrøm and Hasle (1995), Belnap et al. (2001), Rumberg (2016), Werndl (2016) and Müller and Placek (2018). As we pointed out in Section 2, in some of these areas the debate received a crucial input from the technical results in Nishimura (1979a,b), where alternative structures to represent time were compared in terms of formal languages describing their properties. Nishimura's articles are not the first works on logic in which alternative structures to represent time *in general* (whether deterministic or not) are compared. For instance, in Prior (1967), there is an extended presentation of models of time for various systems of tense logic, motivated also by Prior's personal correspondence with Saul Kripke. However, Nishimura is the first author who explicitly points out some crucial differences between various representations of indeterminism; in particular, between those structures that in the present article we called history-based and moment-based.

Indeterministic scenarios discussed in the literature in connection with Nishimura's works can be regarded as attempts to integrate his results with some philosophical intuitions, in order to argue in favour or against the adequacy of the various representations. Indeed, a central question in the debate has been the following: can we establish some criterion of preference among alternative representations of indeterminism on the basis of the different classes of scenarios that they are able to model? Before commenting on how the present article contributes to answering such a question, it is useful to review some answers provided in the literature; our focus will be on *metaphysical arguments* based on the analysis of specific scenarios.

For instance, Thomason (1984, pp. 151f.) shows some skepticism towards indeterministic scenarios that have no models over moment-based structures. More precisely, he reflects on intuitions about the end of the world and argues that the two claims below cannot be held consistently:

C1 inevitably, life on earth will come to an end at some date in the future;

C2 for every date in the future, it is not inevitable that life on earth will come to an end by that date.

Analogies between C1-C2 in Thomason's *end-of-life-on-earth scenario* and P1-P3 in the machine scenario are easily discovered; as a matter of fact, C1 resembles property P3, while C2 is related to properties P1-P2. Since C1-C2 give rise to a scenario which has no moment-based model, then their claimed inconsistency represents a reason for Thomason to prefer moment-based structures over history-based ones. In his words, the adequacy of a certain representation of indeterminism "has to do with intuitions about what should be valid" and he feels that "the natural notion is that of a possible future —not that of a possible course of events". Translating to our terminology, this means that moments have ontological priority over histories.

Belnap et al. (2001, pp. 199f.) reflect on a scenario involving the radioactive decay of an atom A (radioactive decay is the physical process in which an unstable nucleus of an atom loses mass emitting radiation); in particular, they consider the following two claims, made under the assumption that time is discrete:

C3 as long as A has not yet decayed, (I) A might decay before the next tick, and (II) A might not decay before the next tick;

C4 at a certain moment it is inevitable that A will decay after a finite number of ticks.

Properties C3-C4 in the *decaying-atom scenario* are very close to properties P1-P3 in the machine scenario (C3 to P1-P2, C4 to P3) and give equally rise to a scenario which is incompatible with moment-based accounts. The authors agree with Thomason that similar scenarios are intuitively inconsistent and that, for such a reason, moment-based structures are all one needs to represent indeterministic scenarios. After all, they claim that in the scenario taken into account "it is a real possibility, not to be ruled out by switching 'logic', that the atom will never decay" (Belnap et al. 2001, p. 201).

A different opinion is discussed in Øhrstrøm and Hasle (1995, p. 269) and Zanardo (2006, p. 394), where it is shown that the perspective can be reversed: the alleged inconsistency of pairs of sentences like C1-C2 and C3-C4 might depend on the assumption that every maximal sequence of moments in a structure is a history. Thus, one could say that history-based structures are preferable, because they can be used to model a larger class of indeterministic scenarios.

It has to be remarked that there is a way to overcome the limits of a moment-based structure in dealing with certain indeterministic scenarios, which consists in restricting the analysis to a subset or *bundle* of its histories satisfying a relevant closure condition, e.g. that every moment of the domain belongs to at least one history in the bundle; this strategy gives rise to the so-called *bundled semantics* (see, e.g., Burgess 1979 and Reynolds 2002) and turns moment-based structures into history-based ones. However, it is a strategy which seems to lack a metaphysical justification: if one starts with a primitive notion of moment and defines histories as maximal chains of moments, then every maximal chain of moments in a structure represents a complete possible development of the world and it is not clear what reason can be invoked for refusing to regard some of these chains as histories. As Zanardo (2006, p. 394) rhetorically puts it, "we cannot pretend not to see" certain histories in a structure; thus, considering a restricted bundle "instead of the set of *all* histories clearly looks like an unmotivated omission".[8]

When one moves from a metaphysical to a semantic perspective, the possible solutions to modify moment-based structures increase. Indeed, one can specify a list of parameters that have to be taken into account in evaluating the truth-conditions of propositions in a structure, and the same structure can be associated with different lists of parameters, giving rise to different truth-conditions. For instance, within moment-based structures one can distinguish between an Ockhamist and a Peircean semantics, depending on whether moment/history pairs or moments alone are used as parameters for evaluating propositions. The crucial difference is that in the Ockhamist semantics it is possible to say that a proposition about the future (e.g., the

[8]As already pointed out, this debate relies on a notion of possibility that is broader than the one arising from standard probability theory. Indeed, branching-time structures include, in general, infinite sequences of outcomes whose standard probability is zero.

proposition expressed by the sentence 'there will be a sea-battle') is true with reference to a *specific* history h to which the moment of evaluation m belongs, while this is not possible in the Peircean semantics, where the truth of a proposition about the future always involves either a universal or an existential quantification over the set of histories to which the moment of evaluation m belongs. Some semantic approaches make also use of parameters that we have not been discussing here, such as contexts, worlds, transitions and branches. To deepen this topic, see Belnap et al. (2001) and Rumberg (2016).

In the present article we introduced a novel scenario to feed the debate on alternative representations of indeterminism and reconstructed the formal side of the debate, aiming at a rigorous description of the structures involved. Our scenario is computationally-oriented: it concerns the problem of providing a machine with a set of instructions encoding certain properties of its *expected* future runs (P1-P3). Since it follows from Nishimura's results that history-based structures can model a larger class of indeterministic scenarios than moment-based structures, our aim was to check whether this difference can be explained in terms of computability properties: is it possible to build a scenario which is computationally unfeasible, in the sense defined in Section 1, and which has models over history-based structures but no model over moment-based structures? The extended version of the machine scenario is an example of such a scenario. The crucial difference between the two versions of the machine scenario is that the extended version relies on a very strong assumption about the possible behaviour of the machine, that is conveyed by property P3 and it is precisely the addition of this assumption that makes the scenario incompatible with moment-based structures.

Now, let us have a closer look at the aspect of our analysis that is more relevant for the computational perspective adopted: what it means to encode properties P1-P3 in programming instructions. It is clear that properties like P1 and P2 can be easily encoded in a set of instructions and thus that one can computationally construe the future as open in accordance with *all* expected runs of the machine in the basic scenario. For instance, a programmer may use conditional constructs like the following:

> *If* the value of time measured by the clock is 6 p.m. of 10 December 2029 or later, *then* toss the virtual coin. *If* tails appear during a coin toss, *then* execute T.

This set of instructions constitutes a *randomized algorithm* to perform task T. At each stage of the computation (an instant measured by the clock after 6 p.m. of 10 December 2029), the behaviour of the algorithm depends on a generator of a random number in the set $\{0, 1\}$ (that is, the coin toss). The algorithm may never lead to the intended output: an infinite sequence of 0s represents a run of the algorithm in which task T is never executed, whereas any other sequence σ represents a run of the algorithm in which task T is performed immediately after the first instant at which number 1 is generated. The algorithm at issue can be mimicked by a probabilistic Turing machine; hence, it can be implemented in a machine whose range of possible computations is a subset of those of a probabilistic Turing machine. This means that the basic scenario is computationally feasible.

By contrast, it is impossible to find any set of programming instructions able to force P3 as an additional property of the future behaviour of the machine. Indeed, the only way of blocking via a programming instruction the possibility that the task will never be performed is to pick an instant t after 6 p.m. of 10 December 2029 and to say that when the time measured by the clock is t, the machine performs the task. Even if t is chosen in a random way, this means that one has to assign from the beginning an end to the sequence of coin tosses and so that P2 has to be falsified, since at t it is no longer possible that the task will not have been performed immediately after. Thus, if taken together, P1, P2 and P3 represent a set of expected runs of the machine that gives rise to a computationally unfeasible scenario.

In conclusion, history-based structures make room for some indeterministic scenarios that cannot be triggered by any set of instructions given to a machine, while moment-based structures rule out these scenarios. One has therefore some ground to argue that, at least as long as a computational perspective is concerned, there is evidence to prefer moment-based structures over history-based ones. It is important to insist on the *restricted perspective* adopted in this article: our analysis does not entail that moment-based structures are the most appropriate models to account for an open future (as a kind of indeterminism) *in general*. Indeed, one could think that the computational perspective on indeterministic phenomena is too narrow; but the point of focusing on such a narrow perspective is that it offers a criterion of preference among alternative representations of time that is not a mere consequence of one's metaphysical intuitions: some indeterministic scenarios that cannot be encoded in a program have only models of a certain kind. Moreover, as we mentioned in Section 1, from a broader metaphysical perspective one could even claim that the future is open not only in terms of how it will unfold, but also in terms of whether it will unfold (will reality still exist in a thousand of years?) (cf. Correia and Rosenkranz 2018, p. 99). Yet, the possibility of a doomsday scenario in which there will be a last moment of time cannot be pictured by moment-based structures, since the absence of further branches from a node represents the absence of any open possibilities beyond that node, not that the end of the world is just *one* of the possibilities beyond that node (cf. Cameron 2015, p. 179). These latter considerations invite one to further explore the limits of moment-based structures in the light of the intuitions at the basis of various "open future-friendly" theories of time available in metaphysics, such as Presentism and the Growing Block Theory of time.

Conflict of interest statement

The present manuscript does not give rise to any conflict of interest.

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