Theory of loss measurements of Fabry–Perot resonators by Fourier analysis of the transmission spectra

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We present a detailed theoretical analysis for the determination of the total internal loss in Fabry–Perot resonators based on Fourier analysis of the emission or transmission spectrum. The observation of higher-order harmonics and their relative height in the Fourier-transformed spectrum allow us to quantify the total resonator loss. Because this new method considers both contrast and shape of the Fabry–Perot fringes it is especially well suited for the evaluation of high-finesse laser resonators such as those of vertical cavity surface-emitting lasers in terms of propagation loss/gain.

The analysis of Fabry–Perot (FP) resonators is central to the understanding of most types of laser. Although the working principle of both passive and active FP resonators is in general well understood and has been published,[1] some of their elementary properties are surprisingly difficult to measure, especially when it comes to experiments with high-finesse resonators. These problems are typical for resonators of semiconductor lasers at transparency and still below threshold but also for high-finesse wavelength-selective elements in gas lasers (so-called FP étalons).

The characterization of laser cavities in terms of propagation loss/gain is an important diagnostic ability and is especially of interest with regard to new semiconductor materials that can emit light in the blue or violet range of the spectrum. For other laser architectures such as vertical cavity light emitters it is of critical importance to have reliable numbers for the finesse. For low-finesse edge-emitting lasers, an established method is based on a contrast measurement of the FP fringes in the spectrum.[2,3] However, this method is no longer suitable for high-finesse resonators, mainly because of the difficulty in making precise measurements of the fringe contrast m, which is defined by $m = (I_{\max} - I_{\min})/(I_{\max} + I_{\min})$. When m approaches unity, an alternative technique, which measures the ratio between the free spectral range and the width of the transmission peaks, might be appropriate.[4] However, this method is valid only for finesse values above 5.

Here we present the theory and applications for a generalized method that can be used to measure the finesse of a FP resonator. The detailed analysis of Fourier-transformed transmission spectra[5,6] allows us to determine the cavity propagation loss/gain in FP resonators accurately. This new method can be used for both low- and high-finesse resonators; in addition, it is equally well suited for passive and active resonators. In the latter case the emission spectrum of the active device contains the relevant information, which is in fact identical to the transmission spectrum of the resonator. As is shown below, the cavity propagation loss/gain is related to the ratio between the Fourier coefficients of adjacent harmonics. Further, the FWHM of the single peaks is inversely proportional to the width of the wavelength range whose light was used in the experiment. This method works from zero device bias all the way up to lasing threshold where the round-trip gain is unity and is limited only by the resolution of the spectrometer employed. It is thus suitable for the analysis of laser structures and of other important devices operating under nonlasing conditions.[7,8]

For generality, we derive an analytical expression for the Fourier transform of a FP spectrum. If the considered FP resonator has a length L and consists of a piece of material with parallel facets, refractive index n, and absorption index k, then the transmitted electrical field amplitude experiences multiple reflections between the facet mirrors. It can be calculated as a geometrical series as given by

$$A(\beta) = [1 - R\exp(2i\psi)]\sum_{m=0}^{\infty}[R\exp(2i\psi)]^m \times \exp[-2kL(m + 1/2)\beta]\exp(2inLm\beta)\,. \quad (1)$$

In Eq. (1), $\beta = 2\pi/\lambda$ is the wave number, $R = [(n - 1)^2 + k^2]/[(n + 1)^2 + k^2]$ is the power reflectance of the facets, and $\psi = \arctan(-2k/n^2 + k^2 - 1)$ is the phase change of the light that is due to the facet reflection. Obviously, the expansion into a converging geometrical series fails if $R\exp(-2kL\beta) \geq 1$; for active resonators this means that the expansion works only below lasing threshold. From Eq. (1) we can calculate the transmitted intensity according to

$$I(\beta) = |A(\beta)|^2 = A(\beta)A^*(\beta) = \frac{(1 - R)^2\exp(-2kL\beta) + 4\sin^2(\psi)}{[1 - R\exp(-2kL\beta)]^2 + 4R\exp(-2kL\beta)\sin^2(\psi + nL\beta)}\,. \quad (2)$$

With the optical path length d as the conjugate variable of wave number β, the so-called $-i$ Fourier transform of $A(\beta)$, calculated for $k \neq 0$, is defined as in Ref. 9 and leads to

$$\overline{A(d)} = [1 - R\exp(2i\psi)] \sum_{m=0}^{\infty} \int_0^{\beta_{\max}} [R\exp(2i\psi)]^m \times \exp\{-2[kL(m + 1/2) + i(\pi d - nLm)]\beta\}\mathrm{d}\beta \,. \tag{3}$$

The upper limit of the integration is intentionally set at $\beta_{\max}$ to prevent the integral from diverging; as is shown below, $\beta_{\max}$ will drop out during further calculations. The lower limit's being zero has physical meaning; it restricts the function $A(\beta)$ to positive wave numbers. The above integration results in

$$\overline{A(d)} = [1 - R\exp(2i\psi)] \times \sum_{m=0}^{\infty} \frac{[R\exp(2i\psi)]^m}{2[kL(m + 1/2) + i(\pi d - nLm)]} \times (1 - \exp\{-2[kL(m + 1/2) + i(\pi d - nLm)]\beta_{\max}\}) \,. \tag{4}$$

The next step is the calculation of the convolution integral $\overline{I(d)} = \overline{A(d)} * \overline{A^*(d)}$, which yields

$$\overline{I(d)} = |1 - R\exp(2i\psi)|^2 \sum_{m=0}^{\infty} \sum_{l=0}^{\infty} R^{l+m} \exp[-2i\psi(l - m)] \times \int_{-\infty}^{\infty} \frac{\{1 - \exp[-2\pi i(x - x_1)\beta_{\max}]\}\{1 - \exp[-2\pi i(x_2 - x)\beta_{\max}]\}}{(2\pi i)^2(x - x_1)(x_2 - x)} \mathrm{d}x \,, \tag{5}$$

where

$$x_1 = \frac{-kL(m + 1/2) + inLm}{i\pi}, \qquad x_2 = \frac{kL(l + 1/2) + i(\pi d + nLl)}{i\pi} \tag{6}$$

are the two singularities of $\overline{I(d)}$. One can simplify the integration in Eq. (5) by making partial fractions and by the fact that always only one of the singularities given by Eqs. (6) contributes to the integration because x_1 and x_2 have imaginary parts of opposite sign. This procedure leads to

$$\overline{I(d)} = |1 - R\exp(2i\psi)|^2 \times \sum_{m=0}^{\infty} \sum_{l=0}^{\infty} \frac{R^{l+m}\exp[-2i\psi(l - m)]}{\pi i(x_2 - x_1)} = |1 - R\exp(2i\psi)|^2 \times \sum_{m=0}^{\infty} \sum_{l=0}^{\infty} \frac{R^{l+m}\exp[-2i\psi(l - m)]}{\{kL(l + m + 1) + i[\pi d + nL(l - m]\}} \,. \tag{7}$$

Equation (7) basically describes a function with peaks that are arranged symmetrically to the origin. The position of these peaks is given by $d = nL(l - m)/\pi$ (l, m integers) unveiling them as higher-order harmonics of the first-order peaks ($d = \pm nL/\pi$). The height of these harmonics decreases exponentially with increasing order. The optical path length d is proportional to the resonator length L, and the proportionality factor is n/π.

Although the complex function in Eq. (7) consists—in general—of amplitude and phase, we plot only the amplitude for the following considerations. Also, we use units of micrometers for the optical path length.

Figure 1 shows the transmitted intensity according to Eq. (2) for $R = 0.3$, $n = 1.77$, $k = 10^{-4.5}$, and $L = 100\ \mu$m in a wave-number range of $15.7 \pm 0.1\ \mu\mathrm{m}^{-1}$. Even though the finesse of this resonator, defined by $F = \pi\sqrt{R}/(1 - R)$, is relatively low ($F = 2.46$), the FP fringe shape already deviates clearly from being sinusoidal. This nonharmonic oscillating behavior is apparent in the Fourier transform by the presence of higher-order harmonics (see the inset of Fig. 1). As a comparison, we show in Fig. 2 the spectral data and the Fourier transform for a higher-finesse resonator with $R = 0.7$ and the same refractive and absorption indices as above ($F = 8.76$). The Fourier transform of this spectrum (see the inset of Fig. 2) shows that the higher-order harmonics are much stronger but still in a constant ratio to each other. In both cases this harmonic amplitude ratio (HAR) is related to the total resonator loss; the latter is a combination of mirror loss and cavity propagation loss/gain. Under the assumption of small absorption $k \ll n$ and a relatively narrow wavelength window ($\Delta\lambda \ll \lambda_0$), the HAR, r, turns out to be as simple as $r = R\exp(-2k\beta L)$ or, with α the absorption coefficient, $r = R\exp(-\alpha L)$. Inasmuch as the facet reflectance, and hence the mirror loss, is usually known, this method allows us to determine the cavity propagation loss/gain.

In Fig. 3 we present a nomograph that reveals the relationship among mirror loss, absorption loss, and HAR. One can thus use the nomograph in Fig. 3, with measured HAR, to deduce the absorption coefficient α given the mirror power reflectance R, or vice versa (see the example below). The advantage of this method is that it can be used for both low- and high-finesse resonators, mainly because the Fourier-transform

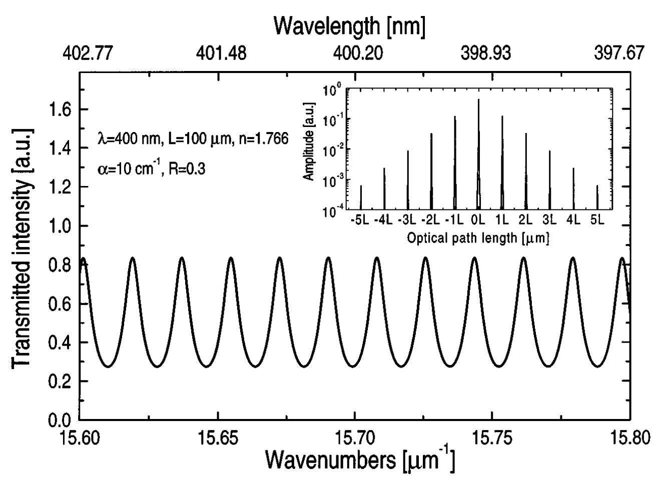

Fig. 1. Analytically calculated transmission spectrum of a FP resonator with 30% reflective mirrors. The numerical Fourier transform of the spectrum is shown in the inset.

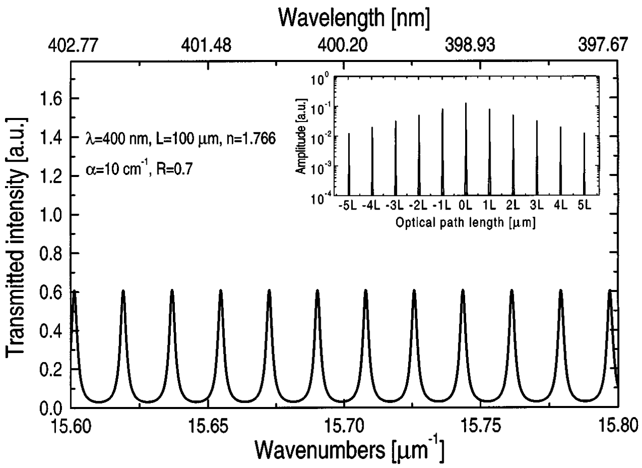

Fig. 2. Analytically calculated transmission spectrum of a FP resonator with 70% reflective mirrors. The numerical Fourier transform of the spectrum is shown in the inset.

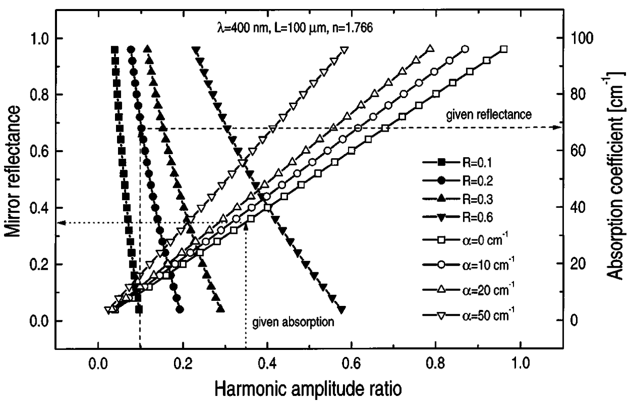

Fig. 3. Absorption loss in an active resonator for the values of mirror reflectance R shown and mirror reflectance for absorption loss α as a function of HAR.

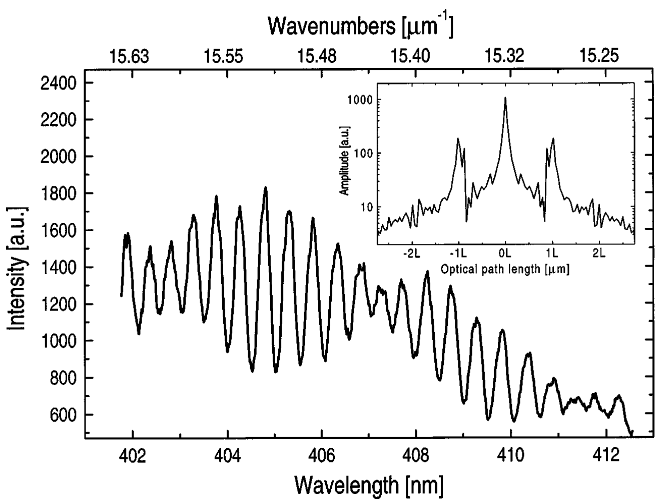

Fig. 4. Emission spectrum of a AlGalnN double heterostructure. The Fourier transform of the spectrum is shown in the inset.

process contains information on the overall shape of the FP fringe patterns instead of only on their contrast or their peak width/separation ratio. Assuming an experimental spectral resolution of $\Delta\lambda_{\text{res}} = 0.045$ nm and the mode separation of the above example ($\Delta\lambda =$ 0.45 nm), we are able to determine classically a resonator finesse of $F = \Delta\lambda/\Delta\lambda_{\text{res}} = 10$. Because the relevant information of the spectrum is in fact the shape of the transmission peaks rather than their width and separation, it is likely that using the same experimental setup as above and the Fourier-transform-based measurement technique should enable us to evaluate even higher-finesse FP resonators accurately.

As an illustrative example, let us analyze the emission spectrum of an AlGaInN double heterostructure grown upon a 100-μm-thick sapphire substrate. The 5-μm-thick epitaxial layers have an average refractive index of 2.51; the refractive index of the sapphire is 1.77. Because of the reflections at the GaN surface, the GaN–sapphire interface, and the sapphire surface, three different Fabry–Perot cavities were formed. However, only two of them show up as closely spaced peaks in the Fourier-transformed emission spectrum of this device. The third peak is too close to the origin to be resolved, mainly because of the large mode spacing ($\Delta\lambda = 5.3$ nm) of the AlGaInN cavity. A typical emission spectrum and its Fourier transform are shown in Fig. 4. By extracting a HAR of 0.1, and assuming an average mirror reflectance of 20%, one can find a cavity propagation loss of approximately 68 cm^{-1} by following the arrows that start at a HAR of 0.1. This example shows the danger of assuming that the presence of pronounced FP modes indicates that a device is close to threshold. In the example, the device has not yet reached the transparency condition ($\alpha = 0$).

In conclusion, we have shown a new powerful method for determination of the propagation loss of a FP resonator. The method is based on Fourier analysis of the transmission or, in active devices, of the emission spectrum and a subsequent measurement of the HAR in these Fourier transforms. In the especially interesting case of semiconductor laser resonators the transparency current level as well as the total internal loss can be determined.

The authors thank Dave P. Bour for crystal growth and Rose M. Donaldson for device processing. This research was supported by U.S. Department of Commerce contract ATP-70NANB2H1241.

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