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Estimating the out-of-hospital mortality rate using patient discharge data

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Summary

This paper explores the hospital quality measures based on routine administrative data such as patient discharge records. Most of the measures used in the literature are based on in-hospital mortality risks rather than post-discharge events. The in-hospital outcomes are sensitive to the hospital's discharge policy, thus could bias the quality estimates. This study aims at identifying out-of-hospital mortality risks and disentangling discharge and re-hospitalization rates from mortality rates using patient discharge data. It is shown that these objectives can be achieved without post-discharge death records. This is an example of the use of public use administrative data for estimating empirical relations when key dependent variables are not available. Using data on the lengths of hospitalizations and out-of-hospital spells, the mortality rates before and after discharge are estimated for a sample of heart-attack patients hospitalized in California between 1992 and 1998. The results suggest that the quality assessments that ignore the variation of discharge rates among hospitals could be misleading. Copyright © 2006 John Wiley & Sons, Ltd.

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Introduction

Assessing quality of health service providers is an important policy issue that has been subject of a great deal of research. Yet, measuring quality from routinely collected data is a challenging question for econometricians. In-hospital mortality has been widely used as a measure of quality of medical care. However, a major concern is that the in-hospital death outcomes do not necessarily reflect the long-term effects. Due to differences in discharge/transfer policies across hospitals the in-hospital mortality could be a biased measure, overstating the quality of hospitals with a relatively high discharge rate especially if low-quality hospitals discharge their patients prematurely or transfer their most severe cases to better hospitals.

In his study of ownership conversion, Sloan [1] reports that while the in-hospital mortality is not affected by conversions, the longer-term mortality probability has increased as hospitals converted to for-profit status. Those findings suggest that hospitals with shorter stays may have higher mortality rates after discharge. A complete measure of hospital-specific mortality risk should therefore include the post-discharge mortality risks. However, longitudinal surveys that follow patients after discharge are expensive, hardly available or limited to certain groups of patients. Alternatively, external sources such as expert evaluations or other independent measures can be used to validate the quality measures based on in-hospital mortality risks [2]. Such validation studies are however very expensive. On the other

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hand, hospital administrative records such as patient discharge data (PDD) provide a large number of observations at relatively low costs. Administrative data are often made available to researchers in public use files, which usually cannot be linked to external data such as death records. For instance, in the PDD public use files, the patient's identification is encrypted with a unique system that allows tracking any given patient only within the discharge data. Such encrypted identifications allow for instance to identify later re-hospitalizations but not any out-of-hospital event.

Therefore, most of the mortality measures used in the literature are based on in-hospital events. In many cases, the long-term effects have been taken into account by complementary measures. Many studies [3–5] have used the probability of re-hospitalization in the future with or without complication, to account for long-term effects. However, other studies [6,7] have found that readmission risks are related to the patient's clinical conditions rather than hospital quality. Moreover, because of negative correlation between mortality and future readmission [4], the estimations based on readmission usually do not provide additional information on hospital quality. Another approach used in the literature is a censored duration model of in-hospital mortality [8–10]. These models to some extent control for the variations in hospital stays across hospitals, but do not give any information about probabilities of discharge, re-hospitalization and post-discharge mortality.

When the deaths out of hospital are not observed, the statistical inference about the out-of-hospital mortality is complicated. Nevertheless, the hospital discharge records can be used to determine the duration of out-of-hospital spells for all patients. For a fraction of these spells that do not end with a second hospitalization, the death outcome occurs but is not observed. This paper shows that the duration of the out-of-hospital spells can be used to derive information about the long-term survival rates after discharge. Given the importance and availability of PDD, an estimation procedure that can accommodate such an analysis can be very useful.

The data used in this paper are taken from the California Patient Discharge Data Version A. This data set contains the records of all individuals who were hospitalized in California from 1992 through 1998. Unlike the other version of PDD, Version A provides a patient identification number. The PDD have been used to measure the quality of hospitals based on survival/death probabilities [3,8,11]. As far as we know, this paper is the first that estimates the mortality rate after discharge from the PDD. We show that out-of-hospital mortality rate is identified, even if deaths after discharge are not recorded. We apply the duration model framework to derive the distribution of hospital spells and out-of-hospital spells for this type of data. The model disentangles the discharge and re-hospitalization rates from mortality rates. In addition, using a simplified version of the derived distribution we evaluate the validity of the quality measures commonly used in the literature. It is shown that in most of the quality measures used in the literature the discharge rate is not disentangled from the mortality rate.

Another complication is that in the public use PDD the exact dates of admission have been omitted. Only the year and month in which the hospital stay started and the length of hospitalization in days are retained. Therefore, the derived distribution of out-of-hospital spells cannot be used directly. In this paper, we develop a statistical framework that deals with both problems, namely the censoring of death outcomes and the omission of exact dates. We show that, at the cost of loss of accuracy, the parameters of interest can be identified from the fragmentary data in the public use file. Therefore, this paper provides an example in which the data imperfections can be dealt with econometric modeling.

The plan of the paper is as follows. The next section introduces the statistical model. We discuss the identification of the out-of-hospital mortality rate if deaths after discharge are not recorded. We also show that most measures of in-hospital mortality that are commonly used in the literature do not fully separate the discharge outcome from survival. It's following section provides more information on the PDD. We also derive the distribution of the spells observed in the PDD. The Penultimate section contains the estimation results and the last section concludes the paper with a brief discussion of the main results.

Identifying the out-of-hospital mortality rate

In this section, we abstract from the problems created by the omission of the exact admission

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dates in the public use file. This shortcoming of the data will be discussed in the following section. Here, we address the problem of unobserved out-of-hospital death outcomes. We assume that for a member of the population the complete hospitalization history during the observation period $[0, T]$ is observed. A complete hospitalization history consists of a sequence of hospital stays and spells outside hospital (Figure 1) or equivalently, of a sequence of transitions between two states: hospitalized (H) and discharged (D). A hospital spell ends if the patient is discharged or if she dies. An out-of-hospital spell ends if the patient is admitted or if she dies. Death is thus considered as a transition to a third absorbing state.

Basically, if the out-of-hospital deaths were known, the problem would reduce to a three-state duration model similar to those models used in modeling unemployment and labor participation [12–14]. In line with this literature we use a proportional hazard framework. In our case however, the time of death is observed only if the patient dies in a hospital. The problem is to estimate the transition rates and in particular, the out-of-hospital mortality rate from the observed hospitalization records. The methodology used here is very similar to the approach used in 'capture–recapture' models for estimating demographic parameters of wildlife populations. Pollock [15] provides a survey of these models.

The hospitalization record has multiple time scales: the observation times (0 is the start of the observation period), the duration of hospital or out-of-hospital spells (0 is the start of the spell), the time since the onset of the disease, calendar time, and age. In the sequel both observation and duration time are used. It is clear from the context which time scale is used.

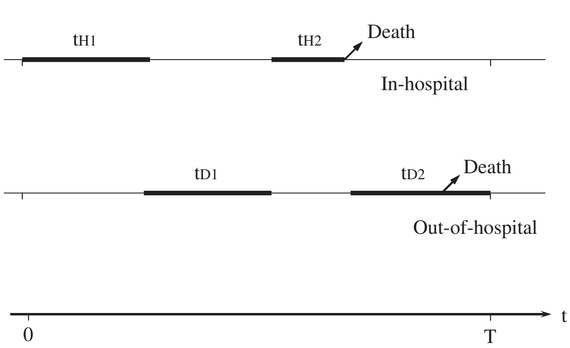

Figure 1. Hospitalization record

The in-hospital mortality and discharge rates

A hospital spell is denoted by t_H. As shown in Figure 1, a hospital spell ends with the death of the patient with intensity $\mu_H(t)$ or with the discharge of the patient with intensity $\lambda_D(t)$. A hospital spell could also end with the transfer of the patient to another hospital. This could be considered by introducing a transfer intensity. Here, only one hospital spell is considered and the patients who have been transferred to another hospital are excluded from the sample. In fact, as the estimated mortality rates are usually used as a hospital quality measure, it is difficult to distinguish the contribution of each one of the hospitals in survival rates. In some administrative records, transfers are not distinguished from other discharges. In such cases, $\lambda_D(t)$ can be considered as a weighted average (with weights depending on the hospital spell) of the discharge and transfer densities.

Let D_H be 1 if the spell ends with discharge and 0 if it ends with the death of the patient. The joint distribution of t_H, D_H has the following pdf:

$$f_H(t, D_H) = e^{-M_H(t)-\Lambda_D(t)} \lambda_D(t)^{D_H} \mu_H(t)^{1-D_H} \tag{1}$$

with $M_H(t) = \int_0^t \mu_H(s)ds$ and $\Lambda_D(t) = \int_0^t \lambda_D(s)ds$. μ_H and λ_D are assumed to be piecewise constant over k intervals $0 = t_0 < t_1 < \cdots < t_{k-1} < t_k = t_{max}$ where t_{max} is the longest hospital stay. μ_H and λ_D are also functions of covariates like patient and hospital characteristics. The covariates are assumed to be constant over time. If X is the vector of covariates, the hazard functions can be written as

$$\mu_H(t) = \exp(X\beta) \sum_{i=1}^{k} \mu_H^i I_{(t_{i-1} < t \leqslant t_i)} \tag{2}$$

$$\lambda_D(t) = \exp(X\gamma) \sum_{i=1}^{k} \lambda_D^i I_{(t_{i-1} < t \leqslant t_i)} \tag{3}$$

where $I_{(A)}$ is the indicator function taking 1 if condition A is satisfied and zero otherwise; μ_H^i and λ_D^i are constants corresponding to interval $(t_{i-1}, t_i]$; and γ and β are the vectors of coefficients corresponding to the independent variables. The pdf given in (1) is the basis for the likelihood function for the in-hospital spells.

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The out-of-hospital mortality and hospitalization rates

For the identification of the out-of-hospital mortality rate the spell spent outside hospital denoted by t_D is considered (Figure 1). This spell starts at the time of discharge from the hospital. It ends if the patient returns to the hospital (not necessarily the same hospital) or if she dies. However, the death is not observed. Let λ_H denote the hospitalization rate and μ_D the mortality rate outside hospital. These rates may depend on the time since the last hospitalization t. For ease of exposition it is assumed that this spell starts at time 0 and that it is censored at time T. The distribution of t_D is mixed discrete-continuous with a positive probability that $t_D \leqslant T$. To show this consider for $t \leqslant T$:

$$\Pr(t_D > t) = e^{-\Lambda_H(t) - M_D(t)} + \int_0^t \mu_D(s) e^{-\Lambda_H(s) - M_D(s)} ds \quad (4)$$

with $\Lambda_H(t) = \int_0^t \lambda_H(s)ds$ and $M_D(t) = \int_0^t \mu_D(s)ds$. The first term on the right-hand side is the probability that by t neither a death nor a hospitalization has occurred. The second term is the probability that during $[0,t]$ the individual has died. In this case, since deaths outside hospitals are not observed, the observed spell is still in progress. In fact, for all patients who die before re-hospitalization the observed spell t_D is of infinite length. This means that the distribution of t_D is defective and the probability of observing an infinite spell is the average of the probability of death before hospitalization, where the average is computed over the duration of the latent out-of-hospital spell, that is: $\int_0^T \mu_D(s) e^{-\Lambda_H(s) - M_D(s)} ds$.

If the observation period is finite, t_D is observed if $t_D \leqslant T$. Otherwise, the event $t_D > T$ is observed. Define D_D as the indicator of the event $t_D \leqslant T$. The probability density of t_D given $D_D = 1$ is written as

$$f(t|D_D = 1) = \frac{\lambda_H(t) e^{-\Lambda_H(t) - M_D(t)}}{\int_0^T \lambda_H(s) e^{-\Lambda_H(s) - M_D(s)} ds}, \quad t \leqslant T \quad (5)$$

Moreover,

$$\Pr(D_D = 0) = \Pr(t_D > T) = e^{-\Lambda_H(T) - M_D(T)} + \int_0^T \mu_D(s) e^{-\Lambda_H(s) - M_D(s)} ds \quad (6)$$

The pdf given in (5) and the probability in (6) are the basis of the likelihood estimation of the out-of-hospital mortality and re-hospitalization rates. μ_D and λ_H are assumed to be piecewise constant over k' intervals $0 = T_0 < T_1 < \cdots < T_{k'-1} < T_{k'} = T$. Similarly, the constant effects of covariates (X) are included in a proportional hazard framework, resulting in the following hazard functions:

$$\mu_D(t) = \exp(X\eta) \sum_{i=1}^{k'} \mu_D^i I_{(T_{i-1} < t \leqslant T_i)} \quad (7)$$

$$\lambda_H(t) = \exp(X\zeta) \sum_{i=1}^{k'} \lambda_H^i I_{(T_{i-1} < t \leqslant T_i)} \quad (8)$$

where μ_D^i and λ_H^i are constant rates corresponding to interval $(T_{i-1}, T_i]$, and η and ζ are the vectors of coefficients.

To show that both hospitalization and mortality rates are identified consider first the special case where both rates are constant over time. In this case, the conditional pdf (5) reduces to

$$f(t|D_D = 1) = \frac{(\lambda_H + \mu_D) e^{-(\lambda_H + \mu_D)t}}{1 - e^{-(\lambda_H + \mu_D)T}}, \quad t \leqslant T \quad (9)$$

which is the pdf of a truncated (at T) exponential distribution with parameter $\kappa = \mu_D + \lambda_H$. Hence, from the distribution of spells that end in hospitalization the sum of mortality and hospitalization rates is identified. Moreover, the probability of re-hospitalization before T is

$$\Pr(D_D = 1) = \frac{\lambda_H}{\lambda_H + \mu_D} (1 - e^{-(\lambda_H + \mu_D)T}) \quad (10)$$

Since $\kappa = \mu_D + \lambda_H$ is identified from (9), λ_H is identified using the probability given in (10). The joint distribution of t_D,D_D has the following pdf:

$$f_D(t, D_D) = \left(\lambda_H e^{-(\lambda_H + \mu_D)t}\right)^{D_D} \left(\frac{\mu_D}{\lambda_H + \mu_D} + \frac{\lambda_H e^{-(\lambda_H + \mu_E)T}}{\lambda_H + \mu_D}\right)^{1 - D_D} \quad (11)$$

The above argument can be extended to piecewise constant rates $\mu_D(t)$ and $\lambda_H(t)$. It suffices to first censor the out-of-hospital spells at T_1 (the first interval). The rates are constant over the interval thus identified using the censored spells. The spells that end with a hospitalization in the interval identify the sum of mortality and hospitalization rates and the fraction of spells that are censored identify the rates separately. Next, consider the

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out-of-hospital spells that end with a hospitalization in the second interval $(T_1, T_2]$. It can be shown that the distribution of these spells is such that $t_D - T_1$ has a truncated (at $T_2 - T_1$) exponential distribution with a parameter that is the sum of mortality and hospitalization rates on the second interval. Hence, this distribution identifies the sum. The hospitalization and mortality rates are identified from the fraction of spells that are censored at T_2. This argument can be repeated for the remaining intervals.

Measures used in the literature

In this section, the quality measures used in the literature are discussed using the proposed model. The measures can be divided into three categories: in-hospital mortality outcome, mortality outcome within a given period after admission, and readmission within a given period after discharge. For ease of exposition it is assumed that all transition rates are constant.

A number of papers [2,3,16] used the mortality outcome at discharge. This measure can be written as a function of in-hospital mortality and discharge rates:

$$\Pr(D_H = 0) = \frac{\mu_H}{\lambda_D + \mu_H} \quad (12)$$

It can be shown that the in-hospital death probability is increasing in μ_H and decreasing in λ_D. To the extent that discharge practices differ across hospitals, this measure cannot be used as a hospital-specific mortality. An alternative used by Geweke *et al.* [11] is the in-hospital death probability within 10 days after admission. This in-hospital mortality probability within period t after admission can be written as:

$$\Pr(D_H = 0, t_H \leqslant t) = \frac{\mu_H}{\lambda_D + \mu_H}(1 - e^{-(\lambda_D + \mu_H)t}) \quad (13)$$

In this case, depending on the chosen value of t, the death probability can be decreasing or increasing in μ_H. Therefore, even assuming a constant discharge rate across hospitals, this cannot be used as a measure of hospital-specific mortality.

Another commonly used measure is the death probability within a given period after admission. These deaths may occur inside hospitals or after discharge. Some studies [17–21] have used mortality within 30 days while others [4,22,23] used longer periods up to one year. This measure may seem appealing because it can represent a relatively long-term outcome that is seemingly independent of discharge rates.

The probability of death within t days after admission can be written as the sum of the probabilities of the in-hospital and post-discharge death before t, that is

$$\int_0^t \mu_H e^{-(\lambda_D + \mu_H)s} ds + \int_0^t \lambda_D e^{-(\lambda_D + \mu_E)s} \mu_D e^{-(\lambda_H + \mu_D)(t-s)} ds$$

It is easy to show that such measures are affected by discharge and hospitalization rates.

Another measure of quality is the re-hospitalization probability within a given period after discharge. Various authors [3,13,18,23] have considered different periods usually varying between 14 days to a few months. The readmission probability within t days after discharge can be written as

$$\Pr(t_D \leqslant t) = \frac{\lambda_H}{\lambda_H + \mu_D}(1 - e^{-(\lambda_H + \mu_D)t}) \quad (14)$$

The problem with this measure is that for short readmission periods (small t), it is not increasing in λ_H, and for relatively large periods the correlation between readmission risk and hospital quality is low [7]. Moreover, as it can be seen this measure depends on the out-of-hospital mortality rate. In fact, for short periods (small t) this measure is a decreasing function of μ_D. In cases where the out-of-hospital mortality is not observed, small rates of re-hospitalization may be associated with high mortality rates, hence not necessarily a higher hospital quality. The above problems provide an explanation to why the readmission measures of quality as used in the literature are inconsistent with other measures of hospital quality [6,7].

Ettner and Hermann [24] used the readmission within 30 days after discharge for psychiatric patients. Given that mortality rates are quite low for these patients, the readmission measure may be appropriate. Assuming that μ_D is close to zero, the probability given in (14) can be simplified as: $(1 - e^{-\lambda_H t})$, which is a non-decreasing function of λ_H, and therefore can be used as a proxy for re-hospitalization rate.

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The patient discharge data

Description of the data

The data used in this paper are extracted from California Hospital Discharge Data. The population considered in this paper are all individuals of 65 years of age or older who were hospitalized during 1992–1998 with Acute Myocardial Infarction (AMI) as their principal diagnosis and who were in an initial episode of treatment. This data set has been merged with data from California Hospital Disclosure Data on hospital characteristics such as size and ownership status. A detailed description of these data has been given elsewhere [25,26].

From the original sample including about 173 000 hospitalizations of 163 000 patients, we excluded the patients older than 95 years old and those who have been transferred from (or to) other hospitals leaving about 132 000 patients. The transferred cases have been excluded mainly because their survival probabilities cannot be related to a single hospital and distinguishing each hospital's contribution is difficult, if at all possible. To further simplify the analysis we also excluded all the patients (less than 3% of the sample) who had multiple hospitalizations in their first admission month or whose first hospital spell was longer than a month. Moreover, since one of the parameters of interest is the effect of ownership status on hospital quality, in order to avoid the reporting errors of ownership changes and their complicated effects in quality [27], we focused our analysis to hospitals that had a stable ownership status over the sample period. The final sample consists of 115 805 AMI patients hospitalized in 387 California hospitals.

AMI is an acute condition and these patients are less subject to selection problems. Systematic selection of patients to specific hospitals may bias the estimates of hospital characteristics on mortality rates. Heart-attack patients are likely to go the closest hospital. Moreover, a considerable part of deaths caused by AMI occur inside hospitals. The elderly age group is chosen because all these patients benefit from Medicare and are less likely to be rejected by hospitals. The identification of post-discharge mortality relies on the assumption that an out-of-hospital spell ends in re-hospitalization or death. A third possibility is that the patient leaves the state of California. The migration is less likely for the elderly patients with an acute condition.

Using the patient identification numbers that are encrypted unique numbers, the patients in the sample have been linked to another data set including all the hospitalizations in California (for any reason) over the sample period. The latter data set including about 10 million patients has been extracted from the PDD files. For each patient in the sample the total lengths of hospitalizations in the first month and in the readmission month were calculated. For each patient, the first month is the month in which her initial hospitalization for AMI has occurred. The second and later hospitalizations need not be for AMI and can be for any condition.

Implementation of the model

The estimation of hospital spells is straightforward and the joint distribution of t_{H}, D_{H} with piecewise constant rates can be directly derived from Equation (1) using (2) and (3). A complete derivation of the joint distribution and the likelihood function is provided in the Appendix. For the out-of-hospital spells, because of the limitations of the data, the exact length of spell is not known. Instead, we derive bounds on the out-of-hospital spells that correspond to these data, and we use these interval data in our estimation. The data consist of a sequence of hospital spells together with the month in which each of these began. A typical realization for a given patient is illustrated in Figure 2. Suppose that the months in the sample period (1992–1998) are, respectively, numbered from 1 to M. Let m_1 denote the number of month in which the patient was first hospitalized for AMI, and m_2 the number of month in which she was re-hospitalized (for any reason) after the initial discharge. Note that m_1 and m_2 have patient-specific values. Note also that a patient can have multiple hospitalizations in a given month, but the first AMI admission is uniquely identified for all patients in the sample.

For the spells that do not end in re-hospitalization the contribution to the likelihood function is given by (6) using expressions (7) and (8). The end of observation period (T) used in (6) is a patient-specific variable. T in days is given by

$$T = 30.5(M - m_1) - t_{\mathrm{H0}} \qquad (15)$$

where t_{H0} is the length of the initial hospitalization.

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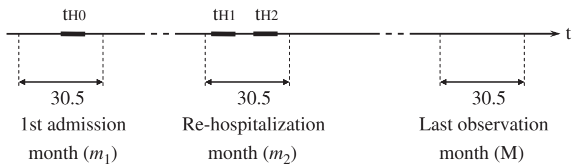

Figure 2. In-hospital spells

As for the cases that end in a re-hospitalization, the out-of-hospital spell t_D can be specified with the following lower and upper bounds:

$$t_D^{\inf} = 30.5(m_2 - m_1 - 1) - t_{H0} \quad (16)$$

$$t_D^{\sup} = 30.5(m_2 + 1 - m_1) - t_{H0} - \left(\sum_{i=1}^{k} t_{Hi} - \max\{t_{Hi}; i = 1, ..., k\}\right) \quad (17)$$

where k is the number of hospital spells that started in the re-hospitalization month m_2, and t_{Hi} ($i = 1, 2, \ldots, k$) is the length (in days) of these hospital spells. Note that the above definitions can be readily extended to cases with multiple admissions in the first month, in which case t_{H0} must be set equal to the first month's longest hospitalization in (15) and (16), and the upper bound (17) must be reduced by the sum of the remaining hospital spells of that month.

The spells that end in re-hospitalization make the following contribution to the likelihood function:

$$\Pr(t_D^{\inf} < t_D < t_D^{\sup}) = \int_{t_D^{\inf}}^{t_D^{\sup}} \lambda_H(s) e^{-\Lambda_H(s) - M_D(s)} ds \quad (18)$$

where the integrals $\Lambda_H(t)$ and $M_D(t)$ are obtained using the expressions in (7) and (8). A complete derivation of the likelihood function is provided in the Appendix.

Estimation results

The data on the first reported hospital spell are used to estimate the in-hospital mortality rate and the discharge rate. The in-hospital sample includes the entire sample of 115 805 elderly patients, hospitalized for an initial episode of AMI. The summary statistics are given in Table 1. The average hospital spell is about 6.4 days and about 17% of the spells end with the death of the patient. For 94 842 patients from this sample the out-of-hospital spells are calculated. Note that the patients who died in hospital are excluded from the out-of-hospital sample. About 65% of these patients were readmitted after their first hospitalization and before the end of the observation period. For these patients the lower bound of out-of-hospital spells varies from 0 to about 2465 days (with an average of 264 days) and the upper bound varies between 7 and 2526 days (with an average of 321 days). Table 2 gives the summary statistics for out-of-hospital spells.

Table 1. Sample statistics for hospital spells ($N = 115\,805$)

	Mean	Standard deviation
Hospital stay (days)	6.356	4.378
Discharged alive	0.828	0.377
For-profit hospital	0.131	0.338
Public hospital	0.111	0.314
Number of beds /1000	0.288	0.167
Male	0.530	0.499
Black	0.048	0.214
Age 70–74	0.221	0.415
Age 75–79	0.214	0.410
Age 80–84	0.191	0.393
Age 85 +	0.184	0.387
Moderate severity	0.380	0.485
Major severity	0.300	0.458
Extreme severity	0.201	0.401
Year 1993	0.144	0.351
Year 1994	0.141	0.348
Year 1995	0.141	0.348
Year 1996	0.141	0.348
Year 1997	0.146	0.353
Year 1998	0.146	0.353

Discharge and in-hospital mortality rates are assumed to be piecewise constant over 5 intervals: 0 to 2 days, 2 to 4, 4 to 6, 6 to 10, and more than 10 days. Table 3 gives a summary of the regression results for the hospital spells. For each listed variable the estimated coefficients represent the variable's marginal effects on the hazard rates of discharge and in-hospital mortality, respectively. For instance, the results suggest that in-hospital mortality hazard rate in for-profit (FP) hospitals is on average 8% higher than in non-profit (NP) hospitals (the omitted category). FP hospitals also show a 5% lower discharge hazard rate compared to NP hospitals. The results also indicate that compared to the base category (non-profit hospitals), public hospitals have higher rates in both mortality and discharge (by about 5%). Hospital size has a significant effect on both mortality and discharge rates with large hospitals having lower rates.

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Table 2. Sample statistics for out-of-hospital spells ($N = 94\,842$)

	Mean	Standard deviation
Re-hospitalized before the end of observation period	0.647	0.478
Lower bound of out-of-hospital spell (days)	264.112	398.564
Upper bound of out-of-hospital spell (days)	321.340	400.398
Spell until the end of observation period (days)	1275.544	735.344
For-profit hospital	0.126	0.331
Public hospital	0.109	0.312
Number of beds /1000	0.292	0.168
Male	0.538	0.499
Black	0.049	0.216
Age 70–74	0.230	0.421
Age 75–79	0.215	0.411
Age 80–84	0.182	0.386
Age 85 +	0.169	0.375
Moderate severity	0.435	0.496
Major severity	0.293	0.455
Extreme severity	0.131	0.338
Year 1993	0.144	0.351
Year 1994	0.142	0.349
Year 1995	0.142	0.349
Year 1996	0.144	0.351
Year 1997	0.150	0.357
Year 1998	0.138	0.345

Table 3. Mortality and discharge rates for hospital spells

	Discharge rate		Mortality rate	
	MLE	Standard error	MLE	Standard error
For-Profit hospital	−0.051*	0.010	0.080*	0.020
Public hospital	0.044*	0.010	0.050*	0.022
Number of beds /1000	−0.490*	0.020	−1.089*	0.049
Male	0.025*	0.007	−0.041*	0.014
Black	0.023	0.015	−0.123*	0.035
Age 70–74	−0.026*	0.010	0.148*	0.026
Age 75–79	−0.042*	0.010	0.257*	0.026
Age 80–84	−0.029*	0.011	0.484*	0.025
Age 85 +	0.014	0.011	0.602*	0.025
Moderate severity	−0.355*	0.010	0.752*	0.064
Major severity	−0.908*	0.011	2.018*	0.062
Extreme severity	−1.733*	0.013	2.806*	0.061
Year 1993	0.111*	0.012	−0.045	0.025
Year 1994	0.241*	0.012	−0.044	0.026
Year 1995	0.327*	0.012	−0.017	0.026
Year 1996	0.421*	0.012	−0.092*	0.026
Year 1997	0.466*	0.012	−0.159*	0.026
Year 1998	0.462*	0.012	−0.060*	0.026
Interval 2–4 days	1.554*	0.015	−0.974*	0.020
Interval 4–6 days	2.220*	0.015	−1.273*	0.024
Interval 6–10 days	2.431*	0.015	−1.411*	0.025
More than 10 days	2.570*	0.016	−1.248*	0.025
Constant	−3.220*	0.020	−4.833*	0.067

*Significant at 5%.

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As expected, both severity and age have a positive effect on mortality. The discharge rate is negatively affected by severity and age, but the age effects on discharge are not uniform. This could be explained by the fact that very old patients might get discharged to nursing homes or long-term care centers. The calendar year effects indicate that there is no significant trend in the mortality rate, but there is a strong upward trend in the discharge rate suggesting a general tendency toward shorter hospitalizations. The significant changes in transition rates over the intervals show that the rates are time-variant. For instance, the mortality rate in the first two days of the spell is significantly higher than in the rest of hospitalization. This result has an important health policy implication pointing to the crucial importance of the immediate stabilization of AMI patients.

The significant variation of discharge rates across hospitals with different ownership status supports the concern that lower in-hospital mortality rates may be associated to higher discharge rates. For instance, the results suggest that part of the difference in mortality between FP and NP hospitals could be associated with different discharge rates across the two hospital types. Therefore, the in-hospital mortality rate does not give a complete picture regarding hospital quality. On the other hand, relatively high discharge rates in the NP hospitals do not represent a lower quality in itself, as long as it does not lead to higher chances of post-discharge mortality.

The estimation results for the out-of-hospital mortality and re-hospitalization rates are given in Table 4. Transition rates are assumed to be constant. This table does not show any significant effect of hospital ownership on the mortality and re-hospitalization rates. The hospital size shows a significant and negative effect on both re-hospitalization and mortality. Combining these results with those of Table 3, one could conclude that mortality rates are on average lower in larger hospitals. As expected, severity and age has a positive effect on both re-hospitalization and mortality rates. The results also indicate a significant growth in both hospitalization and mortality rates. These findings along with those of Table 3 suggest that over time, hospital stays have become on average shorter resulting in lower in-hospital mortality rates but higher re-hospitalization rates and higher out-of-hospital mortality probability.

Another interesting example that can be used to highlight the contribution of the proposed model is the analysis of quality differences between NP and FP hospitals, based on mortality outcomes. A few empirical studies on US hospitals suggest

Table 4. Mortality and re-hospitalization rates for out-of-hospital spells

	Re-hospitalization rate		Mortality rate	
	MLE	Standard error	MLE	Standard error
For-profit hospital	0.021	0.015	−0.002	0.025
Public hospital	−0.001	0.016	0.016	0.026
Number of beds /1000	−0.078*	0.030	−0.249*	0.051
Male	−0.036*	0.010	0.001	0.017
Black	0.128*	0.022	−0.075	0.039
Age 70–74	0.071*	0.015	−0.010	0.026
Age 75–79	0.124*	0.015	−0.068*	0.026
Age 80–84	0.170*	0.016	−0.015	0.027
Age 85 +	0.204*	0.017	0.137*	0.027
Moderate severity	0.337*	0.017	0.086*	0.030
Major severity	0.588*	0.018	0.296*	0.031
Extreme severity	0.829*	0.020	0.608*	0.033
Year 1993	0.079*	0.016	0.152*	0.029
Year 1994	0.165*	0.017	0.394*	0.029
Year 1995	0.267*	0.017	0.697*	0.029
Year 1996	0.416*	0.018	1.051*	0.029
Year 1997	0.655*	0.019	1.557*	0.031
Year 1998	1.037*	0.023	2.243*	0.041
Constant	−6.847*	0.023	−7.768*	0.041

*Significant at 5%.

relatively high AMI mortality rates for FP hospitals [4,28]. This result is similar to our results suggesting relatively high re-hospitalization rate and in-hospital mortality for FP hospitals. However, our findings also show that a major part of the FP hospitals' excess in-hospital death rate might be related to lower discharge rates. Moreover, although FP hospitals show slightly (but not significantly) higher re-admission rates, their out-of-hospital mortality risks are similar to NP hospitals. Finally, the relatively high discharge rate in NP hospitals (Table 3) is not associated with higher probability of out-of-hospital death or readmission for these hospitals. Therefore, the results suggest that though being different in discharge and in-hospital mortality, NP and FP hospitals do not show any quality difference in this regard. However, public hospitals that indicate relatively high in-hospital mortality and discharge rates also have slightly (but not significantly) higher post-discharge death probability. This might be interpreted as a relatively low quality of care in these hospitals.

A shortcoming of the out-of-hospital analysis is that the transition rates are assumed to be constant. A version of the model with piecewise constant rates with one cutoff point has been applied to a similar data set [25]. The high estimation errors of the possible changes in hazard rates in that analysis indicate that with the available data such an extension does not provide any significant improvement over the constant-rate model used in this study. This can be explained by the fact that with the available data we cannot calculate the exact duration of out-of-hospital spells.

Conclusions

This paper has explored the measures of hospital quality based on mortality risks estimated from hospital administrative data. These measures are commonly based on in-hospital death outcomes, which might be affected by hospitals transfer/discharge policies. Using a transition model it has been shown that the out-of-hospital mortality rates can be identified using the patient discharge records without data on post-discharge deaths. This is an example of the use of public administrative data for the estimation of empirical relations when key independent variables are not available in the data. The paper shows that, with certain assumptions, the data on the duration of hospitalizations and out-of-hospital spells can be used to estimate the mortality rates before and after discharge as well as discharge and re-hospitalization rates. The analysis is based on an important assumption that patients do not have access to hospitals outside the sample. The common measures of hospital quality based on mortality risks, used in the literature are studied. Most of these measures do not distinguish discharge from survival. Given the significant variation of discharge rates across hospitals, such measures of quality may be misleading.

The model has been applied to a sample of heart-attack patients hospitalized in California general hospitals from 1992 to 1998. The analysis has been performed for in-hospital and out-of-hospital spells separately. The in-hospital analysis indicates a considerable variation in the discharge rate of AMI patients among different hospital types. For instance, a low incidence of in-hospital mortality in a hospital type could be together with a high rate of discharge. Therefore, the use of such mortality outcomes as a measure of hospital performance, without considering the discharge rates could be misleading. In particular, the results suggest that the relatively high in-hospital mortality rate in FP hospitals is partly due to their low discharge rate. However, public hospitals in the sample show relatively high rates in both in-hospital mortality and discharge.

As for the out-of-hospital analysis, an important complication of this data set is that the admission dates are identified only up to a month. The estimation procedure has been modified to accommodate this lack of data by writing the likelihood function based on upper and lower bounds rather than the exact length of the out-of-hospital spells. This comes at a loss in efficiency, which could potentially result in relatively high estimation errors in the parameters estimates. The results suggest that hospital ownership does not have a significant effect on out-of-hospital mortality or re-hospitalization rates. However, larger hospitals show on average lower incidences in both rates.

There are a few caveats in the present study, which are left for further research. First, in the epidemiological literature [29–31] additive covariate models are generally preferred over multiplicative forms such as proportional hazard framework used in this paper. The application the proposed model in the additive competing risks framework could be an interesting extension. Second, the restriction of piecewise constant hazard rates could be relaxed by using semi-parametric models. Third, the unobserved heterogeneity can be taken into account by introducing stochastic variation in the model's parameters. Finally, and most importantly, a validation study using data with observed out-of-hospital deaths or a Monte Carlo simulation study should be used to validate the adopted methodology regarding the out-of-hospital mortality rates. Pending such validation studies, the results obtained in this paper cannot be directly used for any policy conclusions. Rather, the adopted methodology underscores the potential use of incomplete data for statistical inference about unobserved events.

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Appendix A: Derivation of the likelihood function

In-hospital spells

Using piecewise integration and plugging relations (2) and (3) into Equation (1), the joint probability distribution corresponding to in-hospital spells can be written as:

$$f_{\mathrm{H}}(t, D_{\mathrm{H}}) = \exp\left\{-\sum_{i=1}^{k} I_{(t>t_i)}(t_i - t_{i-1})M^i - \sum_{i=1}^{k} I_{(t_{i-1}<t\leqslant t_i)}(t_i - t_{i-1})M^i\right\} \times \left[\exp(X\gamma)\sum_{i=1}^{k}\lambda_{\mathrm{D}}^i I_{(t_{i-1}<t\leqslant t_i)}\right]^{D_{\mathrm{H}}} \times \left[\exp(X\beta)\sum_{i=1}^{k}\mu_{\mathrm{H}}^i I_{(t_{i-1}<t\leqslant t_i)}\right]^{1-D_{\mathrm{H}}} \quad \text{(A1)}$$

where $M^i = \mu_{\mathrm{H}}^i \exp(X\beta) + \lambda_{\mathrm{H}}^i \exp(X\gamma)$.

The log-likelihood function is obtained by the following summation:

$$\log L(\beta, \gamma, \mu_{\mathrm{H}}^1, \ldots, \mu_{\mathrm{H}}^k, \lambda_{\mathrm{D}}^1, \ldots, \lambda_{\mathrm{D}}^k) = \sum_{n=1}^{N} \log f_{\mathrm{H}}(t^n, D_{\mathrm{H}}^n) \quad \text{(A2)}$$

wherer N is the sample size and superscript n denotes the observation number.

Out-of-hospital spells

The probability that the spell does not end in a re-hospitalization can be obtained from Equation (6) by substituting mortality and re-hospalization rates, respectively, from (7) and (8), and using piecewise integration:

$$\Pr(D_{\mathrm{D}}=0) \equiv \Pr(t_{\mathrm{D}} > T) = \exp\left(-\sum_{i=1}^{k'} I_{(T>T_i)}(T_i - T_{i-1})K_i - \sum_{i=1}^{k'} \mathrm{I}_{(T_{i-1}<T\leqslant T_i)}(T_i - T_{i-1})K_i\right) + \sum_{i=1}^{k'} \frac{\mu_{\mathrm{D}}^i \exp(X\eta)}{K_i} \mathrm{I}_{(T>T_i)}[\exp(-K_i T_{i-1}) + \exp(-K_i T_i)] + \sum_{i=1}^{k'} \frac{\mu_{\mathrm{D}}^i \exp(X\eta)}{K_i} \mathrm{I}_{(T_{i-1}<T\leqslant T_i)}[\exp(-K_i T_{i-1}) + \exp(-K_i T_i)] \quad \text{(A3)}$$

where $K_i = \lambda_{\mathrm{H}}^i \exp(X\zeta) + \mu_{\mathrm{D}}^i \exp(X\eta)$.

Similarly, the probability related to the spells that end in re-hospitalization can be obtained from Equation (18):

$$\Pr(t_{\mathrm{D}}^{\mathrm{inf}} < t_D < t_{\mathrm{D}}^{\mathrm{sup}}) = \sum_{i=1}^{k'} \frac{\lambda_{\mathrm{H}}^i \exp(X\zeta)}{K_i} \mathrm{I}_{(t_{\mathrm{D}}^{\mathrm{inf}} \leqslant T_{i-1} < T_i \leqslant t_{\mathrm{D}}^{\mathrm{sup}})}[\exp(-K_i T_{i-1}) + \exp(-K_i T_i)] + \sum_{i=1}^{k'} \frac{\lambda_{\mathrm{H}}^i \exp(X\zeta)}{K_i} \mathrm{I}_{(T_{i-1} < t_{\mathrm{D}}^{\mathrm{inf}} < T_i)}[\exp(-K_i t_{\mathrm{D}}^{\mathrm{inf}}) + \exp(-K_i T_i)] + \sum_{i=1}^{k'} \frac{\lambda_{\mathrm{H}}^i \exp(X\zeta)}{K_i} \mathrm{I}_{(T_{i-1} < t_{\mathrm{D}}^{\mathrm{sup}} < T_i)}[\exp(-K_i T_{i-1}) + \exp(-K_i t_{\mathrm{D}}^{\mathrm{sup}})] \quad \text{(A4)}$$

where $t_{\mathrm{D}}^{\mathrm{inf}}$ and $t_{\mathrm{D}}^{\mathrm{sup}}$ are, respectively, given in (16) and (17).

The joint likelihood corresponding to out-of-hospital spells can be written as

$$\ell_{\mathrm{H}}(t_{\mathrm{D}}^{\mathrm{inf}}, t_{\mathrm{D}}^{\mathrm{sup}}, T, D_{\mathrm{D}}) = \left[\Pr(t_{\mathrm{D}}^{\mathrm{inf}} < t_{\mathrm{D}} < t_{\mathrm{D}}^{\mathrm{sup}})\right]^{D_{\mathrm{D}}} \times [\Pr(t_{\mathrm{D}} > T)]^{1-D_{\mathrm{D}}} \quad \text{(A5)}$$

where the two probabilities are given in (A3) and (A4). The log-likelihood function of out-of-hospital spells can thus be written as the following summation:

$$\log L(\zeta, \eta, \lambda_{\mathrm{H}}^{1}, \ldots, \lambda_{\mathrm{H}}^{k'}, \mu_{\mathrm{D}}^{1}, \ldots, \mu_{\mathrm{D}}^{k'}) = \sum_{n=1}^{N'} \log \ell_{\mathrm{H}}(t_{\mathrm{D}}^{\mathrm{inf}^n}, t_{\mathrm{D}}^{\mathrm{sup}^n}, T^n, D_{\mathrm{D}}^n) \quad \text{(A6)}$$

where N' is the sample size for out-of-hospital spells, and superscript n denotes the observation number.

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