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Social Interactions and Mathematics Learning

M.L. Schubauer-Leoni,
University of Geneva, Switzerland
A.-N. Perret-Clermont,
University of Neuchâtel, Switzerland

In this contribution we describe the evolution of a field of research in which we have been involved for some 15 years, investigating cognitive and social aspects of mathematics learning. Our aim is not to present a succession of research results but rather the progressive modification of the issue under study. The visible outcome was sometimes answers to the questions raised, but often also of the *modification of the research questions*, and as a consequence the search for changes in *methods appropriate* for their investigation. We will also refer to other work, which is not directly centred on the learning and teaching of mathematics, to which we have contributed and which has played a part in the progress of our studies of children and school mathematics.

A FIRST STUDY: PIAGETIAN VS. SCHOOL TASKS

In the late 1970s we were immersed in the study of the role of social factors in development, bringing to the fore the role of interpersonal interactions as providing the driving force behind cognitive development (Perret-Clermont, 1976, 1980; Mugny, Perret-Clermont, & Doise 1981; Perret-Clermont, 1982; Perret-Clermont & Schubauer-Leoni, 1981). At that time our institutional involvement in a department of educational sciences and our pedagogical concerns induced us to deal more precisely with learning tasks that were meaningful in the school context and hence different from those usually implemented in the studies of the mechanisms of thought in general. We were profoundly interested in the considerations given by psychopedagogists to the connection (and disconnection) between developmental psychology and the teaching of mathematics (Brun & Conne, 1979), and we tried to extend the research paradigm that we were using in the work on the social construction of intelligence to include the testing of the possible benefits of peer interaction on children's learning of elementary school mathematics.

We started this line of research with primary school children solving and representing additive problems of the type a + b – c = x. The task that we used in these studies was as follows. On the table there is a handful of sweets and a non-transparent empty bag. The experimenter (E) invites the child (C) to pay attention to what will happen. E picks up two sweets, shows them to C and then puts them into the bag. Then E repeats the action this time picking up four sweets, shows them to C and puts them into the bag as well. Finally, E takes one sweet back from the bag and asks C : "Is it possible to know how many sweets are now in the bag?" Once the child has answered "five" and made explicit the complete additive/subtractive composition of elements involved, E gives him/her paper and pencil with the following instruction: "You have to explain to a child who has not seen what we have done with the sweets everything that happened with these sweets and how many are left at the end. Try to make him/her understand what we have done with the sweets in order to end up with five in the bag".

The first study (Schubauer-Leoni & Perret-Clermont, 1980), set out to investigate:

1. Whether there is a link between three types of observations: the operative competences of a 7–8-year-old child in a Piagetian task involving the additive properties of number; the ability to respond correctly in school tests of the type "a + ... = b" or "a + b = ..."; and the child's capacity to draw on the "arithmetic code" learned at school when asked to report additive actions on paper.
2. The effects of the interpersonal context in which the written formulation of an addition problem is carried out.

With respect to the first question, we observed that the expected connections were much more complex than we had anticipated, thereby replicating earlier observations by researchers such as Brun (1979) and Vergnaud (1981). Being at the most advanced level in the Piagetian operational tasks appeared to be neither a necessary nor a sufficient condition to solve the gap-filling equations proposed. In none of our problems was operatory competence a sufficient condition for the pupils' use of arithmetic writing at the time of encoding an addition problem. This was for us, at the time, an initial threat to our then rather rigid structuralist understanding of the functioning of the mind, and a disappointment because the implementation of the new mathematics curriculum in Swiss schools relied on the assumption that giving the child adequate opportunities to develop the basic operatory notions would allow for a general grasping of the meaning of numbers and of numerical systems. We were then unaware of Reed and Lave's (1981) pioneering work distinguishing between two classes of arithmetical strategies: those that deal with quantities as such and those that deal with number names and that are reinforced by school socialization. This discussion of the communalities and differences of the arithmetic behaviour in oral or written forms, in everyday life or in school-promoted practice, has been enlarged since then by ethnographic studies (for a review see Nunes, 1992, and Nunes, in press) which produce very interesting evidence to distinguish the uses of (different) symbolic systems from the understanding of cognitive invariants.

Carraher, Carraher, and Schlieman (1985, 1987) as well as Nunes (1992) were concerned with refuting hypotheses about differences in psychological processes between users of different systems of signs, or with observing effects of symbolic mediations in the organisation of human activity as the systems are used. In contrast, our own concern, arising from our earlier studies of the role of socio-cognitive conflicts in cognitive development, was with those characteristics of the social contexts that elicit the use of one or another system of signs to represent quantities and operations and which bring about more or less explicit formulations of these.

In order to test the effects of interpersonal contexts on these encodings by the pupils, we designed an experiment with four experimental conditions: (a) two children interact to formulate their message about the problem for a decoding peer who will then decode their message; (b) two children interact to formulate their message about the problem for a decoding peer without the benefit of feedback from the latter; (c) a child formulates his or her own message for a decoding peer who will then decode the message; (d) a child formulates the message for a decoding peer without the benefit of feedback. Hence two of these conditions involved collective resolutions and two did not. The children's productions varied a lot: some wrote the "orthodox" formal arithmetic formulas but others responded to the experimental demand with drawings or texts, in ways very similar to those reported later by Hughes (1986).

The scenario of the task, the type of problem and the instructions are similar to those described earlier on with only a difference of *content*. This time the experimenter is not presenting sweets to put in a bag but flowers to make a bunch : "you pick two flowers, and then you pick six more; and then you meet a schoolmate and you give him/her three of these. With how many are you left for your bunch?". In this case the *additive* composition is: 2 + 6 – 3 = 5.

During the pretest, the pupils were tested individually outside the classroom and had to produce a first encoding relative to an addition problem of the same type as that later given in the various experimental conditions. The pairs of children who were to work together in the coding task were organized on the basis of the individual productions in the pretest, so as to give rise to cognitive confrontations thought to be beneficial, and also in order to vary the pairs of children as a function of whether or not their members had resorted, in the pretest, to canonical mathematical formulations or to "unorthodox" natural language or drawings. The experimental conditions were followed by individual post-tests in order to measure the effects of these different experimental conditions by comparing the characteristics of formulations produced in the post-test with those of the pre-test.

The results supported our hypothesis in that the pupils who solved the task in pairs and had feedback from a decoding peer (condition *a*) made the most progress: in this group there was a tendency to mention all the quantities and the operations carried out in the situation they had been asked to describe explicitly. When the results of the pre- and post-test were compared, we observed that 93% of the subjects in this experimental condition gave more explicit answers in the post-test than in the pretest. In contrast, this increase in explicitness was observed for 76% of the subjects in the other conditions. Only two children who worked alone and had no feedback from a decoding peer (condition *d*) showed regression on this measure.

It was also observed that pupils who solved the problem in pairs and had feedback from the decoding peer were more likely to use the formal mathematical code (numbers and signs) rather than natural language or drawing in the post-tests: in this experimental condition subjects more often used the formal code (5 times out of 14) than those who worked in pairs but did not receive feedback (2/12), those who worked alone and had feedback (1/6), and those who worked alone and had no feedback from a decoding peer (1/7).

These results are consistent with those obtained previously when the children worked on operative tasks (in the Piagetian sense) either in social settings or alone. However, there are some aspects that are more difficult to explain within the classical experimental research paradigm of the social construction of intelligence. In particular, in this study we observed that the pupils tested individually (experimental condition *d*), who were questioned on three separate occasions (pretest, test, post-test) using the same adult–child face-to-face method, modified their written production more appreciably than expected: either by showing signs of progress, or by producing "regressive" type encodings during the post-test. These results aroused our curiosity. We went back to the audiotapes and listened to all the testing sessions in search not only (as we had done previously) of cues about how the children express their logical thinking, but also in search of the reasons for the unexpected changes in the performance of children who worked alone and without feedback. In our previous research those subjects who worked with a peer showed systematically more cognitive progress than those who worked alone: the latter made very little or no progress. In contrast, in this study we observed two different types of results: working with a peer was better only in condition *a*, where problem solving was followed by interaction with a decoder, and, unexpectedly, subjects performing the task alone also showed progress in the post-tests. Why should the results of this last study differ from the previous ones?

An analysis of the children's reactions led to the following hypothesis about the differences between the task in this last study, which we will refer to as a "mathematical task", and the previous tasks of conservation of number, of quantities, and of spatial relations, the "Piagetian tasks". Piagetian tasks have often been considered as "purely cognitive" tasks, but we are now convinced that they are no less "socio-cognitive" than other mathematical tasks. Certainly they have some specific characteristics that relate to the central interest of Piagetian theory: they involve judgements and reflective skills.

The mathematical task of the present study differs from the Piagetian tasks because it calls on *symbolic* skills, which have not only a reflective but also a primary *communicative* function. This communication does *not take place in a social and historical vacuum* but as a continuation of previous discussions about operations (addition and substraction) that have been approached previously elsewhere (i.e. in the classroom and perhaps at home). It is important to recognise that *mathematical notions* are not the product of the individual mind of a socially isolated child, whose cognitive activity functions *ex nihilo*, but are socio-cultural constructs that pre-exist the child's activity and are made accessible to the child through more or less formal social transmission. Mathematical notions are socially situated within networks of specific social practices, of which classroom practices are an example. One must take this into account when considering whether it is possible for the developed adult mind or for the growing mind of a child to abstract general concepts independent of the social networks in which they are constructed or learned. Symbols and symbolic mediations (writing, formal mathematical code, and even drawing) have their traditions and these are conveyed to the children outside the testing or experimental situation.

The *cognitive* and *social* experience of being asked to make a judgement on the conservation of quantities is different from that of being asked to solve a mathematical exercise. The difference led us to revisit the role of the decoder. Whereas the presence of the peer for the collective production of a notation helped the subjects keep different elements in mind at the same time and confront their evaluation, when the task involved the use of symbolic knowledge, the presence of the decoder appeared to be essential to give meaning to the coding task itself and did not serve only as a feedback. The presence of the decoder stressed the *communicative* function of the notations. Other cues heard in the tape recording of the sessions led us to reconsider the role played by the characteristics of a *mathematical* task in determining the types of social interactions that can take place around it. Through this analysis, we realized that we had underestimated the role of two parameters in the paradigm of this first study: these were *the major impact of the mathematical object* on which the interaction between peers was focused, and *the role of the adult experimenter in the emergence of responses* produced in interaction. The following studies attempted to explore these parameters.

THE MEANING ATTRIBUTED TO THE OBJECT IN TERMS OF THE QUESTIONING CONTEXT

New studies[1] have then both confirmed the superiority of experimental conditions that involve peer collaboration in the production of written formulations of arithmetic operations (Brun & Schubauer-Leoni, 1981) and have also, owing to the establishing of formulation typologies (Brun & Schubauer-Leoni, 1981; Schubauer-Leoni & Grossen, 1984), enabled the expression of subtle differences when analyzing the written formulations produced by pupils at different stages of the experiment. These studies have provided the opportunity to analyse the impact of communication situations between coders and decoders according to whether the decoder "says" or "does" what he understands from a message (Schubauer-Leoni & Grossen, 1984) or by maintaining the exchanges between the coder and decoder until a consensual response is obtained from the two participants (Saada & Brun, 1984). This last study, moreover, provided an opportunity to underline the importance of the prompts provided by the adult in enhancing the coder–decoder interaction. Subsequent analyses (Schubauer-Leoni & Perret-Clermont, 1985) have taken into account the particular nature of the scholastic mathematical object as an object having a social and cultural history but also a scholastic one, which plays a part in the everyday scholastic practices of the pupil. For this reason, during a lesson or a test the child always tackles and relates to the situation in terms of the customs (simultaneously cognitive, social, and didactic) acquired elsewhere at school and possibly at home in relation to this object.

The nature of the object "written formulation of addition problems" is, at this point of our research programme, changed in two ways: first, the mathematical object is not cognitively superimposable on other Piagetian operatory knowledge, and second, its social existence within the realm of scholastic reality means that the child brings with him/her, to the place where he/she is questioned by the psychologist, inherent customs in relation to the scholastic object encountered elsewhere in class.

Consequently we must ask: What is the role played by the adult who poses questions about the pupil's productions? In other words, our initial research questions have now a wider object: *in what sorts of contexts (cognitive, material, relational) is mathematical knowledge displayed or acquired?* What are the *representations which enable the pupil, faced with question acts, to formulate response acts that* are both cognitively satisfactory and relevant—that is, acceptable—in the given socio-cognitive context (Perret-Clermont et al., 1984; Schubauer-Leoni & Perret-Clermont, 1985). Pupils interpret social situations and tasks simultaneously. What are the representations that they construct of their endeavour?

MOVING FROM A BIPOLAR TO A TRIPOLAR MODEL OF KNOWLEDGE CONSTRUCTION

Our first experiments, as mentioned above, followed the Piagetian tradition in that they considered the questioning carried out by the experimenter as neutral and limited to recording visible indications of the child's thinking that were unfolded under his/her eyes. Hence, the descriptions of the patterns of social interactions that took place in the experimental conditions did not take the experimenter into account; and we wrongly considered the pre- and post-tests as "individual" sessions with no awareness that the experimenter–child relation is the locus of a series of social interactions, just as much as the peer–peer relation.

There was another oversimplification in our first approach. The Piagetian bipolar model of knowledge building (subject – object) had been substituted by a primary tripolar model (subject 1 – subject 2 – object). Because of the new importance that we had given to the impact of the interaction between peers, this model risked underestimating the role of the object by stressing the child 1–child 2 bipolarity. But when we started to take the type of task, and in particular the status of the mathematical object, more explicitly into account as the centre of joint activity it became clear that the object played a central role, not only as a "task" but also as a mediation that specified a social system in which the adult is in a high position when asking the questions. The model, which was formerly bipolar (*subject–object*; or *peer–peer*), now becomes clearly tripolar—*questioner–questionee/s–object*—and integrates the experiment as an integral part of the observation. It is interesting to note that this epistemological shift from the conception of the observer as "neutral and external" to a consideration of the observer as constituent of the object under observation took place long ago in other domains such as nuclear physics (see, among others, for the discussion of this point: Prigogine & Stengers, 1979; d'Espagnat, 1979; Perret, 1981).

The different places in which the experiments were carried out (in and out of the classrooms, in school-like formal settings or not) made us aware of the function attributed to the questioner in terms of his/her institutionalised role. Hence our further experiments started to differentiate between the teacher

[1]Carried out in collaboration with J. Brun and psychopedagogists concerned with mathematics (Swiss National Research Foundation, contract No. 1.706.078).

questioner and the psychologist questioner (that is, the experimenter outside the classroom) in order to understand the status of the replies of those pupils questioned by someone having one or the other social role.

Finally, this first stage of work on mathematical contents made us aware of the temporal dimension at work in the pupils' generation of a response. We categorized the experimental productions according to a pretest, intervention, post-test scheme, focusing our attention primarily on changes from the pre- to the post-test. But the consideration of this three-step intervention as a whole, whereby an experimenter acts on three occasions with repeated similar or dissimilar demands addressed to the subject, led us to introduce the concept of the *experimental history of the subject.* This concept allows for a more dynamical approach of the testee's productions because they are considered in a temporal perspective, and opens the way to new questions on the construction and transmission of knowledge. It also raises the question of the link between the social conditions of the appropriation of an object of knowledge and those of its display or use: a point made also, from another perspective, by Forman (1989), who observes that the traditional pre- and post-test decontextualized approach is inadequate to evaluate the success of peer collaboration.

THE EMERGENCE OF THE NOTION OF A "DIDACTIC CONTRACT"

The tripolar dynamics of the approach described above has merged with one developed elsewhere by other researchers in the domain of the teaching of mathematics (Brousseau, 1988; Chevallard, 1988). Brousseau and Chevallard, have already stressed the triangular nature of the teaching relationship, which is bound by an implicit "didactic contract"—that is, a system of reciprocal and specific expectations with regard to the knowledge taught. From our standpoint as social psychologists, we have thus pursued our work in order to show experimentally the workings of a contractual relationship of this sort between, on the one hand, *the teacher, the pupil, and an object of mathematical knowledge* and, on the other hand, between *the experimenter, a subject, and an object of mathematical knowledge.* The didactic contract functions, through representational systems which it activates, as pre-eminently "practical knowledge" (as Bourdieu calls it), knowledge that is not thought of as such and which serves practical ends.

This new phase in our work was carried out in different directions and through diverse approaches. In particular we wanted to study the dynamics of the interaction between peers and with the adult in order to obtain a more detailed description (than previously allowed for, Semin, 1989) of the *simultaneously interactive and representational* nature of the processes involved.

Instead of defining tasks only by their content (for example, the written formulation of addition problems), we considered more broadly the general situation in which subjects were set in the experiment, and we introduced the concept of "experimental staging" (Perret-Clermont et al., 1984). A research design was organized in such a way as to alternate the questioning contexts: first in class, then in face-to-face situations, then once again in class for the third phase. This was done in order to make possible the identification of the meanings that pupils confer on the situation when they try to make sense simultaneously of the task and of what they consider to be the adult's expectations. Obviously pupils are affected in their views by their implicit knowledge learned in the everyday functioning of the didactic contract in the classroom and by the habits that they have acquired in such context (Schubauer-Leoni, 1988). The analysis of protocols of the second phase, where the child is in face-to-face interaction with the adult, demonstrated how social and cognitive elements are interwoven in the subject's understanding of the situation (Schubauer-Leoni, 1986b).

THE STUDY OF THE DIDACTIC CONTRACT VIA THE TEACHER

The studies described up to this point have as a common feature the consideration of the child as the main focus of study. They centred on the child–task (object of knowledge) interactions and on the child–partner (adult or peer) interaction. The widening of the underlying model from bipolar to tripolar made us aware that we had so far concentrated our attention only on certain elements of these triangular relationships, leaving the others in the background: all the previous analyses gravitated around the person in the "lower social position"—as if the adult experimenter or teacher in the "higher position" was not an essential element in the dynamics of these sociocognitive encounters. This led us to set up studies of the didactic contract from the point of view of the teacher's expectations with regard to the pupil's behaviour in certain tasks.

Our next step was a study carried out with a teacher of a class of primary school children aged 8–9 years. It involved several interlinked experimental phases:

- Phase 1: the teacher conceives of and writes up a mathematical task for the pupils (typical task within the current didactic contract) and is invited by the researcher to explain the rationale of the choices that were made.
- Phase 2: the teacher makes predictions as to the behaviour and performance of each pupil with regard to the task.
- Phase 3: the teacher administers the task to the pupils as usual.
- Phase 4: the teacher corrects the answers as usual.
- Phase 5: the researcher submits to the teacher a task that is unusual for, and at the

outer boundaries of, the didactic contract in force. The teacher is asked to determine the interest and feasibility of this task for the class.

Phase 6: the teacher formulates expectations of each pupil's behaviour and performance with regard to this unusual task.

Phase 7: the teacher administers the task to the pupils.

Phase 8: the teacher corrects the answers to this second task.

The research corpus corresponding to these eight experimental phases gave us access to two types of contractual behaviour: those produced under the terms of the usual didactic contract (phases 1 to 4) and those produced in a situation where the contract is broken (phases 5 to 8). The study (described in Schubauer-Leoni, 1986a, 1989, 1991) illustrated the indivisibility of the three terms of the didactic relationship. Indeed we repeatedly observed that the teacher considers the *task*—and hence the mathematical knowledge that it conveys—*in terms of how pupils react to it*; the teacher describes his/her *present pupils* in regard to *how they are expected to tackle the task* according to his/her view of possible solutions. In a later study concerning the teaching of algebra in secondary schools in Switzerland and Italy (Schubauer-Leoni & Ntamakiliro, 1993), the same type of phenomenon was again observed.

Above all we observed that teachers differentiate between cognitive levels of responses to the task according to their representation of each pupil; the didactic contract appears to be the result of differential expectations by the teacher who, without realizing it, creates expectations in mathematics that are part of the sociological characteristics of the pupils in the class. The teachers tended to overestimate the performances of pupils from privileged social backgrounds and underestimate those of the pupils from underprivileged ones.

Through a classic effect of social categorization, the "weaker" pupils (in terms of the actual results on the task and the mark obtained) from the privileged social group were perceived as being more in line with their peers from the same social group; by the same token, the "stronger" pupils from the underprivileged social group tended to be "lowered" in favour of a resemblance between members of their sociological group. It should be noted that this effect becomes even more marked the further the task departs from the usual didactic contract (phases 5 to 8). This differentiation was definitely not intended by the teachers who considered themselves committed to the cause of equal opportunity and democratisation of access to school and knowledge.

The results of this study, and in particular, the differential function of the didactic contract, were confirmed when other corpora, including further data about the same teacher but also other teachers, were taken into account. For example, we also considered the comments written in the school reports addressed to the parents.

A further detailed analysis of the modes of administering the tasks to the pupils was carried out in this as well as in other studies (Schubauer-Leoni, 1989; 1993). It illustrates the complexity of the phase during which the task is transmitted by the teacher to a pupil or a group of pupils. From the very beginning, we find traces of a differential didactic contract in the sense described above. We also observed that pupils look for clues to the meaning of the situation in which they are questioned and try to get confirmation (real or decoded as such) from the teacher or the experimenter in order to define their position as testees. The observation of these adult–child transactions reveals misunderstandings that seem to be due to unshared implicit references about different orders of reality or about supposedly common experiences during the didactic or experimental history. This is indeed the major problem in achieving intersubjectivity between co-actors (Rommetveit, 1974; Grossen, 1988).

THE STUDY OF THE DIDACTIC CONTRACT VIA THE PUPIL

We continued our research into the didactic contract by providing the pupils with the opportunity to play the role of the teacher. The purpose of such an unusual approach is to identify the principal ingredients of the teacher's gestures as decoded by the pupils. Of course this detour requires at first an analysis of the difficulties that pupils will inevitably encounter in trying to occupy a social and cognitive position that is not "normally" their own within the didactic contract.

For this investigation (Schubauer-Leoni, 1986a) we organized role-playing situations during which the pupils (7–8 years old from 2nd primary and 11–12 years old from 6th primary) were first requested to think up mathematical scholastic tasks for another pupil in their class (2nd–2nd) or for younger pupils (6th–2nd). They were then requested to play the role of the teacher giving the task to a pupil.

This study demonstrated three processes: (1) The types of tasks conceived by the "little teachers" are directly based on usual scholastic tasks with a clear emphasis on *calculation activities*: lists of operations to be carried out appear as prototypical representations of what is for them a mathematical task in school. (2) The tasks, provided by the "little teachers" in written form and then directed by them to their "pupils", revealed the difficulty for these children of taking the adult's role and formulating questions. We observed that the young "teachers" systematically constructed their questions on the basis of the replies, hence spontaneously taking their "natural" place of repliers. It was also observed that during the interaction with the "pupil", the "little teacher" is often keen to answer in the place of his/her interlocutor. This strategy is sometimes replaced by the adoption of an attitude of reprobation towards the supposed ignorance of the pupil being questioned! (3) When requested to choose a class mate to give the task to, the "little teacher" tended to choose the pupil whom he/she considered to be the weakest, apparently in order to guarantee a certain distance between the "teacher" and the "pupil", as if there was a prerequisite to be able to consider himself/herself authorised to take on a role that would otherwise have no legitimacy.

All these elements recall the existence of power games in most interpersonal relationships (Goodnow, 1989; Forman, 1992). But above all they underline the function of the knowledge taught as mediating the social relationship in the teaching situation, knowledge that presupposes (or bears on) a system of non-interchangeable social positions.

Thus what seems to matter, and to determine the nature of the games, is the fact that children, when confronted with the task, have to take on a role of expert and a role of novice respectively.

From a methodological point of view, these studies mark the transition from an approach centred on the *effects* of certain interactions to one focusing on the *processes* put into action by those interacting in order to make sense of the task and of the situation into which they are thrown. These two stages of work in the field of mathematics learning, have their parallels in other areas (Schubauer-Leoni et al., 1989; Perret-Clermont et al., 1991; Perret-Clermont, 1992 a, b; Schubauer-Leoni & Grossen, 1993; Grossen, Liengme Bessire, & Perret-Clermont, 1997).

The impact of various social demands on children's drawings (Goodnow, 1989) or on the pupil's cognitive behaviours (Roazzi & Bryant, 1992; Grossen et al., 1996) has been established. In mathematics, in particular, Säljö and Wyndhamn, 1987, and Brossard (1994) deliberately manipulated the interpretational premises of the tasks submitted to the pupils, in order to show the effects of pieces of information about a task on the way in which the pupils handled arithmetical problems.

THE DIDACTIC CONTRACT AND THE INSTITUTIONAL META-CONTRACT

Human communication systems have often been described as based on communication contracts (Rommetveit, 1974; Ghiglione, 1986, 1987) i.e. operating systems of norms and values that are organized in the form of implicit and explicit roles that guide the acts of those interacting (Goodnow, 1989). The studies that we have described therefore show not only traces of the existence of such contracts in exchanges about mathematics, but also the connection between this cognitive activity and the social situation. They also demonstrate that the cognitive activity is never exclusively related to the knowledge contents or to the thought operations that are needed to deal with these contents.

At the same time this cognitive activity relates to an interpretation of the situation as it evolves during the interaction (Grossen, 1988; Perret-Clermont & Schubauer-Leoni, 1989; Perret-Clermont et al., 1990, 1992; Schubauer-Leoni et al., 1992). More specifically, each contract (didactic or experimental) not only governs the meanings of reciprocal behaviour but, in turn, is also under the control of *a meta-contract of an institutional nature*—a contract superimposed on the contract which governs the participants' framing of meaning (Rommetveit, 1974; Hundeide, 1981, 1985; Chevallard, 1989, 1992; Schubauer-Leoni & Grossen, 1993). Relating to an object of knowledge implies relating to those who (re)present it and to the institutions that have set them in this role.

We examined the impact of the rapport that pupils have with the institutional settings (Schubauer-Leoni, 1990; Schubauer-Leoni et al., 1992; Iannaccone & Perret-Clermont, 1993). Two comparable groups of pupils (2nd primary 8–9 years old) were required to produce a written formulation of the same calculation problem either *in class* (collectively under the responsibility of the experimenter, but with an *individual* written response from each pupil in keeping with the way that written work is usually carried out in the class) or *outside the class in a face-to-face* situation with the experimenter. We observed that the pupils' written responses varied as a function of these settings: in the classroom in the presence of their own teacher, the pupils had recourse to conventional arithmetic writing more often, whereas outside the class, face-to-face with an unknown adult, natural language and drawing were more often used to deal with these calculation problems.

These results show an effect on the child not just of the problem but also of the institution in which the problem is posed. Säljö (1991) also produced evidence in this direction in his study of children's use of a postage table and of proportionalities in and out of a mathematics class.

THE ORGANIZED BREAKING OF THE DIDACTIC CONTRACT

Work on didactic (or experimental) interactions in mathematics does not seem adequate to us if it only deals with situations in which the "contract" is functioning "happily". To study only what is taken for granted by the participants in the interaction risks leaving hidden the fundamental basis on which the contract functions. Starting from other work carried out in the field of mathematics didactics and of psychology (IREM de Grenoble, 1980; De Corte & Verschaffel, 1983; Are, 1988; Brissiaud, 1988; Alves Martins & Carvalho Neto, 1990; Carvalho Neto, 1990; Giosuè, 1992) we constructed a study based on the "absurd problem" paradigm (with questions such as: "There are 15 girls and 12 boys in a class. How old is the teacher?"). Our aim was to understand the contractual conditions and the interactional dynamics that make it possible for a pupil to move from trying to solve a problem to rejecting what has been said (Schubauer-Leoni & Perret-Clermont, 1987; Perret-Clermont et al., 1992; Schubauer-Leoni & Ntamakiliro, 1994). The body of work carried out in the field of "absurd problems" agrees in showing the degree of subjugation to the usual didactic contract in that there is a massive production, by the pupils, of replies that are calculations based on the entirely irrelevant information imparted. Only the teacher's explanation of what is "really" expected (i.e. an explicit enlargement of the didactic contract) with regard to problems of this type makes it possible for children to avoid the "taken for granted" arithmetical composition (Giosuè, 1992).

Conversational analysis of the interactions between the experimenter and pupils illustrates the dynamics of the negotiations which allow the experimenter to "extract" responses from a pupil who pays lip service to the adult's demand but does not really change his or her mind about the problem (Perret-Clermont et al., 1992).

This research (Schubauer-Leoni & Ntamakiliro, 1994; Schubauer-Leoni & Grossen, 1993) has provided the opportunity to make progress in understanding the intersubjective, situational, positional, and institutional dimensions of the pupil's reply. This is done through the comparison of two types of "impossible" problems: a manifestly absurd problem (that of the age of the teacher given the number of pupils in the class) and a problem in which the degree of absurdity is more camouflaged ("A farmer has 87 rows of 150 lettuces. What is the surface area of the piece of land?"). The complexity of spatial measurements (Vinh Bang & Lunzer, 1965; Rogalski, 1992; Vergnaud, 1983) and children's relatively late mastery of the notion, is expected to play a leading role in their consideration of an eventual rejection of the problem: given the restrictions imposed by the institutional contract, why not take "a lettuce" as a unit of measurement? Our analyses of the interactive dynamics at the time that the 5th primary pupils (11–12 years old) were handling the two types of absurd problems suggest a conflict of rationalities. The pupils seem torn between their personal rapport (or "private component"; Chevallard, 1992) with the problem which makes them believe that the problem is "absurd", and their institutional rapport ("public component") which makes them write that the teacher is 27 years old and the surface area of the piece of land measures 13050 lettuces!

A PSYCHOSOCIAL THEORY OF SOCIAL INTERACTIONS AND A DIDACTIC THEORY OF DIDACTIC INTERACTIONS

It is useful to differentiate between the objects of study according to the ecological position that they occupy in different scientific debates. In order to make progress in the study of the teaching and learning of mathematics, it is necessary to carve out the object of study: what, of all the complex phenomena that can be observed, is to be studied? Different lines of research make their choice differently depending on their precise ends. Thus it appears to us wise to avoid superimposing fields of research too hastily and to favour a differentiation which, even if it might appear a little formal or arbitrary in the beginning (and it is), should in the long run help us to express the results of these different lines of research in relation to each other. In the area that interests us we observe, in particular, the emergence of two relatively new scientific fields:

- On the one hand, *a "contextual" science of the knowing subject* which is born out of *theories of the subject* that no longer place the subjects in isolation but situate them in relationship with their social and cultural universe. This science endeavours to render an account of the regularities and specificity of the subjects' (in our case, the pupils') socio-cognitive behaviours within structured "community of practice" areas (such as the scholastic institution for example).
- On the other hand, a *theory of situations* aiming to identify *situations which would bring about the emergence of expected knowledge* in the subject. The aim is to understand which situations establish which relations with the object of knowledge, and invite which approaches to the given factors, give rise to which cognitive elaborations, mobilise which know-how, which tools, or which memory. It is a question of contributing to a science of situations which are themselves prone to bringing about the intervention of a rapport with knowledge compatible with the rapport expected in the different institutions (first relative to taught mathematics; but also relative to the mathematicians' community as a whole). Here we find ourselves at the heart of the concerns of didactic theory. Although we are interested in the subject (and more specifically in the *pupil*-subject), the way in which the latter functions becomes an indicator of the possibilities offered by the situation.

These two fields of research differ above all with regard to the direction of their approach: one targets the subject via the situation; the other targets the situation via the subject. These differing focuses of attention are not at all mutually exclusive but have brought with them different methods of approach, observation, and validation on the part of scientists from the two domains. One must take care not to generalize or transpose the "results" from one domain to another without all the necessary precautions. It is, however, particularly interesting to articulate these fields one with the other: the work that we have described was born first from an attempt to understand the subject's cognitive functioning within his or her social interactions, and then focused on the particular, where his or her field of interaction is of a didactic nature. The object of study thus changed to: what understandings do the subjects (be they teacher or pupil) elaborate when they interact "didactically" in a given context?

AN EXAMPLE OF RESEARCH INTO DIDACTIC INTERACTIONS

The methodological transition (described earlier) from an approach centred on the *effects* of certain social interactions to an approach pertaining to *processes* of attribution of meaning to the interactions (and in particular to the participants' question/reply conjunctures) came about in parallel with work carried out elsewhere by mathematics didactitians who wanted to describe precise didactic phenomena (Brousseau, 1990; Chevallard, 1992; Vergnaud, 1983). In particular we were very interested in the emergence, in the didactic field, of a theory of "didactic engineering" of teaching situations (Artigue, 1988). This permitted us to enlarge the study legitimately from "natural" didactic interactions (mainly centred on the understanding of an habitual didactic contract) to "original" situations contrived, by didactic engineering, to provide a more general understanding of "how one induces mathematical cognitive functioning with its points of reference within the general body of mathematics situations, through the setting up of a didactic situation" (Brun & Conne, 1990, p.265). For these authors ". . . it is thus the parameters of this situation, and not individual characteristics, which are our variables" (p.265). But the theoretical goal can be extended to include the study of the interactions between the parameters of the situation and the socio-cognitive functioning elicited from the subjects or brought in with them.

A didactic engineering study in which four pupils interacted in getting to grips with a task pertaining to the notion of distance between two points was the object of two analyses. The first, conducted by Brun and Conne (1990), produced evidence for the function of representation in fostering the contact between the pupils' knowledge and the situation; the other, conducted by Schubauer-Leoni (1994) dealt more specifically with the level of analysis relative to the inter-individual and situational functioning that was permitted by the diverse parameters of the situation.

One of the characteristics of this experiment was the manipulation, during the first stage of the experiment, of the information placed at the disposal of two groups of two pupils in order to promote (during a subsequent stage of the experiment) cognitive confrontation between them. The aim was to demonstrate the notion of distance as being conceptually linked to a certain unit of measurement. The material used consisted of four foam cubes marking out two distances. The teacher placed the cubes on the floor at a distance of four metres for the first couple and at a distance of three metres for the second couple. The first group had at its disposal an unmarked stick measuring 50cm (the pupils are unaware of both the length of the distance and that of the measuring stick); the second group had at its disposal a stick of 25cm. During the first stage of the experiment each couple of pupils attempted to estimate the distance between the two plots placed on the floor with the aid of an unmarked stick; then, during the second stage, these distances were destroyed and the pupils, in a foursome, discussed and decided together which of the two groups had "the longer distance".

This was a difficult task. Indeed, as the measuring sticks were of different length and as the children were not immediately fully aware of what this meant for establishing the relative lengths, they were soon confronted with the puzzling result that the longest distance (4m) measures only "8 sticks" (8 × 50cm = 4m) while the shortest distance (3m) measures "12 sticks" (12 × 25cm = 3m). We observed the meshing of the different socio-cognitive constructions of the four pupils in the course of the argumentative exchange leading them to construct a certain representation of the problem together. We will not report here the detailed analysis of the different sequences (Schubauer-Leoni, 1994), but we can give a broad outline of the results:

- The agreement between the four children was not founded on constructions of the same order: one pupil was able to unravel the conceptual relationships between the variables of the problem and came to a conclusion at this level; but two other pupils could not come round to an acceptance of the difference between the two distances (a difference not in their favour) except by means of a factual procedure aimed at reconstructing the two presumed distances on the floor; the fourth pupil seemed to take a complaisant attitude, only taking part in the physical manipulation of the objects and without taking part in the debate about the relationship between the measurements obtained and the measuring apparatus in use.
- Given the information originally at the disposal of the two pairs of pupils at the time when they were interacting in a foursome, it had been particularly important to grasp the importance of a common construction of the problem and of the need felt by each individual to look for other information to complete that already at their disposal. In terms of intersubjectivity, a first agreement emerged when the pupils discovered a material world taken to be common to all. In this phase the pupils expressed themselves through the following type of formulations: "you had two bits of foam there too . . . and a stick?".
- Once this first level of common reality was constructed, the pupils were then able to engage in communication based on their previous activities and their respective experiences ("we measured with a stick", "what was your result?"), finally allowing for the size of the measuring stick ("was it a big or a little stick?").
- The idea of comparing results: "us twelve, you eight" was then called into play by taking into account the double relationship between the measurements obtained with the help of the two measuring sticks ("eight times of ours [big] that means two times more than with your little

measuring stick"). However, this logical explanation was not taken up by the other partners who preferred: "to do it again with the stick" in order to "see" and to come to a final agreement in this way about "who had the biggest distance".

- A detailed analysis of the protocol shows that it is not possible to give a simple description of the momentum between the respective progress of each child; each pupil ventured his own representation of problems which he tested out as the interaction advanced. This is the same as saying that, at an interactional level, the subjects did not have to put together a complete piece of reasoning in order to be able to introduce possible alternatives to the elaborations of the others, and subjects in the grips of a certain conception of the problem seemed momentarily insensitive to counter-propositions founded on other representational premises. The pupil usually needs to follow an hypothesis through to the end—and to deal with it in terms of his own representation—before being able to take a concurrent representation into account.

This analysis, which took into account certain conversational characteristics of the exchanges (Trognon & Retornaz, 1989; Trognon, 1992), showed that there were different ways of centring on the problem at various stages of working together in groups of four. We concluded that the abilities of the four pupils were important but so also were the didactic possibilities offered by the task itself and by the variables that it calls into play. This recalls the approach of Hoyles and Noss (1992) with which they analyse examples of what they call "a pedagogy built into the structure of the activities themselves" and observe small-group discussion and work processes elicited by this structure.

EPILOGUE

In this contribution we have tried to trace the evolution that our object of study has undergone as we were trying to come to grips with the sociocognitive processes at work when pupils are invited to learn culturally shaped notions such as those taught in mathematics in schools. Starting from the benefits of earlier work in cognitive and social psychology we underwent a shift in the psychological unit of analysis away from the individual thinker more or less affected by social factors of his or her environment (Doise, Mugny, & Perret-Clermont, 1975; Donaldson, 1978; Gilly, 1989; etc.) to an approach of those kinds of transactions that take place between persons socially situated as "novices" or as "experts" (pupils and teachers for instance) in social encounters in which transmission of knowledge or display of competencies is expected and sometimes even part of an institutionalized "contract".

This led us to search for new concepts and paradigms in order to describe the meshing of the cognitive and semantic psychosocial processes at work at the moment in which subjects deal with tasks but also "longitudinally", since previous experience and social roles and routines prestructure the understanding of a given situation and of the possible goals attainable through it.

This shift allows for new research ventures at the cross-roads of other scientific traditions, such as the anthropological approach to cognition and ethnomethodological studies of learning environments. In reviewing them, Cole (1991, p.413) points to the dramatic change that cognitive psychology undergoes "once one adopts the view that cognition refers not only to universal patterns of information transmission that transpire inside individuals but also to transformations, the forms and functions of which are shared among individuals, social institutions, and historically accumulated artifacts (tools and concepts)."

From a didactic perspective new understandings of these socio-cognitive phenomena can inspire the "engineering" of different social settings, which are sometimes disruptive towards the usual institutionalized practices and social routines and which elicit in the learner different types of commitments in the didactic encounters and hence different modalities of socio-cognitive functioning. We can mention the field work of Pontecorvo (1990) and Pontecorvo et al. (1991) on the role of debates in knowledge construction, of Gudmundsdottir (1991) on meaning construction and narrative structures in teaching, and of Lampert (1992) in the specific area of the community of mathematical practice, as very suggestive contributions in this direction. These offer new perspectives which need more exploration.

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