

Encoding of efficient diffractive microlenses

M. Kuittinen* and H. P. Herzig

Institute of Microtechnology, University of Neuchâtel, Rue A.-L. Breguet 2, CH-2000 Neuchâtel, Switzerland

Different coding schemes for diffractive multilevel microlenses are compared. A simple method to code a lens to get the optimum diffraction efficiency is given. Furthermore, a straightforward way to estimate the achievable efficiency of a lens is presented.

Planar diffractive microlenses have become a valuable alternative to refractive lenses. They can be fabricated by well-known mask-based techniques that result in binary or multilevel elements with staircaselike phase profiles.^{1,2} These techniques are attractive because they permit the transfer of computer data into efficient phase elements in rigid materials such as glass and quartz. Almost any structure shape, including asymmetric aspherics, can be manufactured, and this provides all degrees of freedom for the design. High diffraction efficiency (greater than 90%) is predicted for blazed gratings with eight phase levels and more. This technique can be used for diffractive lenses with high f -numbers. The quantization of the phase profile into eight phase levels is simple and not critical. This method is not suitable for diffractive lenses with low f -number ($F/\#$) because of the limited resolution of the lithographic manufacturing process. The diffraction efficiency decreases rapidly for small grating periods if only four or two phase levels are feasible. As a result, the quantization of the continuous lens function becomes important.

The ideal phase distribution after a focusing diffractive lens can be written as

$$\phi(\rho) = (2\pi/\lambda)(f - \sqrt{f^2 + \rho^2}) - \phi_0, \quad (1)$$

where λ is the wavelength, f is the focal length, ρ is the radius in the lens plane, and ϕ_0 is an arbitrary phase factor. The standard way to code the lens phase function $\phi(\rho)$ for the fabrication is by analytic quantization (AQ), in which the phase function is clipped to values between 0 and 2π . Then the transition points for changes from 0 to 2π are determined and regions between these transitions are coded to produce highest possible efficiency toward the focal point. One determines the number of phase values between two transitions by dividing the distance of the transitions by the minimum feature size (mfs) and rounding this value to the integer closest to zero. For the AQ, equally spaced phase levels between 0 and $(N - 1)/N 2\pi$ have to be used with spacing $2\pi/N$. N is determined by the number of lithographic steps n , i.e., $N = 2^n$.

This AQ coding approach works well when it is possible to use at least four phase levels at the edge of the lens. In this case the local efficiency of the lens is 81% at the edge, and it increases gradually toward the center. However, when the $F/\#$ is small,

the wavelength is short, or both, a large area of the lens has only two phase levels, with a low local efficiency of 40.5%. The maximum diffraction efficiency depends strongly on the relative size of the binary area. The fraction of the binary area (A_b/A) in the AQ of the spherical lens can be solved from

$$(A_b/A) = 1 - 4(F/\#)^2 \tan^2[\arcsin(\lambda/3 \text{ mfs})]. \quad (2)$$

In Eq. (2) the phase function is assumed to be binary when the local grating period is less than 3 mfs. The fraction of the binary area is presented in Fig. 1 as a function of the $F/\#$ for four different wavelengths.

To improve the efficiency of diffractive lenses, which include binary regions, Welch *et al.* presented a coding method based on optimization.³ In their method the lens is divided into ring-shaped cells, and then the phase values of the cells as well as the widths and locations are optimized for the entire lens. The disadvantage of this method is the required optimization, the performance of which is computationally a rather heavy task. In what follows, we describe a more straightforward method to perform the coding and show that this method automatically gives the highest possible efficiency for a diffractive lens. We call this method direct sampling (DS). In DS the phase function of the lens, $\phi(\rho)$ [Eq. (1)], is sampled with a constant sample space (=mfs). These sampled

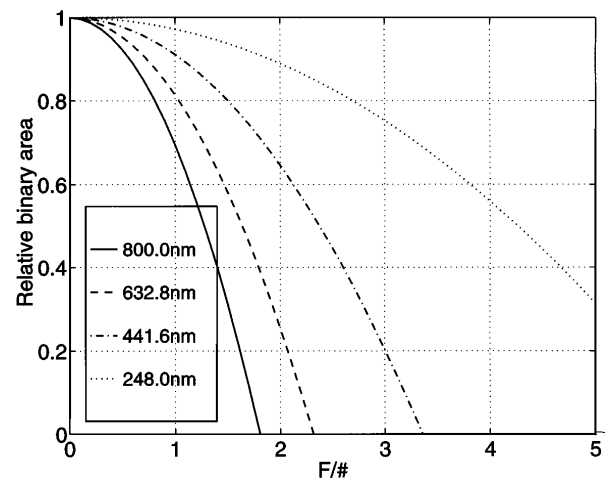


Fig. 1. Relative binary area of multilevel diffractive lenses as a function of $F/\#$ for four different wavelengths. The minimum feature size is assumed to be 1 μm .

phase values are clipped to values between 0 and 2π , and then they are rounded to the closest available phase level. The N phase levels are equally spaced. The advantage of the DS coding is that the method is simple and the phase front is the best possible approximation of the phase front given by Eq. (1). The introduced phase error for each ring-shaped zone is less than half of the quantization step. For example, when the structure becomes binary, the quantization step is π , and the maximum value for the phase front error is then $\pi/2$. These small phase-front errors guarantee a high-quality focal spot, which is difficult to achieve with the radially symmetric iterative discrete on-axis (RSIDO) method, in which only the energy is maximized inside the selected signal window.³ A comparison between the DS and the RSIDO methods is shown in Table 1.

To demonstrate the potential of DS, first we determine the maximum achievable efficiency of the diffractive lens. The first-order diffraction efficiency of a grating with equally spaced phase levels is given by

$$\eta_{1,N} = \left[\frac{\sin(\pi/N)}{\pi/N} \right]^2. \quad (3)$$

One can do the AQ by using in the center of the lens eight phase levels (95% efficiency) as far as possible, then change to four levels (81%), and finally fill the border of the lens, if necessary, with binary gratings (40.5%).³ The efficiency profile of this coding for an $f/1.5$ lens operating at $\lambda = 632.8$ nm is shown in Fig. 2 by a dashed line. Another way to perform the quantization is to use the maximum number of phase levels all the time. This means that instead of dropping from eight to four or from four to two phase levels directly, we use seven, six, five, or three phase levels when possible. Because the selection of phase values for these grating structures has to be done among eight equally spaced phase levels, the efficiency of the structures is reduced slightly compared with that of ideal grating structures. The efficiency profile according to this coding is shown in Fig. 2 by a dashed-dotted line.

The relation between the maximum number of phase levels and the first-order diffraction angle of the local grating is given by

$$N = \frac{\lambda}{\sin \theta \text{ mfs}}. \quad (4)$$

The highest possible efficiency in a specific angle θ is obtained when the number of levels N in Eq. (3) can be varied continuously according Eq. (4). If we now substitute Eq. (4) into Eq. (3) we get an expression for the diffraction efficiency profile as a function of the diffraction angle:

$$\eta(\theta) = \left[\frac{\sin[(\pi \text{ mfs} \sin \theta)/\lambda]}{(\pi \text{ mfs} \sin \theta)/\lambda} \right]^2, \quad (5)$$

where $\theta \in [0, \theta_{\max}]$. The maximum angle θ_{\max} can be solved from $\tan \theta_{\max} = 1/(2F/\#)$. This efficiency profile is shown in Fig. 2 by a solid curve. We stress here that Eq. (5) gives the highest possible efficiency with

an unlimited number of phase levels for a constant sample space (=mfs). In practice the available efficiency is limited by the number of feasible phase levels. In what follows, we estimate the efficiency of lens η_l by integrating $\eta(\theta)$ over the angle. For cylindrical and spherical lenses we can write

$$\eta_l = \int_0^{\theta_{\max}} s(\theta)\eta(\theta)d\theta, \quad (6)$$

where

$$s(\theta) = \begin{cases} 2/d & \text{for a cylindrical lens} \\ 8f \tan \theta/d^2 & \text{for a spherical lens} \end{cases}, \quad (7)$$

where d is the diameter of the lens. For example, a lens with $f/1.5$, $\text{mfs} = 1 \mu\text{m}$, and $\lambda = 632.8$ nm, the estimated efficiencies are 77.1% (cylindrical lens) and 66.4% (spherical lens). The calculated efficiencies are 73.4% (cylindrical lens) and 63.3% (spherical lens). These efficiencies are calculated by application of the Rayleigh–Sommerfeld integral⁴ for wave propagation and with the DS coding of the lens phase function. The differences in the values can be explained because in the DS coding only eight phase levels are used. To compare the

Table 1. Comparison of the Estimated Efficiencies η_l [Eq. (6)], Which Correspond to the DS Coding with 16 Phase Levels and to RSIDO Coding^a

$F/\#$	RSIDO (%) ^b	η_l (%)
1.0	79	87
1.5	89	93
2.0	93	96
5.0	98	100

^aThe lens parameters are $\lambda = 1.3 \mu\text{m}$ and minimum feature size $\text{mfs} = 1 \mu\text{m}$.

^bFrom Ref. 3.

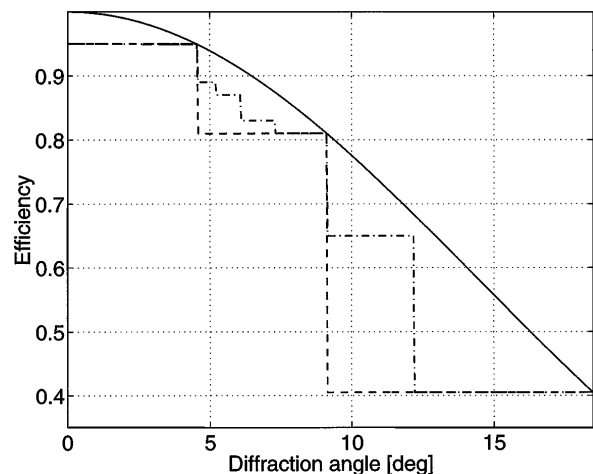


Fig. 2. Optimum diffraction efficiency profile (solid curve) and efficiency profiles when eight, four, and two phase levels are used (dashed line) and when all phase levels are used (dashed-dotted line). The $F/\#$ of this lens is 1.5, and the operating wavelength is 632.8 nm.

Table 2. Diffraction Efficiencies of Cylindrical Lenses Coded with the DS Method and Estimated Efficiencies η_l [Eq. (6)]^a

$F/\#$	DS (%)	η_l (%)
1.0	87.2	87.8
1.5	92.9	93.7
2.0	95.2	96.3
5.0	98.1	99.4

^aThe lens parameters are $\lambda = 1.3 \mu\text{m}$ and minimum feature size $\text{mfs} = 1 \mu\text{m}$.

accuracy of the estimation in the case in which 16 phase levels are used for the DS coding, we calculated the efficiencies of cylindrical lenses with four different $F/\#$'s. The results are presented in Table 2 for a lens of diameter $d = 1 \text{ mm}$ operating at the wavelength $\lambda = 1.3 \mu\text{m}$. For all $F/\#$'s we got good correspondence between the estimated maximum efficiency and the calculated efficiency. The sample space could be chosen smaller than the minimum feature size (mfs). But no increase in efficiency is expected because the values are already close to the maximum. On the other hand, there are some small errors in the optical function that are due to the grid. Those errors could be reduced with a finer grid. However, these wave-front errors are not significant for the examples considered.

Table 1 compares the estimated efficiencies η_l , which correspond to the DS coding with 16 phase levels, with the results achieved with the RSIDO method by Welch *et al.*³ The minimum feature size is $1 \mu\text{m}$, the diameter of the lens is 1 mm , and the operating wavelength is $1.3 \mu\text{m}$. In this case also a Gaussian-beam illumination is assumed with an amplitude value of $1/e$ at the edge of the lens. For low- $F/\#$ lenses the DS coding clearly gives higher efficiencies than the RSIDO method. When the $F/\#$ increases, the efficiency of the two methods becomes almost equal. This means that it is hardly possible to find better solutions for lens coding by optimization.

In conclusion, we have shown that a simple sampling of the phase function of the ideal lens gives a higher efficiency than the traditionally used analytic quantization. Furthermore, we have compared the results of DS coding and RSIDO coding. It turns out that, for low $F/\#$, the DS coding gives higher efficiencies than the RSIDO coding. The main advantage, however, lies in the simplicity of this encoding scheme. Note that scalar theory works well for $F/\#$'s down to $f/2$, which corresponds roughly to a grating period over a wavelength ratio of $\Lambda/\lambda \sim 5$ on the edge of the lens.⁵ Therefore the results for $f/1$ lenses have to be considered with caution. They have been included for a better comparison between the DS method and other encoding methods described in the literature. The direct sampling method is suitable for elements fabricated by standard photolithography with minimum feature size of the order of $1 \mu\text{m}$. For smaller features, a more rigorous investigation is necessary. In addition, we have derived a simple estimation for the efficiency of diffractive lenses based on scalar theory.

M. Kuittinen acknowledges the financial support of the Academy of Finland. Part of this research was supported by the Swiss Priority Program Optique.

*On leave from the Väisälä Laboratory, Department of Physics, P.O. Box 111, FIN-80101, Joensuu, Finland.

References

1. H. Nishihara and T. Suhara, in *Progress in Optics*, E. Wolf, ed. (North-Holland, Amsterdam, 1987), Vol. 24, pp. 3–37.
2. J. Jahns and S. J. Walker, *Appl. Opt.* **29**, 931 (1990).
3. W. H. Welch, J. E. Morris, and M. R. Feldman, *J. Opt. Soc. Am.* **10**, 1729 (1993).
4. J. D. Gaskill, *Linear Systems, Fourier Transforms, and Optics* (Wiley, New York, 1978), pp. 361–390.
5. E. Noponen, J. Turunen, and A. Vasara, *Appl. Opt.* **31**, 5910 (1992).